E3S Web of Conferences **446**, 01003 (2023) https://doi.org/10.1051/e3sconf/202344601003
HSTD 2023

On the Correct Generation of a Single Sound Mode in a Duct with Flow in COMSOL Multiphysics

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Abstract. The paper considers the statement of a single-mode sound generation in a duct with flow in the COMSOL Multiphysics finite element analysis software. Verification of the COMSOL Multiphysics solution is based on a comparison with the analytical solution of a single-mode propagation in an annular duct with uniform flow. The verified statement of a single-mode generation in COMSOL Multiphysics is then used to predict the noise of a JT15D turbofan engine in the far field. The results of computations are compared with JT15D static test data and known numerical simulation results.

1 Introduction

The dominant source of noise in a modern aircraft engine with high bypass ratio (BPR>3) at flyover and approach certification reference points is a fan [1]. In this regard, it is important to be able to predict the fan noise in the far field with good accuracy. Fan noise prediction in the far field can be based on simulation of a single-mode propagation of sound waves with given amplitude coefficients from the fan cross section into the far field, where the modes are then summed [2, 3, 4, 5]. The simulation is often a numerical solution of a mathematical model in a frequency domain with given boundary conditions, which describes the propagation of sound waves in a moving medium to the boundaries, where the velocity of the medium is sufficiently low. Here, the amplitude coefficients of the modes generated by the fan are considered to be known (they can be preliminarily obtained, for example, from simulating the interaction of the oncoming flow with the fan stage [6, 7, 8]. Further, using the Helmholtz-Kirchhoff integral, the acoustic pressure is recalculated into the far field. This approach significantly saves computational resources and time.

The COMSOL Multiphysics finite element analysis software allows us to implement these tasks. An important aspect in solving such problems in the frequency domain is a correct statement of the single-mode generation. Setting the generated mode in terms of acoustic pressure or acoustic potential is incorrect, since it is the sum of the incident mode and the reflected mode, but the amplitude coefficient of the latter is unknown. However, this can be done by setting: the mass flow corresponding to the generated mode and directed through the source boundary inward the computational domain; the difference

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between the total acoustic potential and acoustic potential corresponding to the generated mode. To verify the correctness of such a statement, it is necessary to solve a test problem with a known analytical solution.

Thus, in this paper, we consider the solution of a test problem in COMSOL Multiphysics for generating a single mode in an annular duct with a uniform flow. For verification, the numerical solution is compared with the analytical one. Then a problem with a more complex geometry and non-uniform flow is solved. It is the noise propagation of the JT15D turbofan engine into the forward hemisphere. The obtained results are compared with the experimental results presented in [2] and simulation results from [2, 9].

2 Solution of the test problem

The geometry of the computational domain in the test problem is an axisymmetric model of an annular duct with constant inner and outer radii. At the entrance to the duct, the generation of a single mode is set. Generation of the reflected mode is realized by setting a wall at the duct outlet. The flow is considered to be potential, isentropic, inviscid and compressible. The governing equations in COMSOL Multiphysics are written for a non-uniform flow.

2.1 The governing equations for simulation in COMSOL Multiphysics

The momentum equation in the frequency domain is:

$$i\omega\phi + \mathbf{V}_0 \cdot \nabla\phi = -\frac{p}{\rho_0} \tag{1}$$

where ϕ is acoustic potential; p is acoustic pressure; ρ_0 is mean background density; $\mathbf{V}_0$ is mean background velocity vector; ω is angular frequency; i is imaginary unit.

The mass flow m_n for the quantities of the first order of smallness associated with acoustic excitation is written as:

$$\rho_0\nabla\phi + \mathbf{V}_0\frac{p}{c_0^2} = m_{\mathrm{n}} \tag{2}$$

where c_0 is mean background speed of sound.

Substituting equation (1) into (2) gives:

$$-\mathbf{n}\cdot\left(\rho_0\nabla\phi - \frac{\rho_0}{c_0^2}\left(i\omega\phi + \mathbf{V}_0\cdot\nabla\phi\right)\mathbf{V}_0\right) = m_{\mathrm{n}} \tag{3}$$

where $\mathbf{n}$ is outward normal to source boundary.

The sound hard boundary condition is specified as:

$$-\mathbf{n}\cdot\left(\rho_0\nabla\phi - \frac{\rho_0}{c_0^2}\left(i\omega\phi + \mathbf{V}_0\cdot\nabla\phi\right)\mathbf{V}_0\right) = 0 \tag{4}$$

To obtain an equation describing the propagation of waves in a duct with a non-uniform flow, the continuity equation is used, which, taking into account all above assumptions, is written in the following form:

$$\frac{i\omega}{c_0^2} p + \rho_0 \nabla \cdot \left(\nabla \phi \right) + \mathbf{V}_0 \cdot \left(\frac{1}{c_0^2} \nabla p \right) = 0 \tag{5}$$

Substituting (1) into (5), we get:

$$\frac{i\omega}{c_0^2} p + \rho_0 \nabla \cdot \left(\nabla \phi \right) + \mathbf{V}_0 \cdot \left(\frac{1}{c_0^2} \nabla p \right) = 0 \tag{6}$$

2.2 Analytical solution of the test problem

Let a mode of circumferential order m and radial order n propagates in an axisymmetric duct with an annular cross section. The flow in the duct is considered to be uniform. Then the acoustic pressure at any point of the duct is described by the sum of the incident and reflected waves:

$$p(z,r) = \left(A_{mn} e^{-i\xi_{mn}^{+} z} + B_{mn} e^{-i\xi_{mn}^{-} z} \right) F_m(\eta_{mn} r) \tag{7}$$

where $\xi_{mn}^{\pm} = \frac{-kM_0 \pm \sqrt{k^2 - \eta_{mn}^2 (1-M_0^2)}}{1-M_0^2}$ is longitudinal wave number; $k = \omega / c_0$ is spatial wave number; M_0 is mean background Mach number; $F_m(\eta_{mn} r) = J_m(\eta_{mn} r) + CY_m(\eta_{mn} r)$; J_m, Y_m are Bessel and Neumann functions of order m; $C = -\frac{J'_m(\mu_{mn})}{Y'_m(\mu_{mn})}$; $\eta_{mn} = \mu_{mn} / b$ is radial wave number; μ_{mn} is n-th root of equation $J'_m(\mu_{mn}) Y'_m(\mu_{mn} h) - J'_m(\mu_{mn} h) Y'_m(\mu_{mn}) = 0$; $h = a / b$; a is inner duct radius; b is outer duct radius.

Let us write the formulas for the amplitude coefficients A_{mn}, B_{mn}, assuming that the normalization factor is included in these coefficients. Let the pressure p_{in} be known at the entrance to the duct, where z = 0 and r = b. Then from expression (7) it follows:

$$A_{mn} = p_{in} / F_m(\mu_{mn}) - B_{mn} \tag{8}$$

The reflected mode is generated by a hard wall at the duct outlet, where $z = L$. The sound hard boundary condition can be written in terms of zero mass flow:

$$\rho_0 \upsilon + V_0 \rho = 0 \tag{9}$$

Deriving the acoustic velocity υ from the momentum equation (1) and substituting it into (9), we obtain:

$$p\left[\frac{\xi_{mn}^{\pm} + M_0 \left(k - \xi_{mn}^{\pm} M_0 \right)}{\left(k - \xi_{mn}^{\pm} M_0 \right)} \right] = 0 \tag{10}$$

Substituting (7) into (10), we get:

$$A_{mn} e^{-i\xi_{mn}^{+} L} M^{+} + B_{mn} e^{i\xi_{mn}^{-} L} M^{-} = 0 \quad (11)$$

where $M^{+} = \dfrac{\xi_{mn}^{+} + M_0\left(k - \xi_{mn}^{+} M_0\right)}{k - \xi_{mn}^{+} M_0}$; $M^{-} = \dfrac{\xi_{mn}^{-} + M_0\left(k - \xi_{mn}^{-} M_0\right)}{k - \xi_{mn}^{-} M_0}$.

Substitution of expression (8) into (11) yields:

$$B_{mn} = \frac{p_{in} e^{-i\xi_{mn}^{+} L} M^{+}}{F_m(\mu_{mn})\left(e^{-i\xi_{mn}^{+} L} M^{+} - e^{-i\xi_{mn}^{-} L} M^{-}\right)} \quad (12)$$

2.3 Results of solving the test problem

The geometry of the computational domains in COMSOL Multiphysics is shown in Figure 1. The MAIN is the domain into which the acoustic mode is radiated from boundary 1. A perfectly matched layer (PML) is added behind the source boundary 1 to absorb the reflected wave. The processes in the MAIN and PML domains are described by equation (6). At the boundaries 2-7, the sound hard boundary condition (4) is set. The duct length L is 1 m, the PML length is 0.2 m. The inner radius a is 0.0998 m and outer radius b is 0.2667 m. These values of a and b are chosen by analogy with the source cross section in the JT15D simulation [2]. A flow with a Mach number -0.3 propagates from boundary 3 to boundary 1 (of course, in a real situation, the flow could not be implemented in such a duct, but this is a model situation).

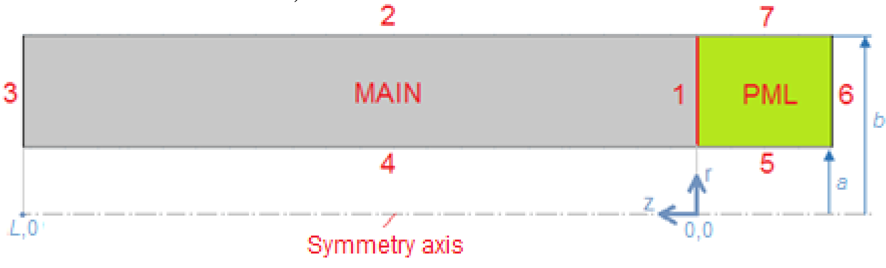

Fig. 1. Geometry of the computational domains of the test problem.

Generation of acoustic mode at the boundary 1, which belongs to the MAIN, is performed by setting the mass flow m_{n}, which is obtained by substituting into (3) the expression for the acoustic potential:

$$\varphi_{mn}^{+}(r) = \frac{A_{mn} F_m(\eta_{mn} r)}{-i\rho_0 c_0\left(k - \xi_{mn}^{+} M_0\right)} \quad (13)$$

Thus, the mass flow is:

$$m_{\mathrm{n}}^{+}(r) = -i\rho_0 \varphi_{mn}^{+}(r)\left[\xi_{mn}^{+} + M_0\left(k - \xi_{mn}^{+} M_0\right)\right] \quad (14)$$

Here, the superscript "+" indicates that acoustic mode propagates in the positive direction of the z-axis.

 https://doi.org/10.1051/e3sconf/202344601003

At boundary 1, which belongs to the PML, it is necessary to set the acoustic potential in the form:

$$\varphi_1 = \varphi - \varphi^+_{mn}(r) \qquad (15)$$

where $\phi^+_{mn}(r)$ is determined by expression (13); ϕ is yet unknown value of the total acoustic potential, which will be found by solving equation (6) by the finite element method.

Let is solve the test problem under the following conditions: m=-13; n=0; f=3150 Hz; p_{in} = 2+1.3i Pa. Expressions (12) and (8) yield B_{mn} = -4.277+8.656i, A_{mn} = 11.448-3.995i. To obtain the analytical distribution of acoustic pressure at the boundary 2 (Fig. 1), we substitute these amplitude coefficients into (7).

Figure 2 demonstrates the comparison of the real and imaginary parts of acoustic pressure at the boundary 2, obtained by solving the problem in COMSOL Multiphysics, with the analytical solution. It can be seen that the solutions are in complete agreement with each other, which indicates the correctness of the single-mode sound generation used in COMSOL Multiphysics for a duct with a uniform flow.

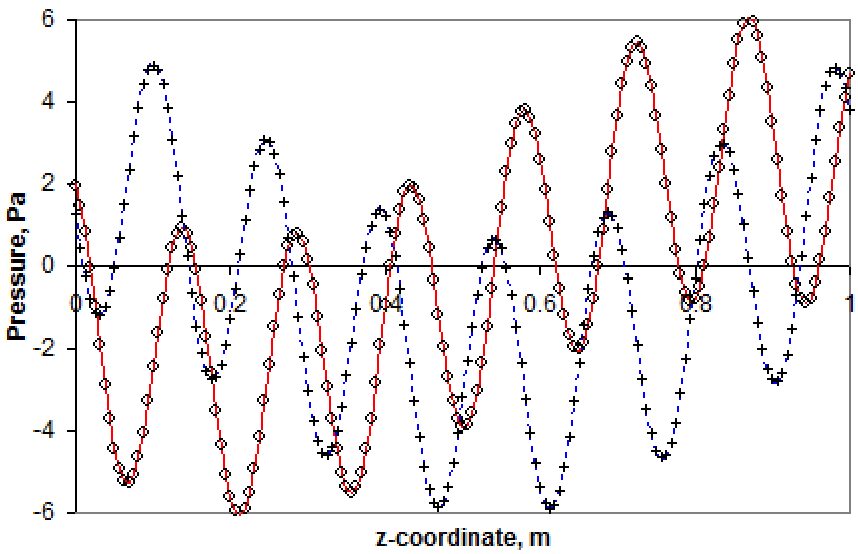

Fig. 2. Acoustic pressure at boundary 2: ◇Re(p) COMSOL Multiphysics; ——— Re(p) analytical solution; + Im(p) COMSOL Multiphysics; - - - - - - Im(p) analytical solution.

3 Simulation of a fan noise propagation into the far field

The statement of sound mode generation and propagation in a duct with non-uniform flow in COMSOL Multiphysics is considered in [10]. The results of numerical simulation for hard wall case are in good agreement with analytical solution for slowly varying duct with mean flow [11]. In our case, to verify the computation of a single-mode sound propagation in a duct of more complex geometry in the presence of a non-uniform flow, data on test of the JT15D turbofan engine [2] are used. Figure 3 shows the geometry of computational domains built for this problem.

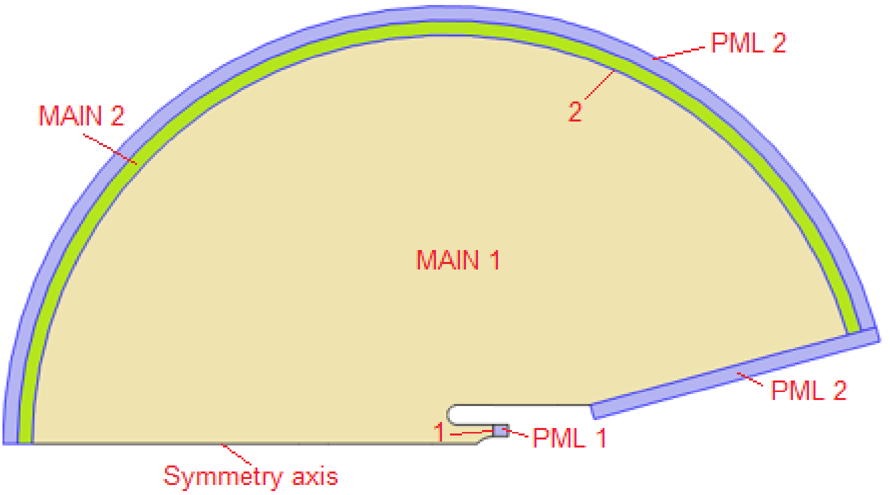

Fig. 3. Geometry of the computational domains.

First, a gas-dynamic computation is carried out for the conditions of JT15D static test. As a result, in the MAIN 1 domain, the velocity and density fields are determined. To do this, the finite element method solves the system of stationary Euler equations for a potential flow:

$$\begin{cases} \left(\frac{|\Phi|^2}{2} + \frac{\gamma}{\gamma-1} \left(\frac{\rho}{\rho_{ref}} \right)^{\gamma} \frac{p_{ref}}{\rho} \right) = \frac{v_{ref}^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_{ref}}{\rho_{ref}}, \\ \nabla \cdot (\rho \nabla \Phi) = 0 \end{cases}$$

where Φ is mean flow velocity potential; ρ is mean flow density; γ is the ratio of specific heats; ρ_{ref}, p_{ref} are reference density and pressure corresponding to normal environmental conditions; v_{ref} is flow velocity at the boundary 1 of the MAIN 1 domain (depends on the fan speed [2]. At the boundary 2 of the MAIN 1 domain, the zero value of Φ is set.

Next, the acoustic computations are performed. Processes in the MAIN 1 and PML 1 are described by equation (6) using the fields $\mathbf{V}_0$, ρ_0, c_0 found in the gas-dynamic computation. At the boundary 1 of the MAIN 1 domain, acoustic mode generation is set by formula (14). At the boundary 1 of the PML 1 domain, the acoustic potential is set in the form (15). Equation (4) is set at the hard walls of the MAIN 1 and PML 1 domains.

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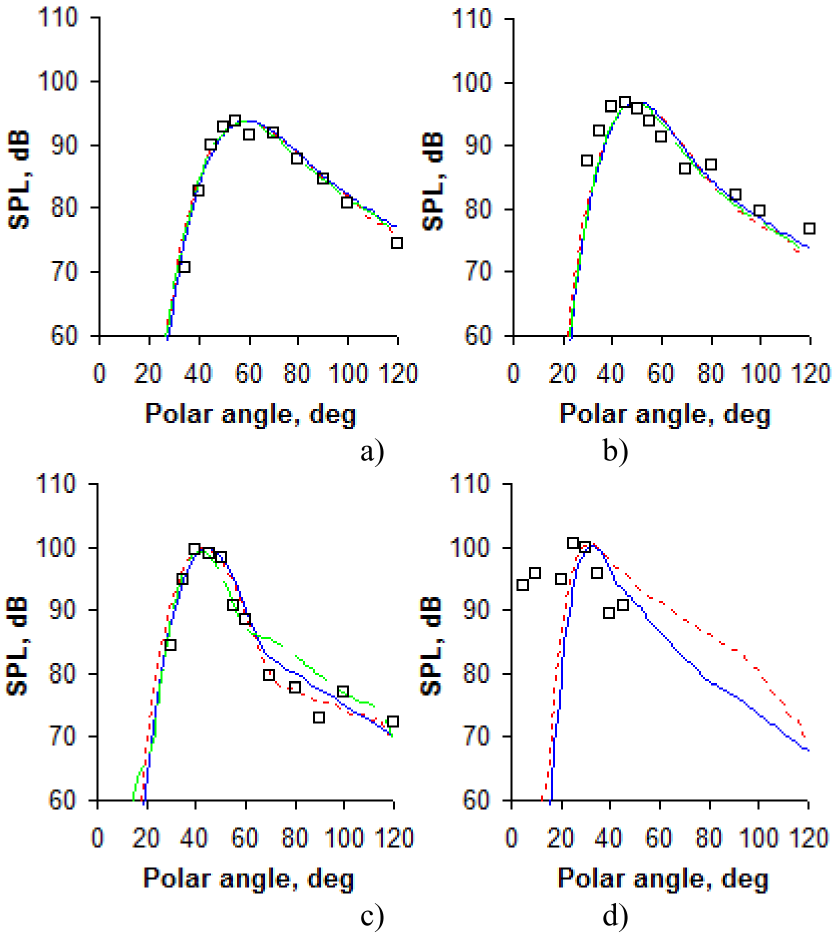

Fig. 4. Comparison of the JT15D noise in far-field: a) 3150 Hz, M = 0.175; b) 3943.3 Hz, M = 0.235; c) 4480 Hz, M = 0.275;d) 6300 Hz, M = 0.412; □ experiment [2]; ------- numerical simulation [2]; —— —— numerical simulation [9]; ———— numerical simulation in COMSOL Multiphysics.

The MAIN 2 and PML 2 are far enough away from the air intake that there is no flow in these domains. Thus, processes in MAIN 2 and PML 2 are described by the Helmholtz equation. At boundary 2, the acoustic pressures relating to the MAIN 1 and MAIN 2 domains are set equal.

After obtaining a finite element solution, the acoustic pressure is recalculated from boundary 2, which belongs to the MAIN 2 domain, to a distance of 30.5 m (according to the JT15D static test conditions [2] using the Helmholtz-Kirchhoff integral. The SPL dericivity in the far-field is calculated relative to the center of the entrance to the air intake.

Figure 4 demonstrates the SPL derictivity for acoustic mode (-13,0). One can see a good agreement between the simulation results in COMSOL Multiphysics and simulation results in [2, 9]. There is also good agreement with experiment at a low fan speed. At a high fan speed (Fig. 4d), the calculated directivity turned out to be shifted relative to the experiment results by several degrees. Perhaps this is due to the peculiarities of the experiment (for example, multimodal sound propagation), which are not taken into account in numerical simulation.

4 Conclusion

The study carried out shows that in COMSOL Multiphysics, the generation of a single sound mode with a known amplitude coefficient can be performed by setting a mass flow corresponding to acoustic excitation at the source boundary, which belongs to the main wave propagation domain. To correctly take into account the effect of the reflected mode in a duct, it is necessary to place a perfectly matched layer in such a way that its input boundary coincides with the sound source boundary of the main domain, and set the difference between the total acoustic potential and acoustic potential corresponding to the generated mode at this boundary. Complete agreement between the results of the numerical and analytical solutions in the test problem for an annular duct with a uniform flow and rigid walls confirms the correctness of such a statement.

The application of the single mode generation to solve a problem with complex geometry and non-uniform flow in COMSOL Multiphysics was also considered using the example of the single mode propagation of noise from JT15D turbofan engine into the front hemisphere under static test conditions. At low fan speeds, a good agreement was obtained with the experiment and with the results of predictions based on numerical simulation published earlier by other researches. At a high fan speed, there is a noticeable discrepancy between the predicted far-field noise and the experiment (as in other authors), which can be explained by the influence of more complex effects that are not taken into account in the numerical simulation.

In general, the considered statement of the single-mode sound generation in COMSOL Multiphysics can be applied to various research problems related to aircraft engine noise (for example, studying the effect of acoustic liner impedance variability on the directivity of fan noise in the far field).

The work was funded by a grant from Perm Region and Russian Science Foundation, grant number 22-22-20087.

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