

# On Edge Proper Interval-Valued Complex Fuzzy Graphs

<sup>1</sup>R. Venkateshwara<sup>1</sup>, <sup>2</sup>R.Sridevi

<sup>1</sup>Ph.D. (Part Time)Research Scholar, PG and Research Department of Mathematics,  
Sri S.Ramasamy Naidu Memorial College(Autonomous),Sattur,  
Affiliated to Madurai Kamaraj University, Madurai

<sup>2</sup>Assistant Professor, PG and Research Department of Mathematics,  
Sri S.Ramasamy Naidu Memorial College(Autonomous), Sattur,  
Affiliated to Madurai Kamaraj University, Madurai

**ABSTRACT:** This article summarizes about edge proper interval-valued complex fuzzy graph. Here, degree of an edge, total degree of an edge, edge proper and edge totally proper interval-valued complex fuzzy were introduced. A necessary and sufficient condition in which these two graphs are equivalent is provided. Many properties of edge proper interval-valued complex fuzzy graphs are examined and they were investigated for edge totally proper interval-valued complex fuzzy graphs,

**Keywords:** degree of a vertex in interval-valued complex fuzzy graph, total degree, proper interval-valued complex fuzzy graph, totally proper interval-valued complex fuzzy graph, edge degree in fuzzy graph, total edge degree in fuzzy graph. A.M.S subject classification: 05C72

## 1. INTRODUCTION

Mathematics is a powerful tool for global understanding and provides an effective way of building new things. In Mathematics, Graph Theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. Graphs are the basis for many new things, ideas, concepts, operations and process in everyday life. In many real applications, the membership degree in a fuzzy set cannot be lightly confirmed. It is more reasonable to give an interval-valued data to describe membership degree. In 2011, Akram and Dudek[1] defined the interval-valued fuzzy graph as an extension of fuzzy graph in which the values of the membership degrees are intervals of numbers instead of the number and described operations on it. R. Venkateshwara and R. Sridevi[10] defined the concept of interval-valued complex fuzzy graph. Throughout this paper  $ivcf$  graph denote interval-valued complex fuzzy graph.

## 2. PRELIMINARIES

**Definition 2.1.** A fuzzy graph  $G: (\sigma, \mu)$  is a pair of functions  $(\sigma, \mu)$ , where  $\sigma: V \rightarrow [0,1]$  is a fuzzy subset of a nonempty set  $V$  and  $\mu: V \times V \rightarrow [0,1]$  is a symmetric fuzzy relation on  $\sigma$  such that for all  $u, v$  in  $V$ , the relation  $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$  is satisfied. The underlying crisp graph of the fuzzy graph  $G: (\sigma, \mu)$  is denoted as  $G^*(V, E)$  where  $E \subseteq V \times V$ .

**Definition 2.2.** An interval-valued fuzzy graph with an underlying set  $V$  is defined to be the pair  $(A, B)$ , where  $A = (\mu_A^-, \mu_A^+)$  is an interval-valued fuzzy set on  $V$  and  $B = (\mu_B^-, \mu_B^+)$  is an interval-valued fuzzy set on  $E$  such that  $\mu_B^-(x, y) \leq \min\{\mu_A^-(x), \mu_A^-(y)\}$  and  $\mu_B^+(x, y) \leq \min\{\mu_A^+(x), \mu_A^+(y)\}$  for all  $(x, y \in E)$ . Here,  $A$  is called interval-valued fuzzy vertex set on  $V$  and  $B$  is called interval-valued fuzzy edge set on  $E$ .

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<sup>1</sup>Corresponding author: [venkisippi@gmail.com](mailto:venkisippi@gmail.com)

Definition 2.3. Let  $G: (A, B)$  be an interval-valued fuzzy graph, where  $A = (\mu_A^-, \mu_A^+)$  and  $B = (\mu_B^-, \mu_B^+)$ . The positive degree of a vertex  $u \in G$  is defined as  $d^+(u) = \sum \mu_B^+(u, v)$ , for  $uv \in E$ . The negative degree of a vertex  $u \in G$  is defined as  $d^-(u) = \sum \mu_B^-(u, v)$ , for  $uv \in E$  and  $\mu_B^+(u, v) = \mu_B^-(u, v) = 0$  if  $uv$  not in  $E$ . The degree of a vertex  $u$  is defined as  $d(u) = (d^-(u), d^+(u))$ .

Definition 2.4. Let  $G: (A, B)$  be an interval-valued fuzzy graph on  $G^*(V, E)$ . If all the edges of  $G$  have the same degree, then  $G$  is said to be regular interval-valued fuzzy graph.

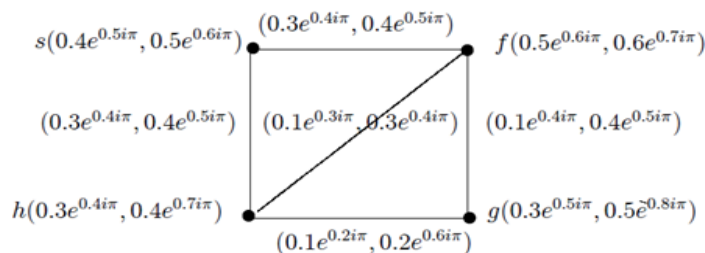
Definition 2.5. Let  $G: (A, B)$  be an interval-valued fuzzy graph on  $G^*(V, E)$ . The positive degree of an edge is defined as  $d^+(uv) = d^+(u) + d^+(v) - 2\mu_B^+(uv)$ . The negative degree of an edge is defined as  $d^-(uv) = d^-(u) + d^-(v) - 2\mu_B^-(uv)$ . The degree of an edge is defined as  $d(uv) = (d^-(uv), d^+(uv))$ . The minimum degree of an edge is  $\delta_E(G) = \wedge \{d(uv): uv \in E\}$ . The maximum degree of an edge is  $\Delta_E(G) = \vee \{d(uv): uv \in E\}$ .

Definition 2.6. Let  $G: (A, B)$  be an interval-valued fuzzy graph on  $G^*(V, E)$ . The total positive degree of an edge is defined as  $td^+(uv) = d^+(u) + d^+(v) - \mu_B^+(uv)$ . The total negative degree of an edge is defined as  $td^-(uv) = d^-(u) + d^-(v) - \mu_B^-(uv)$ . The degree of an edge is defined as  $td(uv) = (td^-(uv), td^+(uv))$ . The minimum total degree of an edge is  $t\delta_E(G) = \wedge \{td(uv): uv \in E\}$ . The maximum total degree of an edge is  $t\Delta_E(G) = \vee \{td(uv): uv \in E\}$ .

### 3. Edge Proper Complex Interval-Valued Fuzzy Graphs

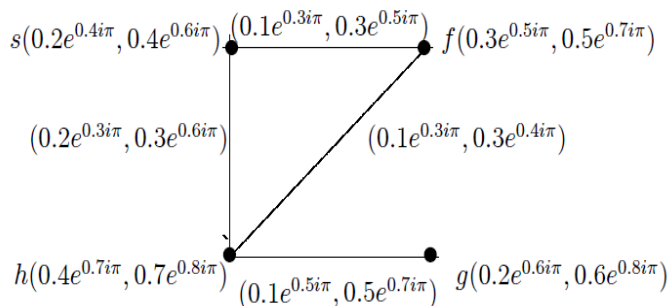
Definition 3.1. Let  $G: (S, T)$  be an interval-valued complex fuzzy graph on  $G^*(F, I)$ . The positive degree of an edge is defined as  $d_I^+(uv) = d_I^+(u) + d_I^+(v) - 2\mu_T^+(uv)e^{i\sum\beta_T(uv)}$ . The negative degree of an edge is defined as  $d_I^-(uv) = d_I^-(u) + d_I^-(v) - 2\mu_T^-(uv)e^{i\sum\alpha_T(uv)}$ . The degree of an edge is defined as  $d_I(uv) = (d_I^-(uv), d_I^+(uv))$ .

Example 3.2. Consider a ivcf graph on  $G^*(F, I)$ .



Definition 3.3. Let  $G: (S, T)$  be an interval-valued complex fuzzy graph on  $G^*(F, I)$ . The positive total degree of an edge is defined as  $td_I^+(uv) = d_I^+(u) + d_I^+(v) - \mu_T^+(uv)e^{i\sum\beta_T(uv)}$ . The total negative degree of an edge is defined as  $td_I^-(uv) = d_I^-(u) + d_I^-(v) - \mu_T^-(uv)e^{i\sum\alpha_T(uv)}$ . The total degree of an edge is defined as  $td_I(uv) = (td_I^-(uv), td_I^+(uv))$ . It can also be defined as  $td_I(uv) = d_I(uv) + T(uv)$ .

Example 3.4. Consider a ivcf graph on  $G^*(F, I)$ .



Definition 3.5. Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . If each edge in  $G$  has the same degree  $(k_1e^{is_1\pi}, k_2e^{is_2\pi})$ , then  $G$  is said to be an edge proper ivcf-graph.

**Definition 3.6.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . If each edge in  $G$  has the same total degree  $(c_1 e^{it_1\pi}, c_2 e^{it_2\pi})$ , then  $G$  is said to be an edge totally proper ivcf-graph.

**Theorem 3.7.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . Then  $T$  is constant function if and only if the following are equivalent

- (i)  $G$  is an  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper ivcf graph.
- (ii)  $G$  is a totally  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph.

**Proof.** Assume  $T$  is constant function. Let  $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$  for all  $gx \in I$ . Suppose that  $G$  is  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ -edge proper ivcf graph. Then  $d_I(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  for all  $gx \in I$ . Now  $td_I(gx) = d_I(gx) + T(gx)$  for all  $gx \in I \Rightarrow td_I(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$  for all  $gx \in I \Rightarrow td_I(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$  for all  $gx \in I$ . Hence  $G$  is a  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ -edge totally proper ivcf graph. Thus (i) imply (ii) is proved. Now assume  $G$  is  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ . Then  $td_I(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$  for all  $gx \in I \Rightarrow d_I(gx) + T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$  for all  $gx \in I \Rightarrow d_I(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  for all  $gx \in I$ . So,  $G$  is  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper ivcf graph. Thus (ii) imply (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, assume (i) and (ii) are equivalent. Let  $G$  be a  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper ivcf graph and totally  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph. Assume  $T$  is not a constant function, then  $T(gx) \neq T(kl)$  for at least one pair of edges  $gx, kl \in I$ . Since  $G$  is  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper ivcf graph. Then  $d_I(gx) = d_I(kl) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) \Rightarrow td_I(gx) = d_I(gx) + T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + T(gx)$  and  $td_I(kl) = d_I(kl) + T(kl) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + T(kl)$ . Since  $T(gx) \neq T(kl) \Rightarrow td_I(gx) \neq td_I(kl)$ . Hence  $G$  is not totally  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph. Which is a contradiction. Now, let  $G$  be a totally  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph. Then  $td_I(gx) = td_I(kl) \Rightarrow d_I(gx) + T(gx) = d_I(kl) + T(kl) \Rightarrow d_I(gx) - d_I(kl) = T(kl) - T(gx) \neq 0 \Rightarrow d_I(gx) \neq d_I(kl)$ . Thus  $G$  is not  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper ivcf graph. Which is a contradiction. Hence  $T$  is constant function.

**Theorem 3.8.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . Then  $T$  is constant function if and only if the following are equivalent

- (i)  $G$  is a  $(a_1 e^{ib_1\pi}, a_2 e^{ib_2\pi})$ - proper ivcf graph
- (ii)  $G$  is an  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper ivcf graph.
- (iii)  $G$  is a totally  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph.

**Proof:** Proof of the theorem is similar to Theorem 3.7.

**Theorem 3.9.** If ivcf graph  $G$  is both edge proper and totally edge proper then  $T$  is constant function.

**Proof:** Let  $G$  be a  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ - edge proper and totally  $((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph. Then  $d_I(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  and  $td_I(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$  for all  $gx \in I$ . Now  $td_I(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi}) \Rightarrow d_I(gx) + T(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$  for all  $gx \in I \Rightarrow (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + T(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi})$  for all  $gx \in I$ . Hence  $T(gx) = ((r_1 + j_1) e^{i(s_1+k_1)\pi}, (r_2 + j_2) e^{i(s_2+k_2)\pi}) - (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$ . Thus  $T$  is constant function.

#### 4. Properties of Edge Proper and Edge totally Proper Interval-Valued Complex Fuzzy Graph

**Definition 4.1.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . If the underlying graph  $G^*$  is a proper graph, then  $G$  is said to be partially proper ivcf graph.

**Theorem 4.2.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$  such that  $T$  is a constant function. Then  $G$  is a proper ivcf graph if and only if  $G^*$  is a partially proper ivcf graph.

**Proof:** Proof is Obvious.

**Theorem 4.3.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . If  $G$  is both edge proper ivcf graph and edge totally proper ivcf graph, then  $G$  is proper ivcf graph if and only if  $G^*$  is proper graph.

Proof: Assume that  $G$  is both edge proper and edge totally proper ivcf graph then,  $T$  is a constant function. Hence by Theorem 4.2, the result follows.

Theorem 4.4. Let  $G: (S, T)$  be a proper ivcf graph on  $G^*(F, I)$ . Then  $G$  is an edge proper ivcf graph if and only if  $T$  is a constant function.

Proof. Let  $G: (S, T)$  be  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  - proper ivcf graph. Then  $d_I(j) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  for all  $j \in F$ . Assume that  $T$  is a constant function. Let  $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$  for all  $gx \in I$ . By definition,  $d_I(gx) = d_I(g) + d_I(x) - 2T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - 2(j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = 2((r_1 - j_1) e^{i(s_1-k_1)\pi}, (r_2 - j_2) e^{i(s_2-k_2)\pi})$  for all  $gx \in I$ . Hence  $G$  is an edge proper ivcf graph.

Conversely, assume that  $G$  is edge proper ivcf graph. Let  $d_I(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$  for all  $gx \in I$ . By definition,  $d_I(gx) = d_I(g) + d_I(x) - 2T(gx)$

$$\Rightarrow (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - 2T(gx)$$

$$\Rightarrow 2T(gx) = 2(r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$$

$$\Rightarrow 2T(gx) = ((2r_1 - j_1) e^{i(2s_1-k_1)\pi}, (2r_2 - j_2) e^{i(2s_2-k_2)\pi})$$

$$\Rightarrow T(gx) = \left( \frac{(2r_1 - j_1) e^{i(2s_1-k_1)\pi}}{2}, \frac{(2r_2 - j_2) e^{i(2s_2-k_2)\pi}}{2} \right). \text{ Hence } T \text{ is a constant function.}$$

Theorem 4.5. Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$  such that  $T$  is a constant function. If  $G$  is proper ivcf graph, then  $G$  is edge totally proper ivcf graph.

Proof. Given that  $T$  is a constant function. Take  $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$ , for all  $gx \in I$ . Assume that  $G$  is proper ivcf graph. Then  $d_I(j) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ , for all  $j \in F$ . Now,  $td_I(gx) = d_I(g) + d_I(x) - T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = (2r_1 e^{i2s_1\pi}, 2r_2 e^{i2s_2\pi}) - (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = ((2r_1 - j_1) e^{i(2s_1-k_1)\pi}, (2r_2 - j_2) e^{i(2s_2-k_2)\pi})$ . Hence,  $G$  is edge totally proper ivcf graph.

Theorem 4.6. Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$  such that  $T$  is a constant function. If  $G$  is proper ivcf graph, then  $G$  is a  $(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  - edge proper ivcf graph.

Proof. Proof is Obvious.

Remark 4.7. The converse of theorem 4.6 need not be true.

Theorem 4.8. Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$  with  $G^*$  is  $k$ -proper. Then  $T$  is a constant function if and only if  $G$  is both proper and edge totally proper ivcf graph.

Proof. Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$ . Let  $G^*$  be a  $k$ -proper. Assume that  $T$  is a constant function. Let  $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$ , for all  $gx \in I$ . Now,  $d_I(j) = \sum T(gx)$ , for all  $j \in F \Rightarrow d_I(j) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$ , for all  $j \in F$

$$\Rightarrow d(j) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) d_{G^*}(j), \text{ for all } j \in F$$

$$\Rightarrow d(j) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) k, \text{ for all } j \in F. \text{ Hence, } G \text{ is proper ivcf graph. Now,}$$

$$td_I(jf) = \sum_{k \neq j} T(jk) + \sum_{k \neq f} T(kf) + T(jf), \text{ for all } jf \in I.$$

$$= \sum_{k \neq f} (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) + \sum_{k \neq j} (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}), \text{ for all } jf \in I.$$

$$= (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) (d_{G^*}(j) - 1) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) (d_{G^*}(f) - 1) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$$

$$= (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) (k - 1) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) (k - 1) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) \text{ for all } jf \in I$$

$$= 2(j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) (k - 1) + (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) \text{ for all } jf \in I.$$

Hence  $G$  is edge totally proper ivcf graph.

Conversely, assume  $G$  is both proper and edge totally proper ivcf graph. Since  $G$  is edge proper,  $d_I(f) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  for all  $f \in F$ . Since  $G$  is edge totally proper,  $td_I(gx) = (h_1 e^{iw_1\pi}, h_2 e^{iw_2\pi}) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - T(gx)$ , for all  $gx \in I$ .  $T(gx) = 2(r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - (h_1 e^{iw_1\pi}, h_2 e^{iw_2\pi}) = ((2r_1 - h_1) e^{i(2s_1-w_1)\pi}, (2r_2 - h_2) e^{i(2s_2-w_2)\pi})$  for all  $gx \in I$ . Hence  $T$  is constant function.

Definition 4.9. Let  $G^*: (F, I)$  be a graph. Then  $G^*$  is said to be an edge proper graph if each edge in  $G^*$  has same degree.

**Definition 4.10.** If  $G$  is both an edge proper ivcf graph and partially edge proper ivcf graph, then  $G$  is said to be full edge proper ivcf graph.

**Theorem 4.11.** Let  $G: (S, T)$  be an ivcf graph on  $G^*(F, I)$  such that  $T$  is a constant function. If  $G$  is full proper ivcf graph, then  $G$  is full edge proper ivcf graph.

**Proof.** Since  $T$  is constant function, Let  $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$  for all  $gx \in I$ . Assume that  $G$  is full proper ivcf graph. Then  $d_G(j) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$  and  $d_{G^*}(j) = r$  for all  $j \in F$ . So,  $d_{G^*}(gx) = d_{G^*}(g) + d_{G^*}(x) - 2 = 2r - 2$ . Hence  $G^*$  is an edge proper ivcf graph. Now,  $d_1(gx) = d_1(g) + d_1(x) - 2T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - 2(j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = 2(r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - 2(j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = 2((r_1 - j_1) e^{i(s_1 - k_1)\pi}, (r_2 - j_2) e^{i(s_2 - k_2)\pi})$ . Hence  $G$  is an edge proper ivcf graph. Thus,  $G$  is full edge proper ivcf graph.

**Remark 4.12.** The converse of the above theorem is need not be true.

## 5. Conclusion

In this paper, we have defined edge proper and edge totally proper ivcf graph and discussed some of its properties. In future we extend this property for improper ivcf graph and try to find real valued application.

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