Comparative efficiency of application of the maximum clique method in searching for critically important objects of the gas industry

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Abstract. The article proposes the use of a model of the "defender - attacker - defender" type to determine the possibilities of the gas industry to reduce gas shortages for consumers in the event of failure of its critical facilities. The calculation scheme of the gas transmission network in this study is presented in the form of a directed graph, in the nodes of which there are objects of production, consumption, storage of gas and nodal compressor stations. All found cliques are analyzed, and a list of measures aimed at reducing possible gas shortages among consumers is compiled. The article presents the results of the analysis. The results of applying the method of determining the maximum cliques to search for critical objects of the gas industry were evaluated in comparison with the results obtained earlier on this topic. Conclusions are drawn about the feasibility of using the proposed method to determine the critically important objects of the gas industry.

1 Introduction

The development of Russia's gas transmission network is associated both with a decline in production at old operating fields, and with the need to enter new areas of gas production. In addition, the network configuration is affected by changes in the volumes of consumption of the country's subjects and the reorientation of export supplies of pipeline gas. Thus, the gas transmission network is constantly changing over time both in terms of its configuration and the load of its elements.

A complex gas transmission network is a critical infrastructure and contains a significant number of facilities, the performance of which critically affects the overall performance of the network and, accordingly, its ability to reliably provide uninterrupted gas supply to consumers. The most large-scale accidents in power systems occur as a result of the failure of various critical objects of these systems [1, 2]. One of the results of such accidents is significant damage to consumers, expressed in the shortage of fuel and energy resources. Therefore, the identification of critical objects and their combinations in power systems in order to develop measures aimed at reducing the significance of these objects is an urgent task.

Currently, various studies are underway in the world to determine the critical elements of energy systems and other critical infrastructures.

A number of works are devoted to the modeling of energy systems as critical infrastructures [3, 4]. Researchers also consider the problems of vulnerability of critical energy infrastructures to terrorist attacks on them and the methodology of risk analysis for systems of interdependent critical infrastructures under various extreme weather conditions [5-10].

Previously, studies were conducted to identify critical facilities in the gas transmission network. A list of intersections of main gas pipelines in the Unified Gas Supply System of Russia has been determined, the disruption of which will lead to a relative shortage of daily gas supplies throughout the system as a whole in the amount of 5% or more [11]. Studies have been carried out to search for and determine combinations of individual sections of main gas pipelines, the simultaneous disruption of which can lead to a significant shortage of daily gas supplies through the system (5% or more) [12, 13]. Studies have been carried out to determine critically important objects in the electric power industry [14]. Taking into account the accumulated experience and on the basis of an analysis of studies currently being carried out in the world, in [11, 15] a methodology was formulated for the formation of lists of critically important objects of energy systems from the standpoint of ensuring the operability of these systems using the example of the Russian gas industry.

In these works, the definition of critically important objects and their combinations is carried out by enumeration. Multi-iteration studies were carried out, in which all elements of the computational network and their combinations were turned off in turn. Thus, those elements of the network and their combinations were determined, the failure of which would lead to the greatest shortage of gas in the system. Due to limitations in the amount of analyzed information, situations with a simultaneous disconnection of three or more network

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elements were not considered. However, due to the specifics of the functioning of the gas transmission network, such emergencies are possible. Therefore, in order to more fully take into account various factors in the study of critical facilities, in this paper, the authors use the “clique” method to find the maximum number of interconnected nodes and main gas pipelines, the failure of which can cause significant damage to the system in terms of reducing gas supplies to consumers.

The problem of a clique is formulated within the framework of the methodology for modeling attacks on infrastructure systems. This technique connects two sides in a single mathematical formulation: the attack side and the defense side. Such models are based on the class of Stackelberg games [16] with prohibitions on networks, in which two actors - the leader and the follower - pursue, as a rule, opposite interests. A detailed description of mathematical models of this type of problems can be found in [17]. We also note that the mathematical description of such problems can be reduced to models of multilevel optimization of the type: attacker-defender, defender-attacker and defender-attacker-defender, which are widely used in scientific research in modeling threats and response measures at various critical infrastructure facilities [18 -20].

2 Statement of the problem of finding the maximum clique

The calculation scheme of the gas transmission network used in this study fully reflects all the technological features of a real gas transmission network and includes 388 nodes, including: 96 consumers corresponding to gas consumption regions and individual large industrial consumers, 33 gas fields, 29 underground storage facilities gas (UGS), 230 nodal compressor stations.

The scheme takes into account the peculiarity of the system, which is that the units corresponding to UGS facilities can be involved in the system both as consumers of gas and as its sources. Communication between the nodes is carried out through 755 lines, reflecting the existing main gas pipelines. The initial data on the functioning of the gas transmission network, such as the volume of gas production in the fields, the volume of gas consumption, the throughput of gas pipelines, are taken in accordance with specialized statistical information [21-23].

To solve the problem of supplying consumers with the required volumes of gas, the statement of the problem of finding the maximum flow is used. It is necessary to find the maximum possible total volume of gas that can be passed through a network with given characteristics of connections between nodes, taking into account the established throughput capacities of the lines, the available production volumes and the given consumption volume. Solving the problem of finding the maximum flow through the network allows you to determine whether the system is able to provide consumers with the required volume of gas. This statement does not take into account the gas consumption for the own needs of the gas transmission network. This gas consumption in the current study is taken into account in accordance with the results of numerous earlier feasibility studies of the functioning of the Russian gas transmission system [24].

The statement of the problem of finding the maximum flow has the following form:

$$\max f,$$

$$\sum_{j\in I}\bar{u}_{ij}x_{ij}=\begin{cases}-f, i=i_O,\\ 0, i\notin\{i_O,i_S\},\ i\in I,\\ f, i=i_S,\end{cases}\tag{1}$$

$$x_{ij}=x_{ji},\quad i,j\notin\{i_O,i_S\},$$

$$0\le x_{ij}\le d_{ij},\quad i\in I, j\in I.$$

After solving problem (1), information about the ability of the gas transmission system to close the load from consumers becomes available. Bottlenecks can also be identified, i.e. sections of the gas pipeline with maximum load, and sections with a large margin in terms of throughput. These data may contribute to making adjustments to the characteristics of the gas transmission system in the direction of increasing or decreasing the throughput of certain gas pipelines, taking into account the load from consumers.

For the Russian gas transmission system, it is proposed to define the most vulnerable objects for attack as combinations of nodes in which each node is connected to each node. In other words, we pose the problem of finding a clique of maximum size [25]. For clarification, we use the terminology from graph theory.

The gas transportation system can be represented as a directed graph, in the nodes of which there are fields, gas consumers, underground storage facilities, and compressor stations. The edges of such a graph are gas pipeline lines. The attacker's main goal is to cause maximum damage to the gas transmission system, i.e. reduction of the maximum gas flow through the gas pipeline network by attacking and incapacitating the key or linear objects of the system.

In this case, the attacking side solves the problem of finding a clique of the maximum size to disable the largest number of interconnected objects of the gas transmission system. Below is a list of the main notation for the mathematical description of the problem of finding the maximum clique.

G = (V, E) - arbitrary undirected and weighted graph,
V = {1, 2, ..., n} - set of vertices (nodes) of a graph G,
E = V x V - set of graph edges G,
w = (w_1, w_2, ..., w_n) - scale vector, $w_i > 0$, i = 1,..., n,
G = (V, E) - padded graph to graph G,
$E=\{(i,j): i,j\in V, i\neq j, (i,j)\notin E\}$.

To find a clique of the maximum size, it is necessary to solve the following problem:

$$\max_{y\in Y}F(y),$$

$$F(y)=\sum_{i=1}^{n}w_i y_i,\tag{2}$$

$$Y=\left\{y\in\{0,1\}^n, y_i+y_j\le 1\quad\forall(i,j)\in\bar{E}\right\}.$$

The found solution of problem (2) with w_i = 1 determines the set of objects of the gas transmission system that form a clique of the maximum size. In the case of a non-unique solution, we will get a whole set of such cliques, which can serve as the basis for further more detailed attack planning with maximum damage on the part of the attacker.

Taking into account (1) and (2), the attacker-defender problem for the Russian gas transmission system can be represented as follows:

$$\begin{gathered}
\min_{y}\max_{x} f, \\
y \in Arg\max\{C(y) : y \in Y\}, \\
r_{ij} = r_{ji} = \begin{cases} 1,\ (i,j) : y_i + y_j \ldots 1,\ u_{ij} = u_{ji} = 1, \\ 0,\ (i,j) : y_i + y_j < 1,\ u_{ij} = u_{ji} = 1, \end{cases} \quad i,j \in I \setminus \{i_O, i_S\}, \\
r_{i_O,j} = r_{j,i_S} = 0, \qquad j \in I, \\
\sum_{j\in I} \overline{u}_{ij} x_{ij} = \begin{cases} -f,\ i = i_O, \\ 0,\ i \notin \{i_O, i_S\}, \quad i \in I, \\ f,\ i = i_S, \end{cases} \\
x_{ij} = x_{ji}, \quad i,j \notin \{i_O, i_S\}, \\
0 \le x_{ij} \le d_{ij}(1 - r_{ij}), \quad i \in I, j \in I,
\end{gathered} \tag{3}$$

y {0,1} – a vector that defines the nodes to which the attack is directed (the attacker's plan); *Arg max{C(y) :y ∈ Y}* set of cliques of maximum size; r_{ij} , *i,j ∈ I* — parameters that define the edges that enter or exit the attacked nodes and are removed from the network along with them (the consequences of the attack). In accordance with the statement, the attacker, based on the solution of problem (3), chooses a clique whose removal from the network causes maximum damage to the throughput of the gas transmission network. The nodes included in the clique are removed from the network along with all edges adjacent to them. Thus, an attack on a node, firstly, disables the gas transmission network facilities located in this node, and, secondly, eliminates the possibility of gas transit through this node.

In case of disruption of gas supplies to consumers due to the shutdown of the click network, “bottlenecks” are formed, i.e. the most gas-loaded sections of the network. An increase in the throughput capacity of such sections, hypothetically, can reduce the shortage of gas among consumers. Such a short-term increase in the throughput capacity of a section of the main gas pipeline is possible with an increase in operating power at large main compressor stations [24]. The result of such an increase in power will be an increase in the working pressure in the gas pipeline, due to which the throughput of the section of the main gas pipeline can be increased within 10%. As a result (by using the technical capabilities of the gas transmission network), the problem of minimizing gas shortages for consumers in the event of a failure of the click network is solved.

To take into account the possibilities for increasing the throughput of sections of the gas transmission network, the original model (1)-(3) is modified into a model of the "defender-attacking-defender" type as follows.

After solving problem (3), the obtained optimal flows along the gas pipeline lines are fixed, we denote them by x^{cl} . The received maximum flow is also fixed f^{cl}.

Next, we consider new variables for flows x with given two-sided constraints. Additional restrictions are also introduced for the maximum flow of a new task. Bilateral limits for line capacities have the following form :

$$x^{cl} \le x \le 1.1x^{cl}. \tag{4}$$

These restrictions make it possible to take into account an increase in the volume of natural gas transportation through a certain line by 10%.

The minimum length flow is considered as the objective function:

$$z = \sum_{i\in I}\sum_{j\in I} L_{ij}(x_{ij} - x_{ij}^{cl}) \to \min, \tag{5}$$

L_{ij} – line length *(i,j), i ∈ I, j∈I.* In this way, the transportation distance of the additional volume of natural gas is minimized.

An additional condition for the maximum flow is introduced:

$$f \ge c_k \cdot f^{cl}. \tag{6}$$

In order to increase the maximum flow with a minimum "penalty" for its transportation, we formulate the problem according to the following principle: the minimization of the "penalty" is carried out in the objective function, and a suitable increase in the maximum flow is carried out using the introduced restriction $k{*}f^{cl}$.

The constant c_k indicates the possible increase in the maximum flow.

The Russian gas transportation system model (3)-(6) is described by means of the AIMMS algebraic modeling system [26]. In this system, the problem of the maximum flow and the maximum clique of nodes was also solved. The calculations were carried out on a computer equipped with an 8-core AMD FX-8350 processor (clock frequency of each core is 4 GHz) and 8 GB of RAM.

3 Calculation results

With the help of the developed mathematical models on the updated calculation scheme, as a result of a number of calculation studies, the following results were obtained.

The maximum clique for the Russian gas transmission network is 3 nodes, i.e. up to a maximum of three objects of the gas transmission system, are physically interconnected (each with each other) by gas pipelines.

Combinations of interconnected nodal compressor stations forming cliques were determined [27]. A total of 45 cliques were found, consisting of three elements. Of the received cliques, six were selected, the failure of which will lead to a gas deficit of 11–39%. It should be noted that some cliques contain critical objects found in previous studies [15].

989 pairs of cliques were received, the exclusion of which creates a shortage of gas for consumers. The failure of the largest of these pairs creates a significant synergistic effect: the first clique creates a gas deficit for consumers of 39%, the second - 11%, both cliques are unrelated. With their joint exclusion from the network, a gas deficit of 61% arises in the system as a whole (gas consumers in such a situation will not receive about 1.3

billion m^3 of gas per day). 11 pairs of cliques were found, a joint exclusion from the network, which leads to a gas deficit of 50-53% for the system as a whole. 64 cliques pairs with a possible gas deficit of 40-49%, 211 pairs of cliques with a possible gas deficit of 30-39%, 318 pairs of cliques with a possible gas deficit of 20-29%, 384 pairs of cliques with a possible gas deficit of 14-19 %.

A series of calculations was carried out to determine the possibilities for reducing gas deficits for consumers in the event of a clique failure. As the calculation scenarios, the situation with the successive failure of each of the previously received cliques was adopted. Thus, using model (3)-(6), 45 calculations were carried out with the aim of minimizing gas shortages for consumers as much as possible at the lowest cost.

As a result, in 9 cliques, it was possible to provide consumers with gas in full. These cliques are characterized by their low significance relative to the previously found cliques; the failure of each of them leads to a gas deficit of up to 5% in the system as a whole.

It was almost impossible to bypass the two cliques with the largest gas deficits (located in the areas of gas supply to the network from the largest operating fields), the decrease in the deficit in these cases reached 1% and 5%, respectively, with gas deficits from the failure of cliques of 39% and 28 %.

For 10 cliques, there was a slight decrease in deficits, up to 1%. Also, by 21 cliques, an increase in the throughput capacity of busy sections of the network by up to 10% did not lead to a decrease in gas shortages among consumers.

Only in three cliques it was possible to achieve a significant reduction in the gas deficit among consumers in the range of 7-12% for the system as a whole.

When analyzing the consequences for consumers from disruptions in the operation of critical facilities in the current configuration of the Russian gas transmission network, 27 sections were found where it is expedient with a high degree of invariance (from 25 to 48% of the scenarios under consideration) to expand capacity. In contrast to the study, which determined invariant measures to reduce the importance of critical network facilities [28], in the case of cliques, it was almost impossible to obtain invariant measures aimed at reducing gas shortages in consumers when cliques fail.

4 Conclusion

Identification of the problem of combining objects in the gas transmission network makes it possible to plan measures to ensure the reliability of gas supply to consumers. The article considers the application of methods for determining the maximum cliques to search for specific sections of the main gas pipelines of the gas transmission network.

As a result of the study, all connected sections of the network were identified and an analysis of the possible consequences for the systems in case of their occurrence was carried out.

First, it should be noted that the maximum sizes of cliques are 3 and 7 elements, respectively, for nodal and edge cliques.

Secondly, gas deficiencies arising from the occurrence of cliques found from the composition, reducing the gas deficit arising from the emergence from the totality of system compounds and their combinations [12, 15].

The results of calculations of applications of the results of previous studies, performed by various methods, models of element-by-element enumerations [12, 15], methods for determining synergistic effects [13].

Also, in the network under consideration, in order to obtain combinations consisting of three objects, it is necessary to carry out 9660036 calculations by element-by-element enumeration. With an average calculation time of one option of 6 seconds, it will take about 680 days to obtain the results of such a study. The study of the gas transmission network using the maximum clique search method was carried out much faster.

Thus, the presented study and calculation results confirmed the applicability of the maximum demand method to attract attention in the gas industry. This method can be used when looking for item captures in other network infrastructure captures.

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