

# Sensor system for determining the risk of track distortion in the continuous welded rail

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**Abstract.** The article considers the peculiarities of load formation in the continuous welded railroad tracks depending on temperature factors, geometric parameters and local curvature. The approach is proposed to determine the risk of distortion of the railway track depending on a combination of force and geometric factors on the ability of the ballast to keep the rail sleepers from moving laterally in relation to the direction of the track.

## 1 Introduction

The implementation of continuous welded rail with the use of long rail strings is a promising trend in the development of railway tracks in the Russian Federation. However, according to JSC Russian Railways, between 2014 and 2016 there were 122 incidents of instability on the road network, resulting in 31 derailments of rolling stock caused by track distortion. The operation of continuous welded rails in climatic conditions causes a number of operational and technological challenges. Such problems are primarily related to temperature stresses and the prevention of a loss of track stability when it is deformed due to drastic temperature fluctuations. For Krasnoyarsk, the interval of extreme rail temperatures for the railway network ranges from -53 to +58 °C. In spring and summer, longitudinal compression forces arise in the railway strings. A particular concern is the irregularity of the railroad track in track layout, which is a stress concentrator for the lateral load on the ballast that keeps the track from being distorted. A track irregularity is a local point of change in rail curvature where the radius drastically changes from its characteristic value, and sections of track with irregularities are concentrators of lateral forces [1-14]. In the case of the sleeper shifts in the ballast due to lateral forces, it drags neighbouring sleepers, resulting in track distortion. Obviously, the stability of a rail-sleeper track is ensured if one of the sleepers is stable and resists being moved due to forces.

The management of the technical condition of continuous welded rail track associated with failures due to thermal distortion includes the development of new approaches in defining and evaluating the functional safety performance of railways. In addition, it is necessary to create and implement a methodology for analysing actual data on the technical condition of continuous welded rails during periods of extreme temperature rise, which would allow advance measures to ensure the safety of trains. In addition, there is a need for

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early detection of a location with excessive longitudinal, compression force causing a loss of stability.

First of all, the safety of the railway track is guaranteed by strength and stability calculations, which determine the relationship between external influences and the geometric parameters of the structure as well as their mechanical properties. These ratios represent inequalities limiting the area of the safe state of the structure.

Therefore, a digital sensor system is developed to monitor the pre-failure condition of the railway tracks. The main feature of the system is the continuous monitoring of the rail sections by means of a strain-sensitive composite material. Strain-sensitive materials are nowadays widely used in various industries. Materials with integrated functional components allow deformation and temperature monitoring in real time. Nanotubes, piezoelectric materials, optical fibres, conductive polymers and other materials can be used as functional materials. Thus, the new generation of intelligent materials is able to perform certain functions autonomously in response to changes in external parameters. Furthermore, a principal feature of the use of strain-sensitive materials in critical structures is that they can be manufactured to suit specific applications with physical and operational characteristics and different overall dimensions.

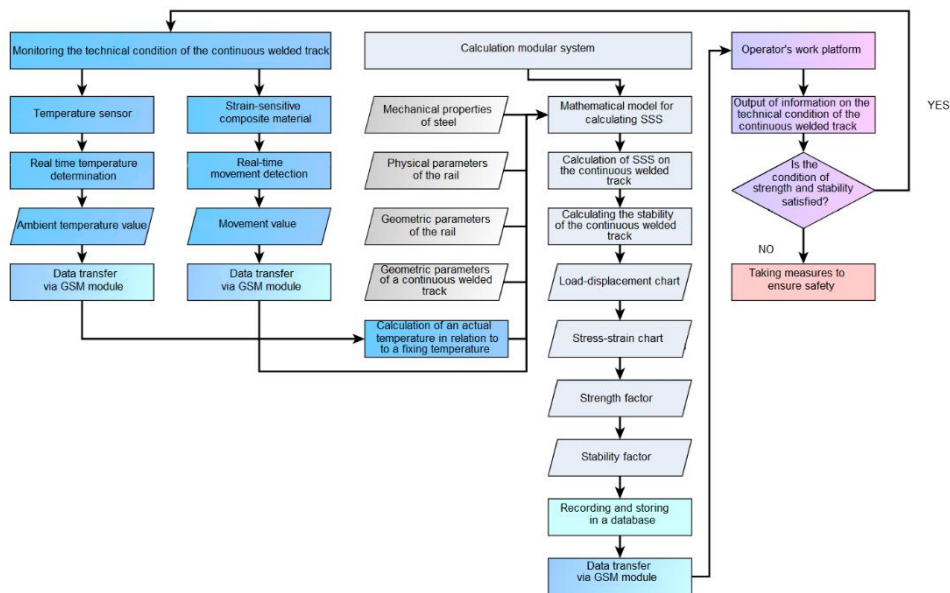
## **2 Development of an approach to monitoring the technical and pre-failure condition of continuous welded rails**

The digital sensor system involves the development of a strain-sensitive composite material that captures specific resistance in proportion to elongation and compression of the material. This material is attached to a continuous welded rail section. When the dimensions change, the strain-sensitive composite material detects movements and transmits information via GSM module, which provides wireless data transmission. In addition, the ambient air temperature is also recorded and the actual temperature is then recalculated in relation to the rail fixation temperature. The information collected is transmitted to the main calculation module system.

In the computational modular system, based on the mathematical model and the developed algorithms typical for the railway track system, calculations are made to determine the loads, deformations, stresses, endurance strength and stability factor. The results of the calculation are recorded and stored in a database for later transfer to the operating platform via GSM module.

A special developed software will be used on the operating platform with a convenient interface to visualise the data obtained, which will enable the technical condition of the continuous welded track to be monitored in real time. In the case of a critical incident or an abnormal situation detected on the railway track, a notification will be sent in order to take the appropriate safety measures.

A block scheme of the developed approach is presented in Figure 1.



**Fig. 1.** Developed approach to monitoring the technical condition of continuous welded rails.

In this regard, it is worth setting an irregularity selection criterion: the length of the irregularity is shorter than the reference value and the beam is larger. Mechanical properties, geometric parameters.

### 3 Mathematical model for calculating the stress-strain state of the rail strings and stability

Therefore, it is necessary to develop an approach for calculating the pre-failure condition of continuous welded rails according to the stability criterion under the influence of thermal compression stresses. Lateral displacement of a rail sleeper is a direct threat to train safety. High longitudinal and lateral forces can cause a rail-sleeper shift in the ballast [2,3]. The resistance of the ballast to shifting of sleepers across the track is one of the crucial factors counteracting the deformation of the rail-sleeper grid in the horizontal plane.

The study of the stress-strain state of continuous welded rails makes it possible to assess their stability and thereby evaluate the technical condition of the track. In this regard, there is a need to develop a mathematical model for calculating the stress-strain state of the sting during temperature changes.

### 4 Calculation of thermal stresses

The stress state of a rail in a continuous welded rail is characterised by the presence of an initial stress caused by temperature stress proportional to the temperature change [1-3] excluding stress compensation due to longitudinal deformation, taking into account the ballast resistance to sleeper movement in the longitudinal direction. The stress state in the rail changes due to axial forces, which can lead to a loss of stability. Therefore, there is a need to measure the actual stress and displacement resistance of the rail track. The

resistance of a sleeper rail to movement along the track axis in the calculations is accounted by the value of resistance per unit length  $r$ , which is defined as the resistance to sleeper shift  $R$  distributed per unit length of sleeper span  $l$  (the distance between the axes of neighbouring sleepers). An approach is to determine the axial force  $F$  through the displacement of point  $x$ .

The following stresses occur in the rail:

- constant uniform compression stresses along the entire length from the exposure to temperature  $\sigma T$ ;

- stresses from the axial force  $P$ ;

- stresses from resistance forces  $q$ ;

Rails are considered to be an ideal elastic element of the permanent way:

$$F = EA \frac{d\lambda}{dx}$$

where  $F$  is the longitudinal force;  $E$  is the track elasticity module;  $A$  is the cross-sectional area;  $\frac{d\lambda}{dx}$  is the relative deformation

From the equilibrium condition of the elementary rail section  $dx$  with the incremental force  $dF$  on its length:

$$R = dF/dx$$

Then a differential equation for longitudinal deformation is obtained:

$$D^2\lambda / dx^2 = r(\lambda) / EA$$

In this case, the length  $x$  of the end section where longitudinal movements will occur is determined by the formula

$$x = F / r$$

where  $r$  is the resistance per unit length to rail-sleeper grid movement, kN/m. In winter (with frozen ballast) with normative stress of terminal and insert bolts, the value of  $r$  is 25000 N/m [2-4].

Magnitude of thermal displacement at the end of the rail:

$$\lambda = F^2 / 2EAR$$

The longitudinal temperature force in the middle of the rail string is found according to the formula:

$$F_T = \alpha \cdot E \cdot A \cdot \Delta T = 2,478 \cdot \Delta T$$

where  $\Delta T$  is the rail temperature in relation to the fixation temperature, °C;  $\alpha$  is the coefficient of linear thermal expansion (0.0000118 1/°C).

The admissible compression and tensile force can be determined from the following equation:

$$[F]_t = 2 \cdot 2,478 \cdot A \cdot [\Delta T]_t$$

$$[F]_c = 2 \cdot 2,478 \cdot A \cdot [\Delta T]_c$$

where  $[\Delta T]_t$  and  $[\Delta T]_c$  are the admissible tensile temperature differences according to the strength condition, which are defined according to the standard.

The admissible temperature differences are determined according to the formulas:

$$\Delta T_t = T_f - T_{min}$$

$$\Delta T_c = T_{max} - T_f$$

where  $T_f$  is the fixation temperature;  $T_{min}$  is the minimum temperature;  $T_{max}$  is the maximum temperature.

The value of the admissible rises and falls in the temperature of the rail sections can be found in the paper [5-12].

The following formula [5,6] is presented for the application of the empirical dependence of admissible longitudinal compression thermal forces as a function of track curvature for different characteristics of the permanent way:

$$[F_c]_R = K_d \cdot [F_c]_\infty - \beta/R$$

where Cd is the coefficient considering the sleeper density: for 1840 pcs/km = 1; for 2000 pcs/km = 1.085; R is the track curvature radius;  $[Fk]_{\infty}$  is the admissible longitudinal compressive force (on tangent railway track);  $\beta$  is the angle factor of the empirical dependence  $[Fk]R$  on R.

The values of  $[Fc]R$  and R for different permanent way structures are given in paper [6-7]. For a continuous welded rail construction with type R65 rails and sleeper density of 2000 pcs/km:

$$[Fc]R = 1,085 \cdot 2,25 - 396 / R = 2,44 - 396 / R \text{ (MN)}.$$

## 5 Strength calculation

When strength calculations are performed, it must be ensured that the actual total temperature and edge stresses in the rails do not exceed the allowable values:

$$\begin{aligned} K_s \sigma_e + \sigma_t &\leq [\sigma] \\ \sigma_{tc} &\leq [\sigma] - K_s \sigma_{ks} \\ \sigma_{tt} &\leq [\sigma] - K_s \sigma_{ks} \end{aligned}$$

$\sigma_t$ -stresses caused by tensile thermal forces;  $\sigma_c$ -stresses caused by compressive thermal forces;  $[\sigma]$  is the admissible stress, 400 MPa for heat-strengthened rails, 350 MPa for unhardened rails.

The largest admissible reduction in rail temperature compared to the fixation temperature, depending on the strength of the rail:

$$\Delta T_t = \frac{([\sigma] - k_n - \sigma_n)}{(\alpha E)} = \frac{[\sigma] - k_n - \sigma_t}{2,478}$$

$K_s$  is the safety factor (1,3 for rails of first wear-life; 1,4 for rails which have passed standard tonnage and proposed without grinding);  $[\sigma]$  is the admissible stress (400 MPa for heat-resistant steels, 350 MPa for unhardened ones);  $\sigma_t$  is the stresses in the rail bottom edges from bending and torsion under load from the wheels of rolling stock, MPa (~200 MPa).

The stress tolerance of the rails must be taken from the endurance limit, not the yield stress.

## 6 Calculating the curvature of the rail-sleeper grid, taking into account the irregularity in the track layout

A track irregularity is a place where the curvature of the rail track changes locally, where the radius changes dramatically from its nominal value, and sections of track with irregularities are places where lateral forces are concentrated [7-8].

Rail irregularity usually becomes stressed when rails are heated by sunshine in summer and are at a track-fixing temperature below the optimum fixing interval. In this case, the track resistance decreases under the passing train and the rails with sleepers move across the track axis.

The actual curvature Ra of the track curve in the layout with a passport curvature of  $1/R_0$  and an existing irregularity of  $1/\omega$  on it is determined by the following formula:

$$1 / Ra = 1 / R_0 + 1 / \omega$$

where  $\omega$  is the roughness of the track in the layout.

The detection of excessive compressive longitudinal forces in a continuous welded rail is controlled by three parameters affecting the stability of the rail-sleeper grid:

- Beams of the rail-sleeper grid in the layout f.
- Rail irregularity length  $l/\omega$ .
- Track irregularity amplitude  $A\omega$ .

- External force  $F$  (longitudinal force  $N$  and lateral force  $Q$ ).

Two relationships  $l(f)$  and  $F(f)$  causing instantaneous loss of stability can be obtained from these parameters [8-9], which correspond to a stressed irregularity.

The approach does not take into account the time factor and is based on statistical theory for calculating a continuous welded rail:

$$F = k_1 \sqrt{\frac{EI \cdot q}{f}}$$

$$f = \frac{k_2 q l_\omega^4}{EI}$$

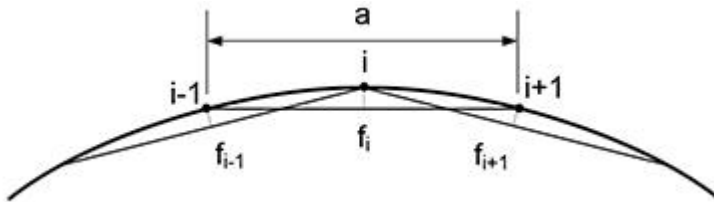
where,  $EI$  is bending stiffness of rail-sleeper grid in a horizontal plane;  $q$  is resistance per unit length of sleeper shift in ballast across the track axis ( $q = 12$  kN/m for compacted ballast,  $q = 7$  kN/m for uncompressed ballast);  $k_1$  is a factor depending on adopted configuration of track bending in a horizontal plane;  $k_2$  is a factor depending on adopted configuration of track bending in loss of stability

In order to ensure lateral stability of the track, it is necessary that the lateral displacement resistance forces of the track are greater than the sum of the lateral forces.

For the calculation, the normalised versine of the rail-sleeper grid is set in the layout and the change in the longitudinal force is determined according to the temperature difference [9-10]. However, the use of this approach does not allow determining which track irregularity in the layout will be stressed, as there are many occasional factors that have a large variance [10-11]. This methodology can be improved by identifying local irregularities, and the resistance rates will be a function of the deformation rate.

The condition of the track on curves in the layout is assessed by means of versines (Fig. 2). In order to measure the curvature, marks are usually placed on the track at 10 m intervals.

The marks are placed on a straight track section and are also completed on a straight track, so that at least two points are placed on the straight track. The bend is measured in the middle of the drawn chord. Accordingly, in an ideal curve condition, the versines within a circular curve will have a constant value.



**Fig. 2.** Scheme for measuring the bend versine.

According to the calculation scheme, the curve radius is related to the bend radius as following:

$$f_0 = \frac{a^2}{8R}$$

where  $f_0$  is the versine;  $a$  is the chord length;  $R$  is the curve radius.

The law of the rail-sleeper grid as a function of time can be determined by the following formula:

$$f = f_0 \exp \frac{F^2 \tau}{4 \cdot EJ \cdot \xi}$$

where  $\tau$  is the time index,  $s$ ;  $\xi$  is the viscosity factor of the ballast displaced by the sleeper across the track ( $\xi = 108 \text{ kN}\cdot\text{s}/\text{m}^2$ [13-12]);  $EJ$  is the section bending stiffness;  $F$  is the longitudinal compressive force ( $F = 400, 800, 1200, 1600 \text{ kN}$ , which corresponds to the temperature difference between the rails regarding their fastening 10, 20, 30 and 40 °C, respectively).

To determine the stability conditions, the most essential characteristic of it is the growth rate of the rail versine in layout, i.e., the derivative over time is taken for the point with the maximum versine.

The ballast is known to be a non-linear elastic-viscoplastic material, which determines its rheological properties. A considerable number of viscoplastic models with elastic properties in the initial stage of deformation have been proposed to describe the processes of resistance to sleeper displacement. The simplest models [11-13] have a linear elastic characteristic for sleeper shifts up to 0.5-2 mm and in the case of larger shifts have a constant resistance, depending on both the vertical load and the type of sleeper and the condition of the ballast bed.

## 7 Calculating the stability of a continuous welded track

Monitoring and further assessment of continuous welded rail system stability is based on the data obtained with the help of digital sensor system. The value of complex index of pre-failure condition of a continuous welded rail is determined by a formula and depends on a group of factors, which characterize the presence of temperature stresses in rail sections and retention properties of a rail-sleeper grid as coefficients:

$$K_k = f(K_a; K_i)$$

where  $K_i$  is introduced if there is a track irregularity in the layout.

In order to assess the stability of continuous welded strings in a particular section, it is necessary to introduce the concept of "track basic condition" which is characterised by the current stability norms. A change in the parameters of any of the factors in relation to the basic ones leads to a reduction in the bearing capacity of a section of a continuous welded rail. A change in the parameters of any of the factors in relation to the basic ones leads to a reduction in the bearing capacity of a section of a continuous welded rail track. As a basic value usually the track deviation parameters are taken into account: length of irregularity equal to 10 m; irregularity beam equal to 5 mm [4-14].

$\Delta I_{\text{limit}}$  is the limit value of rail distortion.

The admissible compressive force according to the stability condition is determined by the formula:

$$[F]_c = F_c / K_s$$

where  $F_c$  is the critical longitudinal compressive force, N;  $K_s$  is the stability factor ( $K_s = 1.1$ )

Then, the admissible compressive force operating in the rail throughout the year will be determined as following: (i.e., the permissible compressive force in the rail):

$$[F]_{\text{year}} = 2 \cdot 2,478 \cdot F \cdot T_a$$

$$T_a = T_{\text{max}} + T_{\text{min}}$$

$T_a$  is the amplitude of the temperatures.

The value of the pre-failure condition coefficient can be expressed in terms of force:

where  $F_c[\Delta t_s]$  is the critical compressive longitudinal force for the relevant track structure in the standard state corresponding to the temperature;  $[\Delta T_s]$  is the admissible increase in temperature of the rails relative to their fastening.

Based on  $K_a$  and  $[\Delta T_s]$ , the actual admissible temperature is calculated:

$$[\Delta T]_a = [\Delta T_s] \cdot K_a$$

The value of the pre-failure condition coefficient can also be expressed in terms of displacement:

$$K_a = (5 \cdot \Delta l) / \Delta l_{pre-f}$$

$\Delta l$  is the measured actual displacement of the rail track, mm.

Coefficient that characterises the safety factor for rail track creepage. It is assumed to be 5 according to the ranking matrix, which includes five assessment conditions:

- At  $K_a = 2.5-3$ , a speed limit of 120 km/h is set for the line.
- At a value of  $K_a = 3-4$ , a speed limit of 60 km/h is set.
- At a value of  $K_a = 4-4.5$  a speed limit of 25 km/h is set and a warning is issued regarding a possibility of derailment.
- At a  $K_a = 4.5$  and above a speed limit is set requiring closure and repair work.

The limit values for rail track displacements are shown in Table 1.

**Table 1.** Limit values for displacements depending on curvature radius of rail sections.

| Radius of curvature, m  | straight track sections and curved sections with $R > 2000$ | $400 \leq R \leq 2000$ | $350 \leq R \leq 399$ | $300 \leq R \leq 349$ | $250 \leq R \leq 299$ |
|-------------------------|---|------------------------|-----------------------|-----------------------|-----------------------|
| Limit displacements, mm | 40  | 35                     | 30                    | 25                    | 20                    |

## 8 Force balance equation

The stability calculation is an empirical method. In order to obtain these coefficients, a calculation must be performed to find the critical values of the parameters. To determine the value of the critical compressive longitudinal force, the force balance equation must be solved. The first parameter of the equation is the lateral force (distributed load) generated by the temperature stress and concentrated at the local non-linearities (beams) due to a sharp reduction in the radius of curvature. At local curvature sections, it can be conventionally assumed that the entire longitudinal load in the rail is converted here into a lateral load pushing the sleepers on which this section is fixed to the section of stone base between the sleeper and the bottom slope in the layout [15]. In addition, the local non-linearity takes shape with a central half-wave and two half-waves at the border of this non-linearity. The peripheral half-waves also push the stone ballast, but in the direction towards the centre of the radius of curvature of the beam. Since the ballast mobility in this direction is significantly lower and the thickness of the ballast layer is many times greater, the lateral forces generated in the peripheral half-waves serve to push the rail track away from the ballast, and the ballast in this case does not move. Hence, the forces occurring in the central and peripheral half-waves of the local non-linearity vectorically sum together into a total lateral force and expel the stone ballast jointly from the end of the sleepers holding the stringline in the area of local curvature in the direction from the centre of the curvature radius of the stringline. The second parameter of the equation is the force (distributed load) from the resistance of the ballast to the displacement of sleepers in the lateral direction in relation to the track axis. The equilibrium condition of these two forces is the distortion case. For reliable track operation, the resistance force of the ballast must exceed the lateral force with a certain surplus. This margin can be taken into account by means of a pre-failure factor  $K_a$ .



## 9 Determining the lateral distortion force at the maximum beam angle

The lateral force must be determined for locations where beams with maximum angles of curvature or maximum ratio of beam size in the layout to the length of the beam arise.

The lateral force, without considering the concentration of lateral load at the local curvature, is determined according to the formula [5-6]:

$$N_a = \alpha(P + P_0) \times [1 + \beta \times S \times \Delta\theta (1 + R_0/R)] \times (K/K_0)^{1/8} \times [\varepsilon(EI)^{1/4}/(EJ)^{1/8}],$$

where  $\alpha(P + P_0)$  is the minimum resistance of the reference track;  $P$  is the axial load;  $P_0$  has a value of  $3 \div 4 \times 10^4$ , N;  $\alpha$  is a factor that depends on the degree of ballast stabilisation;  $[1 - \beta \times S \times \Delta\theta (1 + R_0/R)]$  is a factor that accounts for the influence of temperature  $\Delta\theta$  and track curvature  $1/R$  on  $N_a$ ;  $S$  is the rail cross-sectional area,  $m^2$ ;  $\Delta\theta$  is the increase in rail temperature compared to neutral,  $^{\circ}C$ ;  $R_0$  is the minimum admissible curve radius;  $\beta$  is the empirical coefficient equal to  $0.125 \text{ m}^2 / ^{\circ}C$ ;  $(K/K_0)^{1/8}$  is the multiplier taking into account the influence of track elastic modulus on  $N_a$ ;  $K_0 = 2 \times 10^7$  is the track modulus in loosened ballast condition,  $N/m^2$ ;  $K$  is the measured track modulus,  $N/m^2$ ;  $\varepsilon(EI)^{1/4}/(EJ)^{1/8}$  is the multiplier for the influence of track structure stiffness in the horizontal and vertical planes;  $\varepsilon = 0.225 \text{ N}^{-1/8} \text{ m}^{-1/4}$  is the factor determined on the reference track experimentally;  $EI$  is the bending stiffness of the rail in the horizontal plane,  $N/m^2$ ;  $EJ$  is the bending stiffness of the rail in the vertical plane,  $N/m^2$

The load concentration at the non-linearities must be taken into account depending on the ratio of the boom to the length of the non-linearities or, in other words, on the curvature angle of the non-linearities. The load concentration coefficients of the non-linearity have to be calculated numerically for different curvature angles, beams and lengths of the non-linearity.

## 10 Determining the lateral reaction of the ballast supporting pole

The lateral resistance force of the ballast is determined by the formula:

$$F_b = aW + bpG_e + cpG_3,$$

where  $a$ ,  $b$  and  $c$  are constant coefficients;  $W$  is the linear weight of the rail-sleeper grid;  $\rho$  is the ballast density;  $G_e$  is the ballast volume which resists its displacement behind the sleeper end;  $G_3$  is the same on the side surfaces of sleepers. On a railway track with reinforced concrete sleepers and crushed stone ballast:  $a = 0.75$ ,  $b = 29$  and  $c = 1.8$ .

The value of the critical compressive longitudinal force  $F_c^{[\Delta t_s]}$  is determined for loading conditions under which the equality  $H_a = F_b$  is satisfied. If this condition is met, the parameter  $P$  in the equation is equal to the value of  $F_c^{[\Delta t_s]}$

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