

# Determination of the Best Modes of the Electric Power System Containing Thermal and Hydro Power Plants Using the Method of Stepped Optimization

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**Abstract.** The paper presents an approach to determining the best modes of an electric power system using the stepped optimization method. The features of modeling and optimization of heat sources included in electrical power systems are described. The problem of coordinated optimization of average monthly regimes of the design year is considered, including taking into account the features of hydroelectric power plants with reservoirs of multi-year regulation when optimizing long-term operating regimes of electric power systems. The features of the stepped optimization method are given. As an example of optimization of long-term modes of the electric power system, the electric power system of the Yakutia Republic is considered.

## 1 Introduction

In many electric power systems (EPS) of the Russian Federation, a significant part of the electricity generating capacity consists of thermal power plants (TPP) that carry out the combined production of electrical and thermal energy. At these TPP, as a rule, additional sources of thermal energy are installed - steam and/or hot water boilers. Therefore, when optimizing the operating modes of such systems according to the criterion of minimum fuel costs, it is necessary to take into account both fuel consumption for the combined production of electrical and thermal energy, and for the production of only thermal energy.

In addition to a significant share of the power of TPP, the electric power systems of the eastern regions of the Russian Federation are characterized by the presence of hydroelectric power stations (HPS), and most of these HPS have reservoirs of annual and multi-year regulation [1]. At the same time, in order to coordinate the electrical power of a hydroelectric power station during the calculation period (year), joint optimization of characteristic EPS modes related to different moments of this period is required (for example, joint optimization of all average monthly modes of the year). Among these HPS, some have larger reservoirs that provide multi-year regulation. The difference between reservoirs of annual and multi-

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year regulation is that in the first case the reservoir is released at the beginning of its filling (at the beginning of the flood) to a minimum level, and in the second case the level of filling of the reservoir at the specified moment may be different. This allows you to store water in high-water years and use it in low-water years.

An important feature of the Eastern regions, which have a large area and low population density, is the transmission of relatively small electrical power from HPS over considerable distances. Moreover, in high-water years, energy output from hydroelectric power plants is limited not by the availability of water and not by the total power of installed hydraulic units, but by the capacity of power transmission lines, which are determined by the design characteristics of the lines and the flows of active and reactive power transmitted in the electric power system. Based on the above, the mathematical model of the electric power system should provide calculation of heat flows, flows of active and reactive electrical power for each characteristic mode of the calculation period, as well as calculation of water balances of hydroelectric power station reservoirs. Due to the complexity of the design electrical circuits of EPS, the large number of electric generators, electrical transformers with controlled transformation ratios, controlled reactive power sources and other elements in these systems, the problems of optimizing long-term modes of such systems are very complex nonlinear optimization problems. Experience has shown that an effective method for solving nonlinear large-scale optimization problems is the stepped optimization method developed at ESI SB RAS [2-4]. As an example of the proposed approach, this paper considers the Yakutia electric power system.

## **2 Approach to modeling and optimization of heat sources**

It is assumed that two types of heat sources can be used in EPS: a) thermal power units using organic or nuclear fuel, carrying out the combined production of thermal and electrical energy; b) steam or hot water boilers that burn organic fuel and produce only thermal energy. It is assumed that all heat supply at nuclear TPP is carried out from combined heat and power steam turbines, and at fossil fuel TPP, heat can be supplied both from combined heat and power steam turbines and from hot water boilers. For each heat supply system (located within the centralized power supply system), in which the combined production of heat and electricity is carried out, the dependence of the current heat load on the current outside air temperature, the estimated heat load of consumers, the estimated outside air temperature and some other parameters is constructed. In addition, a mathematical model of heat sources of such a heat supply system is constructed, designed to solve the problem of optimal distribution of thermal and electrical loads of a thermal power plant at a given outside temperature and a given total electrical load. This problem is solved according to the criterion of minimum fuel costs, taking into account technological restrictions in the form of equalities and inequalities for the operation of equipment and with various combinations of electrical load and outside air temperature, which determines the thermal load. Based on a series of optimization calculations, polynomials are constructed that determine fuel consumption and electrical power for own needs depending on the electrical load of the thermal power plant and the thermal load of consumers. In addition, polynomials are constructed with the help of which constraints-inequalities are found that determine the admissibility of the equipment operating regime (for example, the difference between the actual steam flow into the condenser of a steam turbine and the minimum permissible steam flow, which should not be less than zero, and also should not be less than zero difference between the maximum possible and actual amount of heat removed from the exhaust gases of a gas turbine in a waste heat boiler).

To determine the coefficients of each polynomial based on the results of optimization calculations using models of heat sources, two linear programming problems are solved. In

the first problem, the modulus of the maximum deviation of the values of the determined quantity calculated by the mathematical model and its values obtained using a polynomial is minimized. In the second problem, among all solutions that meet the optimality condition of the first problem, a solution is sought that provides the minimum sum of the modules of the indicated deviations. This approach allows, when solving the general problem of optimizing EPS parameters, to use simple models in the form of polynomials to describe heat sources, providing high performance compared to fairly detailed initial models.

### 3 Coordinated optimization of characteristic (monthly average) and EPS

The basis of the calculation scheme of the optimization model of the EPS is the normal electrical scheme of connections of its elements. The mathematical models of electrical network elements (power lines, transformers, etc.) used are based on the calculation methods outlined in [5]. In this case, a group of similar, parallel-connected elements is replaced by one “group” element, for which the number of elements in the group is specified. In calculations, it is assumed that elements in a group are loaded equally. The permissible ratios of active and reactive powers of electric generators are determined using constraints-inequalities constructed on the basis of generator diagrams. For each node of the electrical scheme, lower and upper limits on the voltage module are set. Restrictions for power lines on the maximum permissible heating of wires by current are taken into account. Average monthly operating regimes are considered. For each consumer in such modes, the average monthly electrical and average monthly thermal loads are set.

To optimize the long-term modes of an EPS, which includes a hydroelectric power station with a reservoir of multi-year regulation, it is necessary to carry out a significant number of coordinated optimization calculations of the average monthly regimes of the calculation year. Moreover, in each such calculation the following are specified: a) the water level in the reservoir at the beginning of the calculation year; b) variant of water inflow into the reservoir by month; c) water level in the reservoir at the end of the calculation year.

The mathematical formulation of the problem of coordinated optimization of average monthly regimes of the accounting year is formulated as follows

$$\min U_{fl}^{yr} = \sum_{k=1}^{12} U_k^{hr} T_k, \quad (1)$$

under conditions

$$G(x_{opt_k}^{tr}, x_{opt_k}^{sh}, x_{opt_k}^{el}, d_k, S_k, h_k^{beg}) \geq 0, \quad (2)$$

$$\varphi(x_{opt_k}^{tr}, x_{opt_k}^{sh}, x_{opt_k}^{el}, d_k, S_k, h_k^{beg}) \geq 0, \quad (3)$$

$$h_k^{end} = W(x_{opt_k}^{tr}, x_{opt_k}^{sh}, x_{opt_k}^{el}, d_k, S_k, h_k^{beg}), \quad (4)$$

$$U_k^{hr} = \psi(x_{opt_k}^{tr}, x_{opt_k}^{sh}, x_{opt_k}^{el}, d_k, S_k, h_k^{beg}), \quad (5)$$

$$h_{k+1}^{beg} = h_k^{end}, h_1^{beg} \in \{H^1, \dots, H^N\}, h_{12}^{end} \in \{H^1, \dots, H^N\}, \quad (6)$$

$$S_k \in \{S^1, \dots, S^M\}, k = 1, \dots, 12, \quad (7)$$

where  $U_{fl}^{yr}$  – annual fuel costs;  $U_k^{hr}$  – average monthly hourly fuel costs of the  $k$ -th month;  $T_k$  – number of hours in the  $k$ -th month;  $G$  – system of constraints in the form of inequalities;  $x_{opt_k}^{tr}$  – true independent optimized parameters for the  $k$ -th month;  $x_{opt_k}^{sh}$  – optimized

parameters used to connect iterations in the mathematical model in the EPS scheme for the  $k$ -th month;  $x_{opt_k}^{el}$  – optimized parameters used to connect iterations in the elements of the mathematical model of the EPS scheme;  $d_k$  – vector of initial information for the  $k$ -th month (electrical and thermal loads of consumers, etc.);  $S_k$  –  $k$ -th component of the inflow vector  $S$ ;  $h_k^{beg}$ ,  $h_k^{end}$  – water levels in the reservoir at the beginning and end of the  $k$ -th month;  $\varphi$  – system of constraints in the form of equalities;  $W$  – dependence determining the water level at the end of the month;  $\psi$  – dependence determining hourly fuel costs;  $H^1, \dots, H^N$  – discrete set of considered water levels in the reservoir at the beginning and end of the calculation year;  $S^1, \dots, S^M$  – the considered set of inflow vectors.

As you can see, the parameters of the connection between the EPS regimes are water levels at the end of the  $k$ -th month ( $h_k^{end}$ ), equal to water levels at the beginning of the  $k + 1$  month where  $k$  varies from 1 to 11.

#### **4 Taking into account the features of hydroelectric power plants with reservoirs of multi-year regulation when optimizing long-term operating regimes of EPS**

The water level in the reservoir of multi-year regulation after the end of its operation before the start of the flood varies from year to year within certain limits. This complicates the calculations of EPS operation in various regimes and when coordinating these regimes throughout the year. It is possible to carry out a coordinated optimization calculation of the characteristic regimes of the year (for example, average monthly regimes in an electric power plant that has one hydroelectric power station with a reservoir of multi-year regulation if the water levels in the reservoir are specified at the beginning of the calculation year and at the end of it (the beginning of the calculation year coincides with the beginning of the flood), as well as water inflows into the reservoir by month. To optimize long-term modes of EPS, which include HPS with reservoirs of multi-year regulation, the following steps must be performed.

1. Assign a possible range of changes in water levels in the reservoir at the beginning of the calculation year (it will also coincide with the possible range of changes in levels at the end of the calculation year).
2. Set several discrete level values that evenly cover the ranges of level changes at the beginning (and end) of the calculation year.
3. Set several discrete options for water inflow into the reservoir, and in each option, inflows are specified for all characteristic regimes, for example, the average inflows of each month. The probabilities of realizing each inflow option are specified, and the sum of the probabilities of all options should be equal to 1.
4. For each combination of a discrete water level at the beginning of the calculation year, a discrete option of water inflow into the hydroelectric power station reservoir and a discrete water level at the end of the calculation year, a coordinated optimization of the characteristic (monthly average) regimes of the year is carried out according to the criterion of minimum annual fuel costs. As a result, each combination of water level at the beginning of the calculation year, water inflow into the reservoir and the level at the end of the calculation year is associated with optimal annual fuel costs.
5. It is accepted that the current state of the EPS, which includes a hydroelectric power station with a reservoir of multi-year regulation, is determined by the water level in this reservoir at the beginning of the current accounting year. The random process of

the system transition to a state at the beginning of the next accounting year is determined by two factors: the probabilities of water inflow options and the conditional transition probabilities (for a given initial level and inflow option) to each possible water level at the end of the current accounting year (or at the beginning of the next accounting year). Note that for all transitions from each combination of the initial level and the inflow option, the sum of the indicated conditional probabilities is equal to 1. If the inflow probabilities and conditional probabilities do not change with time, then the random process under consideration is described by a homogeneous Markov chain with a discrete state and discrete time. The probabilities of the system being in each state and the probabilities of transitions do not depend on the initial state of the system and on time, and are determined by the system of linear algebraic Kolmogorov–Chapman equations. If we accept that for each combination of the initial level and the supply option, a transition to only one level must be chosen (its conditional probability is 1, the conditional probabilities of transitions to other levels are equal to 0), then a discrete-continuous linear programming problem can be formed on minimizing the mathematical expectation of annual fuel costs for EPS. As a result of solving this problem, where the optimized parameters are the probabilities of the system being in each state (continuous optimized parameters) and the conditional probabilities of transition to each discrete level for a given initial level and inflow option (discrete optimized parameters equal to 0 or 1), for each combination the initial level and inflow option will determine the optimal water level at the end of the current accounting year. This problem takes into account constraints in the form of equalities specified by the Kolmogorov–Chapman system of equations and supplemented by the integer conditions for a number of optimized parameters.

The mathematical formulation of the specified continuous-discrete linear programming problem has the following form

$$\min \sum_{i=1}^N \sum_{j=1}^M \sum_{l=1}^N P_{FULL}^{ijl} u^{ijl}, \quad (8)$$

under conditions

$$P_{FULL_i}^{ijl} = P_{BEG}^i P_{INFL}^j P_{USL}^{ijl}, \quad (9)$$

$$\sum_{l=1}^N P_{USL}^{ijl} = 1, \quad (10)$$

$$P_{USL}^{ijl} \in \{0, 1\}, \quad (11)$$

$$\sum_{l=1}^N P_{BEG}^i = 1, \quad 0 \leq P_{BEG}^i \leq 1, \quad (12)$$

$$P_{END}^l = \sum_{i=1}^N \sum_{j=1}^M P_{BEG}^i P_{INFL}^j P_{USL}^{ijl}, \quad (13)$$

$$P_{BEG}^i = P_{END}^i, \quad (14)$$

$$i = 1, \dots, N, \quad j = 1, \dots, M, \quad l = 1, \dots, N, \quad (15)$$

where  $N$  – the number of discrete values of the reservoir level at the beginning (and at the end of the calculation year);  $i$  – reservoir level index at the beginning of the calculation year;  $M$  – number of vectors of inflow values into the reservoir;  $j$  – inflow vector index;  $l$  – reservoir level index at the end of the calculation year;  $P_{FULL}^{ijl}$  – the total probability of implementing the option with the  $i$ -th level in the reservoir at the beginning of the calculation year, the  $j$ -th inflow vector and the  $l$ -th level of the reservoir at the end of the calculation year (let's call it

option  $ijl$ );  $u^{ijl}$  – annual fuel costs for EPS for option  $ijl$ ;  $P_{BEG}^i$  – total probability of the EPS being in a state with the  $i$ -th initial level;  $P_{INFL}^j$  – full probability of implementation of the  $j$ -th inflow vector; the transition to the  $l$ -th reservoir level will be carried out at the end of the calculation year;  $P_{END}^l$  – the total probability that at the end of the calculation year the system will be in a state with the  $l$ -th discrete value of the water level (based on the stationarity of the random process, it is assumed that  $P_{BEG}^i = P_{END}^i$ ).

## 5 Using the stepped optimization method

The stepped optimization method considers an initial nonlinear mathematical programming problem involving constraints-equalities and constraints-inequalities. In this case, in the future, each constraint-equality of the original problem is replaced by two constraints-inequalities imposed on the residual of the original constraint-equality. In this case, the relative residual is considered, which is obtained by dividing the absolute residual by its maximum permissible value. In this case, one inequality limits the growth of the residual if its value is positive, and the other, if its value is negative. In all these constraints-inequalities, one (non-negative) auxiliary parameter is introduced. The larger this parameter, the wider the corridor of acceptable values, determined by two inequalities corresponding to one equality. If the auxiliary parameter tends to zero, then the corridor shortens and in the limit tends to the dependence determined by the constraint-equality. At each iteration of the stepped optimization method, two nonlinear mathematical programming problems are solved. When solving the first problem, the auxiliary parameter is fixed and the objective function of the original problem is minimized, taking into account the true constraints-inequalities of this problem and the constraints-inequalities that replace the constraints-equalities of the original problem. When solving the second problem, the auxiliary parameter is minimized while taking into account the same constraints-inequalities as in the first problem. In this case, additional conditions are introduced that prevent the auxiliary parameter from decreasing too much. It should be noted that the optimal point of the first task of the  $i$ -th iteration is taken as the starting point of the second task of the  $i$ -th iteration, and the optimal point of the second task of the  $i$ -th iteration is taken as the starting point of the first task of the  $i+1$ -th iteration. The selection of the starting point of the first problem at the first iteration is carried out as a result of solving an auxiliary nonlinear programming problem, in which the optimized parameters of the original problem and the auxiliary parameter are selected so that all constraints-inequalities of both the original problem and those that replace the equality constraints are strictly satisfied (i.e. the starting point of the first problem at the first iteration is the internal point of the entire system of constraints-inequalities). To solve these nonlinear programming problems, the method of possible directions is used [6]. At each step of this method, two auxiliary linear programming problems are solved to determine the “direction of descent.” In the first problem, the linearized (at the current point) objective function of the nonlinear problem being solved is minimized under linearized (at the current point) constraints-inequalities. As a result of the solution, an estimate “from below” of the optimal value of the objective function and an assessment of its possible improvement are determined. In the second problem, the direction of descent is sought, which, when rejecting a given share of the possible improvement of the objective function, provides the maximum (in a sense) movement “into the depths” of the admissible region. In this case, linearized constraints are also used.

Using a variant of the possible directions method based on solving two linear programming problems turned out to be very effective. The optimization process is organized

in such a way that the movement is carried out along the internal points of the feasible region of the current problem.

Features of the stepped optimization method are as follows:

1. All iterative processes for solving systems of equations are moved from the model level to the optimization level.
2. It is not required that at the starting point the system of constraints in the form of equalities has a solution.
3. A sufficiently accurate solution to the system of constraints in the form of equalities is achieved only in the vicinity of the optimal point.
4. The optimized parameters include true optimized parameters that determine optimal solutions and parameters responsible for iterative processes, transferred from the level of mathematical models of the circuit as a whole and its elements to the optimization level.

When solving linear programming problems, the method of interior points by A. Fiacco – G. McCormick (method of unconditional sequential minimization with a logarithmic penalty function) was used [7]. It turned out to be more resistant to the presence of linearly dependent constraints than the Simplex method. It is important to note that its efficiency compared to the Simplex method increases with increasing problem dimension. Moreover, the greatest effect was obtained when solving problems of unconditional minimization of the logarithmic penalty function using Newton's method, using the matrix of second derivatives of the logarithmic penalty function. Choosing the direction of descent in this method comes down to solving a system of linear algebraic equations. The dimension of this system is equal to the number of optimized parameters. With a relatively small number of optimized parameters (up to 500), the computer time spent on solving linear programming problems is a relatively small part of the total time spent on solving the original problem of optimizing EPS modes. However, when the number of optimized parameters was 1000-2000, the cost of solving linear programming problems began to consume a large share of computer time (this is due to the fact that the time spent solving a system of linear algebraic equations is proportional to the third power of the dimension of this system). Moreover, the use of various available solvers for systems of linear algebraic equations did not allow us to achieve a significant reduction in calculation time. With an increase in the number of optimized parameters to 7000 or more, the original nonlinear programming problems became practically unsolvable (it took several tens of days to solve one such problem on 16-24 core processors. For example, this applied to the problem of coordinated optimization of twelve average monthly modes of the EPS of Yakutia considered below. Analysis of the matrix of second partial derivatives of the logarithmic penalty function in the problem of coordinated optimization of several operating modes of an EPS with a hydroelectric power station showed that with the correct construction of a mathematical model of an EPS, it is possible to reduce the time for solving systems of linear algebraic equations in Newton's method by tens of times and, accordingly, the time for solving the entire original nonlinear programming problem. To do this, the water levels in the hydroelectric power station reservoir at the end of months from 1 to 11 (parameters of connection between regimes) must be included in the independent optimized parameters. In order for these levels to correspond to the water balances of the corresponding months, additional residuals are introduced equal to the differences in levels at the end of the specified months, determined from water balances and levels specified as optimized parameters (in the future, each such residual is replaced by two inequalities). In this case, the water balance of the  $r+1$  month is compiled based on the level at the end of the  $r$ -th month, which is an optimized parameter (a parameter of the connection between regimes). As a result, the constraints-inequalities of the  $r$ -th regime are affected only by the internal optimized parameters of this regime and communication parameters. The internal optimized parameters of other regimes do not affect the restrictions of the  $r$ -th



regime. Therefore, the matrix of second derivatives of the logarithmic penalty function takes on a structure in which all second derivatives with respect to the internal optimized parameters of two different regimes are equal to zero. The second derivatives of the logarithmic penalty function with respect to the internal optimized parameters of one regime, the internal optimized parameter of some regime and the optimized communication parameter, and the two optimized communication parameters have non-zero values. This allows us to reduce the solution of a “large” system of equations in Newton’s method to the inversion of twelve “small” matrices, the dimension of which is equal to the number of internal optimized parameters of one mode, the solution of one linear system of equations, the dimension of which is equal to the number of optimized communication parameters, and the performance of a certain number of multiplication and addition of matrices of specified dimensions. This approach, which can be called a structural approach to solving “large” systems of linear algebraic equations, made it possible to reduce the time of solving linear programming problems by tens of times using the interior point method of A. Fiacco - G. McCormick.

As an example of optimization of long-term EPS regimes, the EPS of the Republic of Yakutia is considered. The system includes two gas turbine Yakutsk TPP; a large coal-fired Neryungrinskaya TPP of three blocks (one condensing and two heating power units); a small coal-fired Chulman TPP; a small gas-fired steam turbine Yakutsk TPP; Vilyuiskaya HPS 1, 2 with a common dam and multi-year regulation reservoir and unfinished Svetlinskaya HPS with small-capacity reservoirs. An electrical network with voltages of 110 kV and 220 kV is being considered.

The design scheme of the EPS of Yakutia includes power equipment (elements of the EPS scheme) presented in Table 1. It should be noted that some equipment of the same type is combined in a scheme as group.

**Table 1.** Elements of the Yakutia EPS scheme.

<b>Name of equipment (elements of EPS scheme)</b>	<b>Total number of EPS scheme elements, pcs.</b>	<b>Number of groups of similar equipment, pcs.</b>
Power lines	110	73
Turbines:	29	12
-HPS	11	2
-gas	9	3
-steam	9	7
Hot water boilers	13	6
Waste heat boilers	9	3
Compensators	34	19
Transformers:	134	65
- communication	52	24
- hanging	82	41

As mentioned above, when creating a mathematical model of an EPS as a whole, mathematical models of the generating equipment of its energy sources are used, constructed using polynomials, the data for which were obtained as a result of a series of optimization studies of the same equipment using detailed mathematical models.

Thus, based on the created mathematical models of the elements of the EPS of the Republic of Yakutia: gas turbine units, combined cycle gas turbine units, boilers, power lines, transformers, etc. a mathematical model of the EPS of Yakutia as a whole was created. The number of optimized parameters of the mathematical model for one regime (one month) is 162, the number of iteratively refined parameters is 161, the balanced parameters are 254, the residuals on the balanced parameters are 254, the constraints are 404. During the

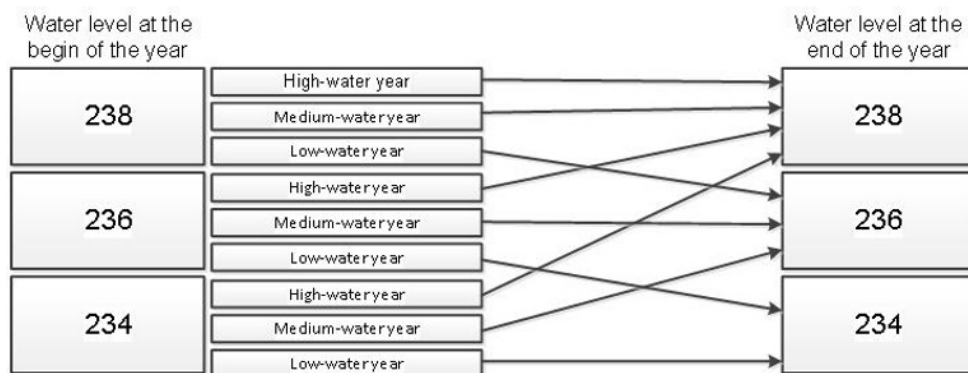


optimization studies of the EPS, the following basic initial data were accepted: the number of water levels - 3 (238 m, 236 m, 234 m), the number of inflow options – 3, of which: low-water (probability 20%), medium-water (probability 60%) and high-water (probability 20%). Optimization calculations were carried out for all accepted water levels and inflow options. The total number of calculations performed is 27.

In Table 2 as an example, some results of optimization calculations of a long-term regime are presented at an initial water level of 238 m, a medium-water year and a final level of 238 m.

The values of fuel costs obtained as a result of optimization calculations for 27 coordinated operating regimes of the Yakutia EPS are presented in Table 3.

Fig. 1 shows the optimal options for operating a hydroelectric power station with a multi-year regulation reservoir (Vilyuiskaya HPS-1 + Vilyuiskaya HPS-2) as part of the Yakutia EPS.



**Fig. 1.** Optimal options for operating a hydroelectric power station with a multi-year regulation reservoir (Vilyuiskaya HPS-1 + Vilyuiskaya HPS-2) as part of the Yakutia EPS.

The probability of an initial level value of 234 m is 0.10, a value of 236 m is 0.4, and a value of 238 m is 0.5. The mathematical expectation of annual fuel costs is 10,385 million rubles.

## 6 Conclusion

This paper describes a methodological approach to determining the best EPS regimes using the stepped optimization method. Features of modeling and optimization of heat sources included in the EPS are presented. The problem of coordinated optimization of average monthly regimes of the design year is considered, including taking into account the features of HPS with reservoirs of multi-year regulation when optimizing long-term operating regimes of EPS. The features of the stepped optimization method are described. As an example of optimization of long-term regimes of the EPS, the EPS of the Yakutia Republic is considered. Based on the criterion of the minimum value of fuel costs, optimization calculations were carried out and 27 coordinated (according to 12 average monthly regimes) long-term operating regimes of the EPS of Yakutia were obtained for various combinations of water level in the reservoir of Vilyuiskaya HPS-1, 2 at the beginning and end of the calculation year and the level of water inflow in this reservoir. Based on these calculations, taking into account the probability of low-water, medium-water and high-water inflows, for each combination of the water level in the reservoir at the beginning of the calculation year and the inflow option, the optimal water level in the reservoir at the end of the calculation year was determined.

**Table 2.** The main results of optimization calculations for a long-term regime (at an initial level of 238 m, a medium-water year and a final level of 238 m).

Options	Month, No.											
	5	6	7	8	9	10	11	12	1	2	3	4
Water inflow into the reservoir of Vilyuiskaya HPS 1-2, m <sup>3</sup> /s	2563	2715	679	345	491	191	39	16.5	5.1	3.2	2.8	4.6
Level top reservoir of Vilyuiskaya HPS 1-2 at the begin of the calculation interval, m	238.10	240.86	243.53	243.76	243.64	243.52	243.04	242.33	241.52	240.66	239.81	238.91
Level top reservoir of the Vilyuiskaya HPS 1-2 at the end of the calculation interval, m	240.86	243.52	243.76	243.64	243.52	243.03	242.32	241.52	240.66	239.81	238.91	237.99
Total electrical power of units, MW												
- Vilyuiskaya HPS 1-2	278	237	253	238	316	317	339	344	344	354	321	317
- Svetlinskaya HPS	107	87	91	85	114	115	124	127	129	135	124	124
- Neryunginskaya TPP (Power unit T-180; bus 110 kV)	90	36	44	36	91	93	92	95	95	88	93	94
- Neryunginskaya TPP (Power unit T-180; bus 220 kV)	94	39	47	38	63	82	120	146	149	130	106	73
- Yakutsk TPP (gas turbine GTE-45)	87	48	47	48	90	122	122	122	122	122	122	117
- New Yakutsk TPP (gas turbine LM6000)	163	134	160	135	160	157	163	163	163	163	163	153
Total heat supply, Gcal/h												
- Neryunginskaya TPP (Power unit T-180; bus 110 kV)	98	48	47	48	118	132	137	139	140	136	136	135
- Neryunginskaya TPP (Power unit T-180; bus 220 kV)	98	52	53	52	78	117	176	211	217	199	154	105
- Yakutsk TPP (waste heat boilers)	110	63	63	63	114	148	132	124	122	126	144	140
- New Yakutsk TPP (waste heat boilers)	81	46	46	46	84	109	113	105	104	107	119	103
Active power of power lines at input/output, MW												
AS400 Rayonnaya-Gorodskaya	186/18 0	188/18 2	196/19 0	188/18 2	209/202 2	209/20 3	224/21 6	228/22 0	229/22 0	235/22 6	215/20 8	215/20 8
AS 300 Tommot-Maya	-127/ -134	-118/ -123	-116/ -122	-119/ -125	-126/ -133	-136/ -143	-119/ -125	-107/ -112	-106/ -110	-110/ -115	-130/ -137	-130/ -137
Reactive power of power lines at input/output, Mvar												
AS 400 Rayonnaya-Gorodskaya	-41/ 2	-38/ 3	-40/ 0.85	-38/ 3.5	-49/-13	-48/ 13	-52/ 21	-53/ 23	-53/ 24	-46/ 21	-49/ 15	-51/ 17
AS 300 Tommot-Maya	-41/ 57	-45/ 55	-47/ 54	-45/ 55	-43/ 55	-39/ 54	-46/ 56	-50/ 57	-51/ 57	-52/ 51	-42/ 55	-42/ 55
<b>Fuel cost, thousand \$/t</b>	<b>797</b>	<b>504</b>	<b>554</b>	<b>504</b>	<b>763</b>	<b>871</b>	<b>1039</b>	<b>1140</b>	<b>1154</b>	<b>1092</b>	<b>980</b>	<b>836</b>
Total cost of fuel, thousand \$/t	10237											

**Table 3.** Values of annual fuel costs in 27 coordinated operating regimes of EPS, million rubles.

Inflow options for year	Water level values in May of the calculation year (beginning of the period)	Water level values in April of the calculation year (end of the period)		
		234	236	238
Low-water	234	12195.7	13146.4	Invalid regime
	236	11173.4	12101.0	13125.1
	238	10055.1	10822.6	11999.0
Medium-water	234	10623.0	11316.3	12346.0
	236	9722.8	10368.9	11411.9
	238	9421.9	9596.5	10365.9
High-water	234	9296.4	9544.6	10141.6
	236	9186.0	9567.0	9533.2
	238	9197.4	9222.1	9563.4

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