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Stochastic Mechanism of Shock Disturbances of Rocket Sleigh Supports during High-Speed Track Testing of Aviation Equipment

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Abstract. Tests of ballistic aircraft on rail tracks in conditions close to real ones are an economically viable alternative to flight tests. Rocket rail tracks exist in the USA, Canada, France, China and a number of other countries. The article con-siders potentially self-oscillating mechanical systems that have a threshold mechanism for the transition from a noise vibrational stable state to a self-oscillatory one, depending on the mode parameters. An algorithm for estimating the damping coefficient of the harmonic components of a particular mode of a complex process is proposed. As an indicator of energy loss of the oscillatory component during resonant interaction, the value of the density of the power spectrum related to the width of the frequency range is taken. Based on the solution of the Fokker-Planck equation for a stochastic process, estimates of the "drift" and "diffusion" coefficients are obtained. A technique for estimating the state of potentially self-oscillatory systems in the form of a set of probabilistic stochastic characteristics is proposed. According to the experimental data of vibration accelerations for the structural elements of the rocket sled with the test object, according to the proposed method, it is possible to make the contribution of the self-oscillations of the elements, if they are pre-sent in a complex vibration process, and also to make estimates of their vibration strength at resonances.

1 Introduction

The rail track is an experimental installation with a two-rail and (or) monorail track of various lengths, for example, in the USA, the track is more than 12 km long. The existing track in the FKP "GkNIPAS" named after L.K. Safronov has a vertical profile, consisting of three sections: an accelerating section with an angle of attack, a straight section, and a descent section designed to slow down a moving sled with the equipment under test. The total length of the track is 3500 m. The movable rail sled is an all-welded rigid frame with three transverse beams. The rear and front beams are pivotally connected to the sliding supports (shoes) and lodgements for fastening rocket engines of solid fuel, based on the rear

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and middle beams. The test object is a cylindrical body with a conical nose fairing. It is attached to the middle and front beams. Three-axis vibration acceleration sensors have recently been placed on rocket sledges and test objects, and signal recorders have been placed on the upper base of the carriage. Fire launches are carried out regularly, however, since the vibration sensors were installed, it has been noted that under the same acceleration modes, in some cases, shock disturbances on the shoes are realized without a response from the elements of the rocket carriage, and in other cases, resonances with large amplitudes are observed, both on the sliding supports and at some points of the carriage. The shoes, when installed on rails, have side gaps. Such shock disturbances on sliding supports lead to dynamic instability of the rocket sleigh and to bending vibrations of the protruding cantilever part of the test object, to limit vibration loading of the electronic equipment of the test object. In order to ensure the directional stability of the movement, the problem of the stochastic transition of the vibroacoustic mode of a potentially nonlinear system into a self-oscillating one is considered [1-2].

2 Rocket sled dynamics

Any systems that have mass and elasticity, loaded with body forces and moments, are dynamic oscillatory systems with an infinitely large number of degrees of freedom. To analyse the elastic oscillations of such systems, we use the d'Alembert method, in which equivalent inertia forces are used instead of body forces in differential equations that describe the equilibrium of systems. In this case, the solutions of these equations are presented as a product of coordinate functions and time functions that change according to a harmonic law. Solutions in the form of functions of coordinates with zero right-hand sides and homogeneous boundary conditions are modes of free vibrations, and solutions in the form of a function of time describe motion as principal coordinates.

When analysing the vibration loading of products placed on track sleds during ground testing, approximations are used in which the real system is replaced by a conditional system with lumped parameters with equivalent mass and elasticity. Since the components of the sled in the theoretical analysis are represented by beams of various profiles, and the vibrations of the structural elements are directed along the three axes X, Y and Z, then the complex vibration field can be represented in a simplified, generalized way, without mode indices and without the direction of the acting vibrations. In what follows, we will introduce notation, if necessary.

In distributed systems, the parameters that change over time and natural oscillations turn out to be related to each other, as a result of which parametric excitation of several harmonic oscillations synchronized with each other is possible. In the case of proximity or coincidence of the spectra of a stationary system with an equidistant velocity spectrum, then a periodic change in time of its parameters can lead to excitation of oscillations of a pulsed form [3-8].

The stochastic state of a dynamical system is described by differential equations of order higher than the second. The threshold transition from an unorganized stochastic state of an open system to a self-oscillating one for various modal configurations of structural elements can be described by a second-order dissipative dynamic system excited by a random broadband vibration [1-9]:

$$\frac{d^2u_j}{dt^2} + 2k\delta_j\left(\Pi_j, y_j\right)\frac{du_j}{dt} + \omega_{0j}^2 u_j = \omega_{0j}^2\zeta\left(t\right),$$
$$\frac{\omega_{0j}}{k\delta_j} >> 1, k\delta_j\left(\Pi_j\right) = \left(k\delta_{jd} - k\delta_{jg}\right) > 0 \tag{1}$$

(one equation (1) for each mode of normal vibrations of a rigid body).

Here: t – time;

u_j is the time realization of the narrow-band random process;

ω_{0j} is the circular frequency of natural oscillations for each mode of normal oscillations (without damping);

$k\delta_{jd}$, $k\delta_{jg}$ are, respectively, the coefficients of dissipation and generation of energy of the jth mode vibrational configurations, which are functions of the parameters (Π_j, y_j) of the sleigh movement mode;

ζ (t) – stationary normal random broadband action.

Within the framework of the dynamic model (1), the decrement of small oscillations $d_j = k\delta_j T_j$ in the vicinity of the resonant frequency $f_j = 1/T_j$ is a diagnostic indicator of the linear stability margin:

$$d_j = k\delta_j T_j = \frac{dE}{3E_s dt} = \frac{E_d - E_g}{2E_s}, \tag{2}$$

where T_j is the oscillation period; E_d is the energy dissipated by the oscillatory system during the oscillation period; E_g is the energy generated by the oscillatory system during the oscillation period; E_s is the energy stored by the system during the oscillation period.

From the point of view of diagnosing the mode of harmonic oscillations, it is important that if there is a random source ζ(t) on the right side of equation (1), simulating a broadband quasi-harmonic effect, the model parameters, namely: natural circular frequency ω0j and damping coefficient $k\delta_j$ can be determined from the observed realizations ξ_j (t). In particular, with a linear signal formation mechanism in the vicinity of the resonant frequency of the jth mode of normal oscillations, the peak width D_f of the power spectral density S(f) of the signal ζ (t) at the level of $0.5S_{max}$ is proportional to the decrement d of the $_j$ th mode.

$$d_j = k\delta_j T_j = \frac{\pi\Delta f_j}{f_j}, \tag{3}$$

where Tj is the oscillation period of the j-*th* mode.

On fig. 1 shows a graphical illustration of the estimation of the damping decrement of quasi-harmonic oscillations

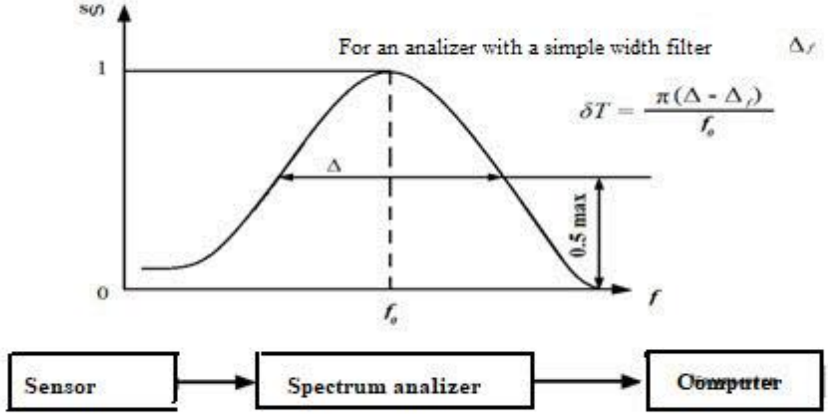

Fig. 1. Graph of the algorithm for estimating the oscillation damping decrement.

Markov processes are determined by the initial distribution and the transition probability equal to the conditional probability density of the transition from the previous state to the next one (process without aftereffect) for any instants of time [2, 9-13]. Using the conditions for the smallness of the change in amplitude and phase over the oscillation period and applying the well-known averaging method for one variable uj, from formula (1) we can obtain the following evolution equation describing the dynamics of the jth normal mode of a potentially self-oscillating system

$$\frac{dy_{\mathrm{j}}}{dt} = -\delta_{k\mathrm{j}}\left(y_{\mathrm{j}}\right)\cdot y_{\mathrm{j}} + \omega_{0\mathrm{j}}^{2}\frac{S_{\mathrm{n}0}}{8y_{\mathrm{j}}} + \Delta\left(t\right), \quad (4)$$

where: yj(t) is the envelope of the oscillation amplitude of the jth mode,
δ_{kj} (yj) is the coefficient of dissipation-generation of vibrational energy, is a function of the oscillation amplitude yj of a specific j-th mode,
ω_{0j} is the natural (resonant) circular frequency of the j-th mode,
S_{n0} is the spectral intensity of random noise exposure in the vicinity of the resonant frequency ω_{0j};
Δ(t) is a normal random delta - correlated function with zero mean.
In order to simplify the notation, we will further omit the index j, which shows that the model parameters belong to the jth mode of normal vibrations of the object under consideration, due to the fact that we use the single-mode approximation. Let us turn to model (1) with a random term ζ = Δζ(t) on the right side. Taking into account that ζ(t) is white noise, and y(t) is a slowly varying function compared to the period of oscillations, the evolution equation (4) describes a one-dimensional Markov random process with “drift” coefficients D^1, further we denote

$$a\left(y_{\mathrm{j}}\right) = -\delta_{k}\left(y_{\mathrm{j}}\right)\cdot y_{\mathrm{j}} + \omega_{0}^{2}S_{\mathrm{n}0}/\left(8y_{\mathrm{j}}\right) \quad (5)$$

and "diffusion" D^2 we denote (D^1,D^2 ... D^k k =1,2, … - means the superscript. Introduced in [9-12] for a one-dimensional process.)

$$b\left(y_{v}\right) = \frac{\omega_{0v}^{2}\cdot S_{\mathrm{n}0}}{4} \quad (6)$$

The stochastic differential equation (4) can be put in correspondence with the Fokker - Planck - Kolmogorov equation for the stationary probability density of the amplitude $P_{\text{ст}}(y)$ [9-14]

$$\frac{d}{\partial y}\left[b(y)\cdot P_{\text{ст}}(y)\right]-2a(y)\cdot P_{\text{ст}}(y)=-2G \tag{7}$$

with boundary conditions $G(0,t)=G(\infty,t)=0$,

where G is the probability flux through the boundaries of the vibrational hysteresis).

The solution to equation (7) is the dependence

$$P_{\text{ст}}(y)=C\cdot y\cdot\exp\left[2\int_0^x\frac{a(z)}{b(z)}dz\right]=C\cdot y\cdot\exp\left[-\frac{8}{\omega_0^2S_{n0}}\int_0^x z\cdot\delta_k(z)dz\right]= \\ =C_1\cdot y\cdot\exp\left[-\frac{1}{\delta_{k0}\sigma^2}\int_0^x y\delta_k(y)dy\right] \tag{8}$$

Here

$$\sigma^2=\frac{\omega_0^2\cdot S_{\text{n}0}}{4\delta_k} \tag{9}$$

The coefficient C1=const is found from the normalization condition $\int_0^\infty P_{\text{ст}}(y)dy=1$.

In the particular case of the linear model δk = δk(y) = const, distribution (9) leads to the experimentally observed Rayleigh distribution of the oscillation amplitude y

$$P_{\text{ст}}(y)=\frac{y}{\sigma^2}\exp\left[-\frac{y^2}{2\sigma^2}\right] \tag{10}$$

Nonlinear diagnostic model (7) has the property of identifiability of its parameters, in particular, the functional dependence of the damping coefficient on the oscillation amplitude. The dependence $\delta_k = \delta_k\ (y)$ can be estimated based on the expression obtained from (11)

$$P_{\text{ст}}(y)=C_1\cdot y\cdot\exp\left[-\frac{1}{\delta_{k0}\sigma^2}\int_0^x y\delta_k(y)dy\right] \tag{11}$$

Where $\sigma^2=\frac{\omega_0^2\cdot S_{\text{n}0}}{4\delta_k}$.

Dependence (11) makes it possible to estimate δk = δk(y) from the function $P_{\text{ст}}(y)$, determined from the experiment [1-2].
The results of the experiment and calculations are presented in the form of graphs in Figures 2 and 4.

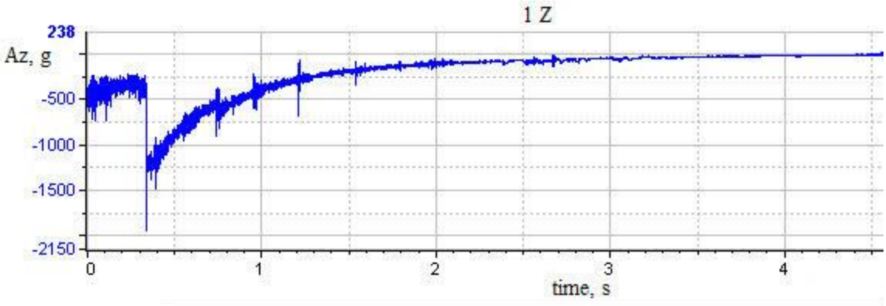

Fig. 2. The signal of the vibration overload sensor along the Z axis, perceived by the sliding support at a speed of 360 m / s of the rocket sled. On the *y*-axis, the dimension of the overload is in *g*.

Identical in form and synchronous signals along all three axes, but the magnitudes of shock disturbances are different. The maximum values of the amplitude of vibration overloads are realized in the horizontal plane along the Z axis, directed perpendicular to the movement. On fig. 3 shows the dependence of the density of the power spectrum of vibration overloads along the Z axis on the frequency

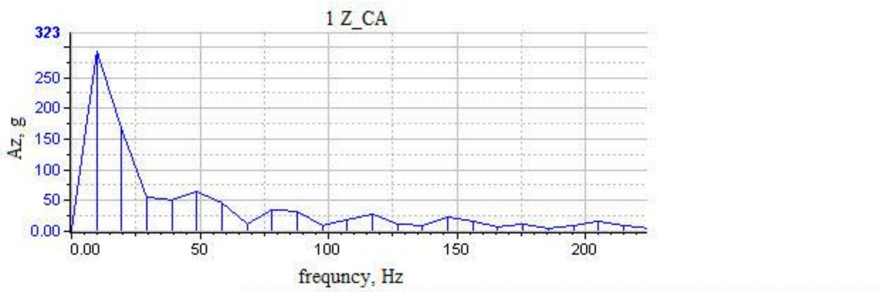

Fig. 3. The distribution of the density of the spectrum by frequency. Z-axis encoder signal.

It follows from the graph that significant energy of vibration overloads is localized in the low frequency range from 5 Hz to 25 Hz. On fig. 4 shows graphs of mutual correlation functions of vibration overload signals along the X and Y, X and Z axes.

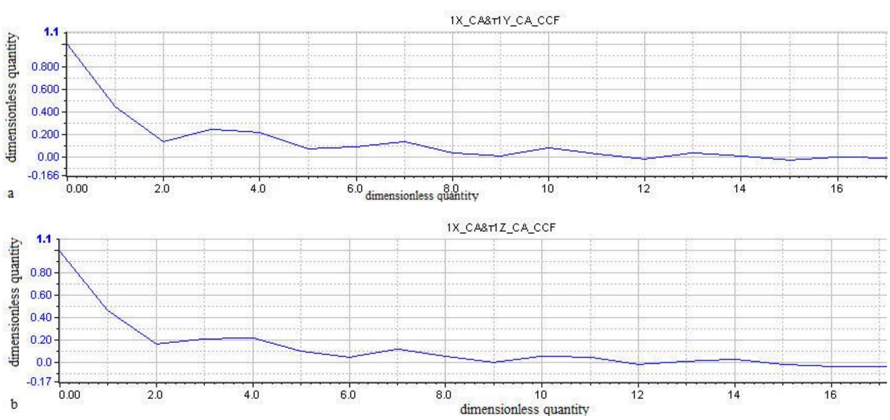

Fig. 4. Mutual correlation functions (CF).

a - Sensor No. 1 signals (right support) along the X and Y axes.

b - Signals of sensor No. 1 (right support) along the X and Z axes.

The graphs indicate that in a complex vibrational process there are harmonic components with a very low attenuation coefficient. On different axes, the harmonic components have a slight phase shift and a slightly different decay rate. Synchronized harmonic components of a complex vibrational processes directed along the X, Y and Z axes at a frequency close to its own, for the right front sliding supports bearing, lead to a threshold transition to the self-oscillating mode and initiation of pulsed oscillations, while the damping coefficient of oscillations of this system is close to zero [4-8;15-16].

3 Conclusion

The presented algorithm for estimating the damping coefficients of the harmonic components of vibration overloads that arose as a result of an induced threshold transition to a broadband shock disturbance. Due to the accelerated acceleration of the track sleigh, the control parameters of the limit cycle of self-oscillations change and further relaxation of the dynamic system is realized in the form of forced oscillations of the carriage elements. The analysis of complex signals makes it possible to estimate the probability of excitation of impact actions on sliding supports bearings due to a stochastic nature. Considering that the coherence of signals along the X, Y, Z axes is equal to one, then the impulses perceived by the sliding support in the vertical direction led to broadband perturbations along the Z axis in the transverse direction and affect the trajectory stability of the sleigh. Signs that the system under study belongs to potentially self-oscillatory ones with a threshold mode of self-excitation are: the damped form of the autocorrelation or mutual correlation functions, the close-to-Gaussian distribution density of the probability of instantaneous signal values, and the final dependence of the self-oscillation damping coefficient (decrement), which necessarily decreases with increasing amplitude. Prediction of the probable threshold of excitation of the unstable limiting cycle of self-oscillations is carried out by analytical continuation of the smoothed dependence to zero value of the damping coefficient. The ultimate goal of the research is the selection of optimal control parameters for the implementation of vibration protection measures under random stochastic effects.

References

1. V.I. Biryukov, R.A. Tsarapkin, Damping Decrements in the Combustion Chambers of Liquid Propellant Rocket Engines, Russ. Eng. Res., **39** (1), 6-12 (2019)
2. V.I. Biryukov, R.A. Tsarapkin, V.N. Ivanov, Method for Predicting the Stability Limit to Acoustic Oscillations in Liquid-Propellant Rocket Engine Combustion Chambers Based on Combustion Noise. Combust. Explos. Shock Waves, **57**(1), 74-72. Pleiades Publishing, Ltd., 2021.
3. H. Risken, The Fokker - Planck Equation (Berlin: Heidelberg: Springer Verlag, 1989).
4. J. Zhao, Y. Shao, Stability of a three-species cooperative system with time delays and stochastic perturbations, Discrete Dyn. Nat. Published as Electron. Res. Announc. Math. Sci., 1-15 (2021). https://doi.org/10.1155/2021/5577499
5. R. Tribe, A travelling wave solution to the kolmogorov equation with noise, Stoch. Published as Electron. Res. Announc. Math. Sci. Rep., **56**, 317-340 (1996). https://doi.org/10.1080/17442509608834047
6. Z.-C. Wang, Traveling curved fronts in monotone bistable systems, Discrete Contin. Syst., **32**, 2339-2374, (2012) Zbl 1242.35148

7. A.I. Vesnitsky, S.V. Krysov, A.I. Potapov Instability of torsional waves in rods with movable fastenings. - News of the Academy of Sciences of the USSR, OTN. Rigid Body Mechanics, **6**, 128 (1978).
8. A.I. Vesnitsky, S.V. Krysov Pulse generation in distributed systems with moving boundaries. - Letters to ZhTF, **3** (19), 1032 (1977).
9. *Vibrations in technology*: a Handbook. In 6 volumes, Editorial Council: V.N. Chelomey (prev.) (Moscow: Mashinostroenie, 1978) - v.1. Vibrations of mechanical systems / Ed. V.V. Bolotina. 1978.
10. E. Parzen, *Stochastic Processes* (Courier Dover Publications, 2015) ISBN 978-0-486-79688-8.
11. C. Gardiner, *Stochastic Methods*. A Handbook for the Natural and Social Sciences. (Springer, 2010) ISBN 978-3-540-70712-7.
12. P. Bremaud, *Markov Chains: Gibbs Fields, Monte Carlo Simulation, and Queues*. (Springer Science & Business Media, 2013) ISBN 978-1-4757-3124-8.
13. V. I. Tikhonov and M. A. Mironov, *Markov Processes* (Sov. Radio, Moscow, 1977)
14. G. I. Ivchenko and Yu. I. Medvedev, *Introduction to Mathematical Statistics* (Lenand, Moscow, 2017)
15. P.S. Landa, *Self-oscillations in distributed systems* (Moscow, Science. Main edition of physical and mathematical literature, 1983)
16. A.I. Vesnitsky, *Waves in systems with moving boundaries and loads* (Moscow, Fizmatlit, 2001)