

THEORYTICAL APPROACH ON APPLICATION OF GENERALIZED CLOSED SETS ENVIRONMENTAL LIFESTYLE USING r -NEIGHBOURHOOD IDEALSPACES

THARMAR S¹, Veerasivaji R² & Senthil kumar R³

^{1,3}Department of Mathematics, Mohamed Sathak Engineering College, Kilakarai, Tamil Nadu, India.

²Assistant Professor ,Department of Mathematics,Sri Sankara Arts and Science College, Enathur Kanchipuram , India.

Abstract. We introduce in this paper, the new notion of R_1 -g-closed sets and obtain some of its characterizations in ideal r -Neighbourhood space and give nice results on R_1 -g-closed sets with examples. Finally, we discuss application of R_1 -g-closed sets.

Keywords: Relation, I_r -closed set, R_1 -g-closed set, R_1 -g-open set.

1. Introduction

In 1970, the notion of generalized closed sets introduced by Levine [5] and various Generalized concepts in topology were introduced by Levine [5] and various authors were making modifications in generalized concepts. Lin[6] and Yao[9] introduced the concept of rough sets in neighbourhood system and Hosny[2] generated different topologies by the concept of idealization of j -approximation spaces.

In this paper, we introduce the new notion of R_g^I closed sets and obtain some of its characterizations in ideal r -Neighbourhood space and give nice results on R_g^I closed sets with example. Finally, we discuss application of R_g^I closed sets with an example.

1Correspondingauthor: tharmar11585@gmail.com * vracvag@gmail.com,
srisenthil2011@gmail.com

2. Preliminaries

Definition 2.1. [3] A non-empty collection I of subsets of a set U is called an ideal on U , if it satisfies the following conditions.

- (1) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.
- (2) $A \in I$ and $B \subset A \Rightarrow B \in I$.

Definition 2.2. [6] Let U be non-empty finite set and R be an arbitrary binary relation on U . The r -neighbourhood of $x \in U$ ($N_R(x)$) is defined as r -neighbourhood: $N_R(x) = \{y \in U: xRy\}$.

Definition 2.3. [6] Let U be a non-empty finite set and R be an arbitrary binary relation on U and $\sum_r: U \rightarrow P(U)$ be a mapping which assigns for each x_r in U its r -neighbourhood in $P(U)$. The triple (U, R, \sum_r) is called a r -neighbourhood space (in briefly, r -NS).

Theorem 2.4. [6] Let U be a r -NS and $A \subset U$. Then, the collection $\tau_r = \{A \subset U : \forall p \in A, N_R(p) \subset A\}$ is a topology on U .

Definition 2.5. [2] Let U be a r -NS and a subset $A \subset U$ is called r -open set if $A \in \tau_r$ and the complement of r -open set is called r -closed set.

Definition 2.6 [2] Let U be a r -NS and I be an ideal on U . Then, the collections $\tau_r^I = \{A \subset U : \forall p \in A, N_r(p) \cap A' \in I\}$ is a topology on U .

The r -Neighbourhood space with an ideal I is called ideal r -NS.

Definition 2.7 [2]

- (1) Let U be an ideal r -NS and a subset $A \subset U$ is called I_r -open set if $A \in \tau_r^I$ and the complement of I_r -open set is called I_r -closed set.
- (2) $\underline{R}_r^I(A) = \bigcup \{G \in \tau_r^I : G \subset A\} = \text{int}_r^I(A)$ (int_r^I is called I_r -lower approximations)
- (3) $\overline{R}_r^I(A) = \bigcap \{F \in (\tau_r^I)^c : A \subset F\} = \text{cl}_r^I(A)$ (cl_r^I is called I_r -upper approximations).

Lemma 2.8. [2] Let U be an ideal r -NS and $A, B \subset U$. Then

- (1) $\underline{R}_r^I(A) \subset A \subset \overline{R}_r^I(A)$.
- (2) $A \subset B \Rightarrow \underline{R}_r^I(A) \subset \underline{R}_r^I(B)$
- (3) $A \subset B \Rightarrow \overline{R}_r^I(A) \subset \overline{R}_r^I(B)$
- (4) $\overline{R}_r^I(\underline{R}_r^I(A)) = \underline{R}_r^I(A)$.
- (5) $\underline{R}_r^I(\overline{R}_r^I(A)) = \underline{R}_r^I(A)$.

3. RI- g-CLOSED SETS

Definition 3.1 Let U be an ideal r -NS and a subset A_r is said to be R_1 - g -closed set if $\overline{R}_r^I(A_r) \subset O_r$ whenever $A_r \subset O_r$ and O_r is I_r -open. The complement of R_1 - g -closed set is called R_1 - g -open set.

Example 3.2 Let $U = \{\text{Food, Health, Exercise, Sleep}\}$ and we get the relation $R = \{(\text{Food, Food}), (\text{Food, Health}), (\text{Health, Health}), (\text{Health, Exercise}), (\text{Exercise, Exercise}), (\text{Exercise, Sleep}), (\text{Exercise, Health}), (\text{Sleep, Sleep}), (\text{Sleep, Health})\}$ and the ideal $I = \{\emptyset, \text{Health}\}$.

The neighbourhood of Food is $\{\text{Food, Health}\}$, neighbourhood of Health is $\{\text{Health, Exercise}\}$, neighbourhood of Exercise is $\{\text{Health, Exercise, Sleep}\}$ and neighbourhood of Sleep is $\{\text{Health, Sleep}\}$.

The members of I_r -open sets are $\{\text{Food}\}$, $\{\text{Sleep}\}$, $\{\text{Food, Sleep}\}$, $\{\text{Exercise, Sleep}\}$, $\{\text{Food, Exercise, Sleep}\}$, $\{\text{Health, Exercise, Sleep}\}$, \emptyset , U and the members of I_r -closed sets are $\{\text{Food}\}$, $\{\text{Health}\}$, $\{\text{Food, Health}\}$, $\{\text{Health, Exercise}\}$, $\{\text{Food, Health, Exercise}\}$, $\{\text{Health, Exercise, Sleep}\}$, \emptyset , U .

Subset of U (A_r)	$N_r(P), p \in A_r$	$\overline{R_r^I}(A_r)$	$A_r \subset O_r$ and O_r is I_r -open	Member of R_g^I closed
{Food}	{ Food, Health }	Singleton member {Food}	{Food}, {Food, Sleep}, {Food, Exercise, Sleep}	Yes
{Health}	Two members { Exercise, Health}	Singleton member {Health}	{ Exercise, Health, Sleep}	Yes
{Exercise}	{Exercise, Health, Sleep}	{ Health, Exercise}	{Exercise, Sleep} and {Food, Exercise, Sleep} and {Health, Exercise, Sleep}	No
Subset of U (A_r)	$N_r(P), p \in A_r$	$\overline{R_r^I}(A_r)$	$A_r \subset O_r$ and O_r is I_r -open	Member of R_g^I closed
{Sleep}	{Health, Sleep}	{ Exercise, Sleep, Health, }	{Sleep}	No
{ Food, Health}	{ Food, Health } and { Health, Exercise}	{ Food, Health }	U	Yes
{Food, Exercise}	{ Food, Health } and {Health, Exercise, Sleep}	{Food, Health, Exercise}	{Food, Exercise, Sleep} and U	No
{Food, Sleep}	{ Food, Health } and {Health, Sleep}	U	{Food, Sleep} and {Food, Exercise, Sleep}, U	No
{ Exercise	{ Exercise,	{ Exercise,	{Exercise,	Yes

Health }	Health}, {Exercise, Health, Sleep}	Health}	Health, Sleep}, U	
{Health, Sleep}	{ Exercise, Health}, {Health, Sleep}	{ Exercise, Health, Sleep}	{Exercise, Health, Sleep}, U	Yes
{Exercise, Sleep}	{Health, Sleep}, {Health, Exercise, Sleep}	Health, Exercise, Sleep}	{Exercise, Sleep}, {Food, Exercise, Sleep}, {Health, Exercise, Sleep}, U	No
{Food, Exercise, Health}	{ Exercise, Food, Health}, { Health}, {Health, Sleep, Exercise}	{Food, Exercise, Health}	U	Yes
{Food, Exercise, Sleep}	{ Food, Health }, {Health, Sleep} {Sleep, Exercise, Health}	U	{Food, Sleep, Exercise}, U	No
{ Health, Sleep, Exercise}	{Exercise, Health, Sleep}, { Health, Exercise}, {Health, Sleep}	{Exercise, Health, Sleep}	{ Sleep, Exercise, Health}, U	Yes
{ Health, Food, Sleep}	{ Exercise, Health}, {Health, Sleep}, {Food, Health}	U	U	Yes
Subset of U (A_r)	N_r(P), p ∈ A_r	$\overline{R_r^I}(A_r)$	A_r ⊂ O_r and O_r is I_r-open	Member of R_g^I closed
∅	∅	∅	All I _r -open sets	Yes
U	U	U	U	Yes

Theorem 3.3 Every I_r-closed set is R_r-g-closed.

Proof. The indication comes from the reality that $\overline{R_r^I}(A_r) = A_r$.

Remark 3.4 In the Example 3.2, the member {Exercise} in R_r-g-closed set but not in I_r-closed.

Theorem 3.5 Let U be an ideal r-NS and A_r ⊂ U. Then $x_r \in \overline{R_r^I}(A_r)$ if and only if O_r ∩ A_r ≠ ∅ for every I_r-open set O_r containing x_r.

Proof. Let O_r be an I_r-open set such that x_r ∈ O_r and O_r ∩ A_r = ∅. Then A_r ⊂ U - O_r and U - O_r is I_r-closed set and hence $\overline{R_r^I}(A_r) \subset U - O_r$. Since x_r ∉ U - O_r which implies x_r ∉ $\overline{R_r^I}(A_r)$. Conversely, let x_r ∉ $\overline{R_r^I}(A_r)$. Then there exists an I_r-closed set F_r such that A_r ⊂ F_r and x_r ∉ F_r. Hence (U - F_r) ∩ A_r = ∅.

Theorem 3.6 If A_r and B_r are R_r-g-closed then

- (1) $A_r \cup B_r$ is R_I -g-closed.
- (2) $A_r \cap B_r$ is R_I -g-closed.

Proof. The reality, $\overline{R_r^I}(A_r \cup B_r) = \overline{R_r^I}(A_r) \cup \overline{R_r^I}(B_r)$ and $\overline{R_r^I}(A_r \cap B_r) \subset \overline{R_r^I}(A_r) \cap \overline{R_r^I}(B_r)$ gives the proof.

Remark 3.7 The collection of R_I -g-closed sets form a topology.

Theorem 3.8 If A_r is R_I -g-closed and $A_r \subset B_r \subset \overline{R_r^I}(A_r)$ then B_r is R_I -g-closed.

Proof. Let $B_r \subset O_r$ and O_r be I_r -open. Then $A_r \subset U$ and A_r is R_I -g-closed. Therefore $\overline{R_r^I}(A_r) \subset U$ which implies $\overline{R_r^I}(B_r) \subset U$. Hence B_r is R_I -g-closed.

Theorem 3.9 Let U be an ideal r -NS and $A_r \subset U$. Then A_r is R_I -g-open if and only if $F_r \subset \underline{R_r^I}(A_r)$ whenever $F_r \subset A_r$ and F_r is I_r -closed.

Proof. Let A_r be R_I -g-open and $F_r \subset A_r$ and F_r be I_r -closed. Then $U - A_r \subset U - F_r$ and $U - F_r$ is I_r -open. Since $U - A_r$ is R_I -g-closed, $\overline{R_r^I}(U - A_r) \subset U - F_r$ and $U - \underline{R_r^I}(A_r) = \overline{R_r^I}(U - A_r) \subset U - F_r$. Hence $F_r \subset \underline{R_r^I}(A_r)$.

Conversely, let $U - A_r \subset O_r$ where O_r is I_r -open. Then $U - O_r \subset A_r$ and $U - O_r$ is I_r -closed. By hypothesis, we have $U - O_r \subset \underline{R_r^I}(A_r)$ and hence $\overline{R_r^I}(U - A_r) = U - \underline{R_r^I}(A_r) \subset O_r$. Hence A_r is R_I -g-open.

Theorem 3.10 Let U be an ideal r -NS and $A_r \subset U$.

- (1) A_r is R_I -g-closed,
- (2) $\overline{R_r^I}(A_r) \subset O_r$ such that $A_r \subset O_r$ and O_r is I_r -open,
- (3) $\overline{R_r^I}(A_r) \cap F_r = \emptyset$ whenever $A_r \cap F_r = \emptyset$ and F_r is I_r -closed.

The statements (1), (2) and (3) are equivalent.

Proof. (1) \Leftrightarrow (2) Directly we get by the definition 3.1.

(2) \Rightarrow (3) Let $A_r \cap F_r = \emptyset$ and F_r be I_r -closed. Then $A_r \subset U - F_r$ and $U - F_r$ is I_r -open. By (2), $\overline{R_r^I}(A_r) \subset U - F_r$. Hence $\overline{R_r^I}(A_r) \cap F_r = \emptyset$.

(3) \Rightarrow (1) Let $A_r \subset O_r$ where O_r is I_r -open. Then $A_r \cap (U - O_r) = \emptyset$ and $U - O_r$ is I_r -closed. By (3), $\overline{R_r^I}(A_r) \cap (U - O_r) = \emptyset$ which implies that $\overline{R_r^I}(A_r) \subset O_r$. Hence A_r is R_I -g-closed.

4. Application

In the example 3.2, the members of I_r -open sets are $\{\text{Food}\}$, $\{\text{Sleep}\}$, $\{\text{Food, Sleep}\}$, $\{\text{Exercise, Sleep}\}$, $\{\text{Food, Exercise, Sleep}\}$, $\{\text{Health, Exercise, Sleep}\}$, \emptyset , U and the members of I_r -closed sets are $\{\text{Food}\}$, $\{\text{Health}\}$, $\{\text{Food, Health}\}$, $\{\text{Health, Exercise}\}$, $\{\text{Food, Health, Exercise}\}$, $\{\text{Health, Exercise, Sleep}\}$, \emptyset , U .

The members of R_1 -g-open sets are $\{\text{Food}\}$, $\{\text{Exercise}\}$, $\{\text{Sleep}\}$, $\{\text{Food, Exercise}\}$, $\{\text{Food, Sleep}\}$, $\{\text{Exercise, Sleep}\}$, $\{\text{Food, Exercise, Sleep}\}$, $\{\text{Health, Exercise, Sleep}\}$, \emptyset , U and the members of I_r -closed sets are $\{\text{Food}\}$, $\{\text{Health}\}$, $\{\text{Food, Health}\}$, $\{\text{Health, Exercise}\}$, $\{\text{Food, Health, Exercise}\}$, $\{\text{Health, Exercise, Sleep}\}$, \emptyset , U .

Therefore the members in difference of I_r -open sets and R_1 -g-open sets are $\{\text{Exercise}\}$ and $\{\text{Food, Exercise}\}$ and hence the common activity is Exercise and hence Exercise is base for all human activities.

5. Conclusion

Further study about r -neighbourhood space, may give many solutions for the real life problems.

References

- [1] M. E. Abd El-Monsef, A. S. Salama and O.A. Embaby, Granular computing using mixed neighborhood systems, *Jour. of Inst. of Mathematics & Computer Science (Computer Science Ser.)*, 20(2009), 233–243.
- [2] M. Hosny, Idealization of j -Approximation spaces, *Faculty of sciences and mathematics*, 34(2)(2020), 287-301.
- [3] D. Jankovic, T. R. Hamlet, New topologies from old via ideals, *Amer.Math.Monthly* 97 (1990) 295 310.
- [4] A. M. Kozae, On topology expansions by ideals and applications, *Chaos, Solitons and Fractals*, 13(2002), 55–60.
- [5] N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo* (2)(19)(1970), 89-96.
- [6] T. Y. Lin, Granular computing on binary relations II: Rough set representations and belief functions, in: A. Skowron, L. Polkowski (Eds.), *Rough Sets In Knowledge Discovery*, Physica-Verlag, (1998), 121–140.
- [7] A. S. Salama, Some topological properties of rough sets with tools for data mining, *Int. J. Compu. Sci.*, 8(2011), 588–595.
- [8] O. A. E. Tantawy, Heba. I. Mustafa, On rough approximations via ideal, *Inform. Sci.*, 251(2013), 114–125.
- [9] Y. Y. Yao, Rough sets, neighborhood systems and granular computing, *Proceedings of the IEEE Canadian Conference on Electrical and Computer Engineering*, (1999), 1553–1559.
- [10] W. Zhu, Topological approaches to covering rough sets, *Inform. Sci.*, 177(2007), 1499–15.