Mathematical Model of Hot Water Boiler and its Experimental Validation

Sergey Gordin[*], *Ilya* Zaychenko, *Alena* Zhivotova, and *Vitaliy* Stolyarov

Komsomolsk-na-Amure State University, Editorial Department, 681013 Lenina str., 27, Russia

Abstract. Low efficiency of isolated heat supply systems based on coal-fired hot water boilers requires the search of feasible control systems reducing incomplete combustion and exhaust heat losses. The paper considers a mathematical model enabling to calculate burned coal weight and boiler efficiency by a measurement set of various parameters. The proposed model is successfully validated on an experimental unit simulating a heat supply system and can be used to develop various control systems for hot water boilers, including adaptive ones.

1 Introduction

The Far East fuel and energy industry produces most of its heat energy by burning coal in hot water and steam boilers. As a rule, coal consumption for heat generation is determined in accordance with boilers' performance charts basing on specific fuel consumption in proportion to the heat output. But as it is proved in practice, coal consumption accounting on the basis of specific consumption is not accurate and the discrepancy with the actual consumption can be up to 10% [1, 2]. Coal consumption accounting based on the balance between purchases and stocks in storage may be the most accurate, but this method is not prompt and therefore cannot be used to analyze and evaluate energy efficiency solutions.

Coal's physical properties are such that it is difficult to measure its consumption directly, like gas consumption. At boiler houses with manual feeding the consumption of coal is measured in shovels and wheelbarrows, at ones with automatic feeding it is measured through the number of feeds per hour or the number of hopper fills per day. At thermal power plants there are some attempts to determine the coal consumption through the power consumed by mills for its grinding, but the most common method is weighing the coal fed into the hopper on conveyor scales. By such methods it is possible to determine average daily coal consumption, which is suitable for general control, but still insufficiently prompt for control tasks. Therefore, the issue of near real-time determination of coal consumption is relevant for many power engineering facilities [3].

During operational tests and boiler operation for the purposes of coal combustion quality control a gas analyzer is used, which determines so-called combustion efficiency *qA* based on Siegert's formula [4] by composition and temperature of exhaust gas:

---

[*] Corresponding author: gordin@knastu.ru

$$\begin{cases} qA = (FT - AT)\cdot\left(\dfrac{A1}{CO_2} + B\right), \% \\ CO_2 = CO_2{}^{\max}\cdot\left(1 + \dfrac{O_2}{O_2{}^{\max}}\right) \end{cases} \tag{1}$$

where FT is exhaust gas temperature, ^0C; AT is environment temperature, ^0C; $A1, B$ are conversion factors depending on fuel type; O_2 is oxygen concentration in exhaust gas, %; CO_2 is carbon dioxide concentration in exhaust gas, %; CO_2^{max} is maximum CO_2 concentration in exhaust gas for a given fuel type, %; O_2^{max} is oxygen concentration in the air, %.

Siegert's formula allows estimating incomplete combustion and heat loss with exhaust gas, but it is quite approximate [4]. Based on the combustion theory and heating calculations for boilers, a more accurate formula (valid only for coal) was proposed in [5]:

$$\begin{cases} qA = \dfrac{\left[0.205 + 2.783\cdot\alpha + \left(\dfrac{W^p}{C^p} + d^{air}\cdot\alpha\right)\cdot 0.45\right]\cdot FT - 2.757\cdot AT}{81 - 6\cdot\dfrac{W^p}{C^p}}, \% \\ \alpha = \dfrac{1659 - 8.4\cdot CO}{79\cdot\left(21 - O_2 + CO\right) - 0.5\cdot CO} \end{cases} \tag{2}$$

where W^p is moisture concentration in coal, %; C^p is carbon concentration in coal, %; d^{air} is humidity, %; α is excess air coefficient in exhaust gas; CO is proportion of carbon monoxide in exhaust gas, %.

In addition to incomplete combustion and heat loss with exhaust gas, the boiler efficiency is also reduced by losses with ash residue removal, including physical underburning of fuel, as well as losses through the boiler envelope, which in total can reach 5...7% of combusted fuel calorific capacity [6].

The research goal is to develop a mathematical model of conversion of chemical energy of coal burned in a hot water boiler into energy carrier heat and to estimate the actual efficiency of a coal-fired hot water boiler and fuel firing rate by results of mathematical modeling and laboratory tests basing on available measurements of various physical values, which will allow to solve the task of prompt determination of boiler energy efficiency and will create a basis for development of various solutions to improve it.

2 Mathematical model

The mathematical model is based on the general combustion theory [6], and thermophysical parameters of various substances are taken as per the data of the Department of Water and Fuel Technology of Moscow Power Engineering Institute [7].

Let Q^p be the amount of coal combustion heat, Q_1 be the amount of useful heat. Then the boiler efficiency will be equal to

$$\eta = \frac{Q_1}{Q^p} \tag{3}$$

The Q_1 value is commonly determined in boiler houses using a heat meter by measuring boiler's consumption of a heat carrier and the temperatures of the heat carrier at the boiler inlet and outlet. So, determining Q^p is the main task of the mathematical model.

Due to the physical properties of exhaust gas produced by coal combustion (high temperature, many ash impurities), direct measurement of its consumption is much more difficult than measurement of combustion air flow rate.

Therefore, as the main parameters for considered model, the following can be defined:

- boiler inlet air temperature, humidity and flow rate;
- temperature and composition of exhaust gas: *O2*, *CO*.

Measurement of temperature, humidity and air flow rate at the boiler inlet allows to determine the mass of oxygen inflowing to the boiler furnace (ingress rate), while the measurement of temperature and composition of exhaust gas allows to determine its proportion entered into combustion reactions:

$$\begin{cases} C + O_2 = CO_2 + Q_{CO_2},\, Q_{CO_2} = 393.5 \cdot 10^3\, J/mole \\ 2C + O_2 = 2CO + Q_{CO},\, Q_{CO} = 221.0 \cdot 10^3\, J/mole \end{cases} \tag{4}$$

To simplify the model, let us assume that air consists only of nitrogen, oxygen and water vapor, air nitrogen does not enter into combustion reactions, and impurities of other combustible substances in the fuel are not significant. Then it is possible to make a system of equations describing the balance of substances before and after fuel combustion in the boiler furnace:

$$\begin{cases} M^{air} = M_{N_2}^{air} + M_{O_2}^{air} + M_{H_2O}^{air} \\ M^{smoke} = M_{N_2}^{smoke} + M_{O_2}^{smoke} + M_{H_2O}^{smoke} + M_{CO}^{smoke} + M_{CO_2}^{smoke} \\ M_{O_2}^{smoke} = M_{O_2}^{air} - \left(M_{CO_2}^{smoke} + 0.5 \cdot M_{CO}^{smoke} \right) \end{cases} \tag{5}$$

where M^{air} is the mass of combustion air, mol; M^{smoke} is exhaust gas mass, mol; $M_{xx}{}^{air}$, $M_{xx}{}^{smoke}$ are masses of substance *xx* in air and exhaust gas, mol.

Gas analyzer readings of oxygen ($\phi_{O_2}^{smoke}$) and carbon monoxide (ϕ_{CO}^{smoke}) in exhaust gas allow to calculate the proportion, respectively, of oxygen and carbon monoxide in the total mass of exhaust gas. Due to the physical operating principles of different sensors, prior to measurement, exhaust gas is filtered and cooled below the dew point, which causes condensation. Therefore, without compromising accuracy, it can be assumed that the gas analyzer readings are for "dry" smoke, i.e., not including water vapor:

$$\begin{cases} \phi_{O_2}^{smoke} = M_{O_2}^{smoke} / M_{dry}^{smoke} \\ \phi_{CO}^{smoke} = M_{CO}^{smoke} / M_{dry}^{smoke} \\ M_{dry}^{smoke} = M^{smoke} - M_{H_2O}^{smoke} \end{cases} \tag{6}$$

where M_{dry}^{smoke} is molar mass of smoke excluding water, mol.

Equation system (6) allows to calculate the molar mass of "dry" smoke basing on the gas analyzer readings:

$$M_{dry}^{smoke} = \frac{M_{N_2}^{smoke} + M_{O_2}^{smoke} + M_{CO_2}^{smoke}}{1 + \phi_{O_2}^{smoke}} \tag{7}$$

If the gas analyzer is equipped with a carbon dioxide sensor, then

$$\phi_{CO_2}^{smoke} = \frac{M_{CO_2}^{smoke}}{M_{dry}^{smoke}} \tag{8}$$

and, therefore,

$$M_{dry}^{smoke} = \frac{M_{N_2}^{smoke} + M_{O_2}^{smoke}}{1 + \phi_{O_2}^{smoke} - \phi_{CO_2}^{smoke}} \tag{9}$$

Unfortunately, many gas analyzers are not equipped with a carbon dioxide sensor, but calculate CO_2 values, for example, as in Siegert's formula (1). So, another method of $\phi_{CO_2}^{smoke}$ determination will be required that is applicable for use with a gas analyzer without CO_2 sensor.

Combustion reactions (4) and masses of gases M_{CO}^{smoke} and $M_{CO_2}^{smoke}$ allow the mass of combusted fuel to be calculated for a known carbon concentration C^p in that fuel:

$$m^{coal} = \frac{\left(M_{CO}^{smoke} + M_{CO_2}^{smoke}\right) \cdot \mu_C}{C^p} \tag{10}$$

where μ_C is carbon molar mass, kg/mol.

Assuming the fuel is combusted uniformly and at a constant composition, the moisture evaporated from the fuel is proportional to the mass of fuel combusted. This assumption allows us to calculate the mass of moisture evaporated from the fuel and the total mass of water vapor in exhaust gas:

$$M_{H_2O}^{smoke} = M_{H_2O}^{air} + m^{coal} \cdot \frac{W^p}{\mu_{H_2O}} \tag{11}$$

where μ_{H_2O} is water molar mass, kg/mol.

Let us describe the heat balance equation for the boiler:

$$Q^p + Q^{air} + Q^{coal} = Q_1 + Q^{smoke} + Q^{fune} \tag{12}$$

where Q^{air} is heat brought into the boiler furnace with air, J; Q^{coal} is heat brought into the boiler furnace with fuel, J; Q^{smoke} is heat lost with exhaust gas, J; Q^{fune} is heat consumption for moisture evaporation from fuel, J.

$$\begin{cases} Q^{p} = M_{CO_2}^{smoke} \cdot Q_{CO_2} + \frac{1}{2} \cdot M_{CO}^{smoke} \cdot Q_{CO} \\ Q^{air} = \left(M_{N_2}^{air} \cdot \mu_{N_2} \cdot c_{N_2} + M_{O_2}^{air} \cdot \mu_{O_2} \cdot c_{O_2} + M_{H_2O}^{air} \cdot \mu_{H_2O} \cdot c_{H_2O}^{steam} \right) \cdot T^{air} \\ Q^{coal} = \mu_{N_2} \cdot \left(M_{CO}^{smoke} + M_{CO_2}^{smoke} \right) \cdot \left(\frac{1-W^{p}}{C^{p}} \cdot c_{coal} + \frac{W^{p}}{C^{p}} \cdot c_{H_2O} \right) \cdot T^{air} \\ Q^{smoke} = \left(M_{N_2}^{smoke} \cdot \mu_{N_2} \cdot c_{N_2} + M_{O_2}^{smoke} \cdot \mu_{O_2} \cdot c_{O_2} + M_{CO_2}^{smoke} \cdot \mu_{CO_2} \cdot c_{CO_2} + \right. \\ \qquad \left. + M_{CO}^{smoke} \cdot \mu_{CO} \cdot c_{CO} + M_{H_2O}^{smoke} \cdot \mu_{H_2O} \cdot c_{H_2O}^{steam} \right) \cdot T^{smoke} \\ Q^{fune} = \mu_{C} \cdot \left(M_{CO}^{smoke} + M_{CO_2}^{smoke} \right) \cdot \frac{W^{p}}{C^{p}} \cdot \left[\left(373.15\ {}^{0}C - T^{air} \right) \cdot c_{H_2O}^{water} + \cdot c_{H_2O}^{fune} \right] \end{cases} \tag{13}$$

where μ_{xx} is molar mass of substance *xx*, *kg/mol*; c_{xx} is specific heat capacity of substance *xx*, *J/kg·K*; $c_{H_2O}^{steam}$ is specific water vaporization, J/kg·K; $c_{H_2O}^{water}$ is specific heat capacity of water, *J/kg·K*; c_{H_2O} is specific heat capacity of water vapor, *J/kg·K*; T^{air} is air temperature, *K*; T^{smoke} is exhaust gas temperature, *K*.

Combining heat balance equation (12) and system (5), we get formulas for determining $M_{CO_2}^{smoke}$ and M_{CO}^{smoke} values based on measured parameters of air, exhaust gas and useful heat:

$$M_{CO_2}^{smoke} = M_{O_2}^{air} - \frac{1}{2} \cdot M_{CO}^{smoke} \cdot \left(1 + 2 \cdot \frac{\phi_{O_2}^{smoke}}{\phi_{CO}^{smoke}} \right) \tag{14}$$

$$M_{CO}^{smoke} = 2 \cdot \frac{Q_1 + \left(M_{N_2}^{air} \cdot \mu_{N_2} \cdot c_{N_2} + M_{O_2}^{air} \cdot \mu_{CO_2} \cdot c_{CO_2} + M_{H_2O}^{air} \cdot \mu_{H_2O} \cdot c_{H_2O}^{air} \right) \cdot T^{smoke}}{\left(1 + 2 \cdot \frac{\phi_{O_2}^{smoke}}{\phi_{CO}^{smoke}} \right) \cdot \left(\mu_{CO_2} \cdot c_{CO_2} \cdot T^{smoke} - Q_{CO_2} \right) + Q_{CO} +} \cdots$$

$$\frac{- \mu_{C} \cdot \frac{W^{p}}{C^{p}} \cdot M_{O_2}^{air} \cdot \left[\frac{1-W^{p}}{W^{p}} \cdot c_{coal} + c_{H_20} \cdot T^{air} - c_{H_2O}^{air} \cdot T^{smoke} - \left[\left(373.15\ {}^{\circ}K - T^{air} \right) \cdot c_{H_2O}^{water} + \cdot c_{H_2O}^{fune} \right] \right]}{+ \mu_{C} \cdot \frac{W^{p}}{C^{p}} \cdot \left(1 - 2 \cdot \frac{\phi_{O_2}^{smoke}}{\phi_{CO}^{smoke}} \right) \cdot \left[\left(\frac{1-W^{p}}{W^{p}} \cdot c_{coal} + 2 \cdot c_{H_20} \right) \cdot T^{air} - c_{H_2O}^{air} \cdot T^{smoke} - \left(373.15\ {}^{\circ}K \cdot c_{H_2O}^{water} + c_{H_2O}^{fune} \right) \right]} \tag{15}$$

$$\cdots \frac{- M_{O_2}^{air} \cdot Q_{CO_2} - Q^{air}}{- 2 \cdot \frac{\phi_{O_2}^{smoke}}{\phi_{CO}^{smoke}} \cdot \mu_{O_2} \cdot c_{O_2} \cdot T^{smoke} - 2 \cdot \mu_{CO} \cdot c_{CO} \cdot T^{smoke}}$$

Thus, formulas (3), (10), and (14-15) form a system of equations that allow to solve the task of determining the boiler efficiency and fuel consumption for heat generation basing on measurements of exhaust gas temperature and composition, as well as combustion air temperature and flow rate.

To validate the model, we comparatively calculate measurement results of a KVm solid fuel boiler at the coal-fired boiler house in Selikhino village and plot the graphs of *qA* calculation made by gas analyzer Testo 330L (curve 1) and calculation η (curve 2 - calculation without moisture, curve 3 - calculation with moisture in air and fuel) (fig. 1).

As follows from the above data, the general structure of calculation results coincides, but as expected, the gas analyzer calculation gives an overestimated value that is between the calculation not considering losses for moisture evaporation and water vapor heating and the full calculation by the proposed model.

Despite some practical application complexity of the proposed combustion model due to the cumbersome arithmetic, it is more accurate than models implemented in most gas analyzers. From another side, this model has the advantage that it allows determining boiler performance characteristic with a few additional equipment that includes gas analyzer and areometer, and therefore it can be the basis for an adaptive control system of solid fuel boiler.

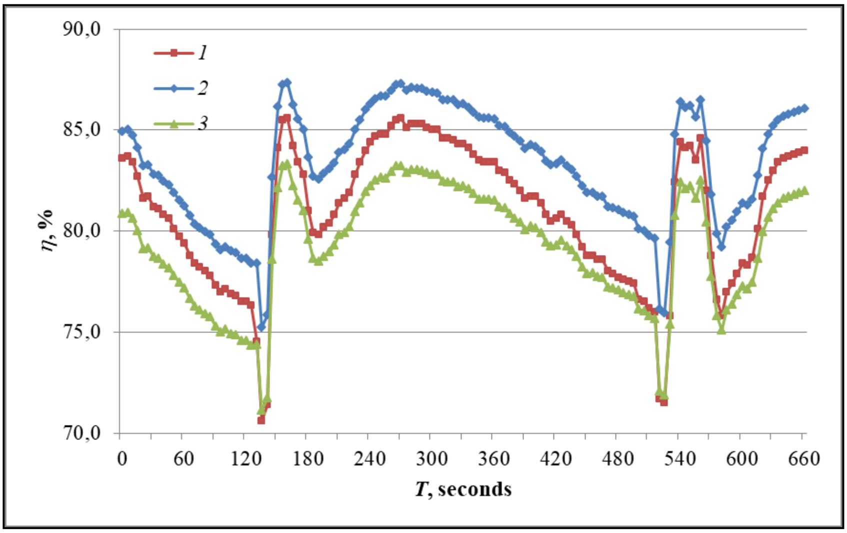

Fig. 1. Comparative calculation of fuel combustion efficiency in the hot water boiler.

3 Simulation results

Modeling is performed for an experimental heat unit simulating an isolated heat supply system and consisting of a solid fuel hot water boiler with manual fuel feeding, a circulation pump with stepped power control, a heat supply network imitator with control of hydraulic resistance and heat losses, as well as an effective load imitator with power take-off control.

Coal consumption is measured by weighing each portion fed into the boiler on table scales, air consumption - by anemometer, temperature and composition of flue gases - by gas analyzer. Other parameters, such as heat carrier temperature and flow rate, are determined by corresponding sensors provided in the unit.

As fuel we use brown coal of BR grade with following parameters (data from the coal certificate): ash content $A^p = 15\%$, humidity $W^p = 30\%$, carbon content $C^p = 1\text{-}A^p\text{-}W^p = 55\%$.

Mathematical simulation is performed for two modes of the experimental heat unit, differing in the positions of air supply regulating dampers and network pump capacity. The results of air, exhaust gas and heat carrier measurements are taken as initial values for the model parameters, as shown in Table 1.

Table 1. Measurement of air, fuel and water parameters.

Parameter	**Experiment 1**	**Experiment 2**
Water flow rate, m_{H_20}, *kg/s*	0.176	0.143
Boiler inlet temperature, $T_{H_20}^{in}$, °*C*	62.5	57.7
Boiler outlet temperature, $T_{H_20}^{out}$, °*C*	78.8	80.7
Average fuel feed rate, m^{coal}, kg/hour	12.1	12.05
Exhaust gases temperature	295.1	314.8
Proportion of O_2 in exhaust gases, $\phi_{O_2}^{smoke}$, %	15.7	14.3
Proportion of *CO* in exhaust gases, ϕ_{CO}^{smoke}, *ppm*	40 796	66 680

Calculation results of boiler efficiency and fuel consumption for heat generation for both modes, as well as verification calculation results of exhaust gas composition are summarized in Table 2.

Table 2. Calculation results.

Parameter	**Experiment 1**	**Experiment 2**
$M_{CO_2}^{smoke}$, *mole/s*	0.063	0.047
M_{CO}^{smoke}, *mole/s*	0.05	0.055
Calculation of O_2 in exhaust gases, $\phi_{O_2}^{smoke}$, %	15.8	14.04
Calculation of *CO* in exhaust gases, ϕ_{CO}^{smoke}, *ppm*	41 170	67 927
Calculated combustion rate, m^{coal}, kg/hour	12.58	12.3
Q^p, kW	35.92	30.65
Q_1, kW	12.0	13.7
η, %	33.4	44.8

Effective load is determined by boiler water flow rate and the temperature difference between the boiler inlet and outlet:

$$Q_1 = m_{H_2O} \cdot \left(S_{H_2O}(T_{H_2O}^{out}) - S_{H_2O}(T_{H_2O}^{in}) \right), W \tag{16}$$

where $S_{H_2O}(T)$ is water entropy at temperature *T*, J/kg.

As shown in Tables 1 and 2, the discrepancy between measured and calculated fuel consumption in both modes is equal to 2...4%, which allows us to conclude that the proposed model is valid.

4 Conclusion

Experimental validation of the hot water boiler mathematical model conducted on the experimental unit, has shown that the model not only accurately enough describes the processes occurring in the system, but also can be used to predict the state of the system. This allows to develop an effective control system for solid fuel boiler based on this model, including the use of such technology as "digital twin".

The study was conducted using a grant from the Russian Science Foundation. No. 22-29-01232, https://rscf.ru/project/22-29-01232/

References

1. Kononov Yu.D., Tirtishny V.N. Influence Assessment of Initial Data Uncertainty on the Efficiency of Power and Fuel Supply Options for Regions in Forecasting Studies, Problems of Forecasting. No.1(136). P.90-94. (2013)
2. Kizil, E.V. Methods for Diagnosing the Level of Regional Innovative Development (on the Example of the Far Eastern Federal District), Advances in Science, Technology and Innovation p. 357-364 DOI 10.1007/978-3-030-90324-4_58 (2022)
3. Sokolov A.D., Muzychuk S.Yu., Muzychuk R.I. Energy and Economic Analysis of the Far East Fuel and Energy Industry~, Journal of the Irkutsk State Technical University. Vol.21 No.1(120). P.141-155. DOI 10.21285/1814-3520-2017-1-141-155 (2017)
4. Gordin S.A., Zaychenko I.V., Sokolova V.S., Bazheryanu V.V. To Calculate Heat Losses With Flue Gases of Coal-fired Boilers~, E3S Web of Conferences, 285, 07028 (2021)
5. Gordin S.A., Kozlova O.V., Zaychenko I.V. Modeling the Combustion Process of Solid Fuel Boilers. Lecture Notes in Networks and Systems. Vol. 200. P. 136-146 (2021)
6. Kagan G.M. Thermal Calculation of Boilers (Normative Method). St. Petersburg, Russia: Vseross. Teplotekh. Inst. (1998)
7. Thermodynamic Properties of Gases, Moscow Power Engineering Institute. URL: http://twt.mpei.ac.ru/MCS/Worksheets/WSP/GasesProperties.xmcd (accessed 07/11/23)