

Theoretical Study on Unsteady Temperature Field and Heat Regulating Zone for Earth-air Heat Exchanger

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Abstract. The earth-air heat exchanger (EAHE) play an important role in building energy efficiency. Most of the previous studies have adopted numerical simulation and experimental methods, and Theoretical research was rarely used. The theoretical analysis method has clear mathematical physical meaning. Due to the few theoretical analysis methods used, there were no general criteria for the value of heat-regulating zone. The inaccurate value of the heat regulating zone will cause calculation errors or waste unnecessary computing resources. In this research, the calculation formula of the unsteady temperature field of the EAHE was derived and the calculation correlation of value of the heat regulating zone was also given. The results show that the derived calculation formula of the unsteady the dimensionless airflow of EAHE is in good agreement with the experiment, and calculation correlation of the value of the heat regulating zone is suitable for various engineering. This research provides a theoretical basis for the design and application of tunnel ventilation and a correlation formula for the value of the heat regulating zone.

1 Introduction

The heat storage characteristics of soil lead to the delay and attenuation of soil temperature relative to the surface air temperature[1]. The essence of soil temperature delay and attenuation is the heat conduction process inside the soil and the convective heat transfer process between the soil and the air flow. This heat transfer process is the basis for the practical application of EAHE, so it is necessary to explore the mechanism of this heat transfer process.

From the perspective of research methods, most of the previous studies used numerical simulation and experimental methods[2-6], and rarely used theoretical research. Numerical simulation methods have the advantages of time saving and convenient operation, and experimental methods are highly reliable, but their mathematical and physical meanings are not clear enough, while theoretical analysis methods have clear mathematical and physical meanings. Therefore, this study will use the theoretical analysis method to study the heat transfer and flow mechanism of EAHE.

In addition, the flow heat transfer mechanism of EAHE is similar to that of tunnel ventilation, so the methods and conclusions of this study are also applicable to the heat transfer process of tunnel ventilation. After investigating the previous literature on the EAHE and tunnel ventilation, it is found that there is no universal standard for the value of the heat regulating zone, which leads to different scholars with different values of heat regulating zone. For instance, Chen Q et al. [7] took the thermal regulation zone radius as 2 times the tunnel radius, and Zhang Z et al. [8] took 5 times the

tunnel radius. If the value of the thermal regulation area is too small, calculation errors will occur, and if the value is too large, unnecessary computing resources will be wasted.

Therefore, this study adopts the method of theoretical analysis to deduce the calculation formula of the unsteady temperature field of EAHE, and on this basis, derives the calculation formula of the value of the heat regulating zone. This study provides a theoretical basis and a formula for the heat regulating zone for the design and application of the EAHE.

2 Theoretical method

2.1 Assumptions

The air enters from the tunnel entrance and conducts convective heat exchange with the surrounding soil and flows out from the tunnel exit. The airflow temperature affects the soil temperature, and the soil temperature also affects the airflow temperature, as shown in Fig. 1. The airflow temperature in the tunnel and the soil temperature are coupled. To establish a mathematical description of this heat transfer process, simplifying assumptions are made as follows:

- (1) The physical properties of the soil around the tunnel are isotropic and constant.
- (2) The airflow in the tunnel and the surrounding soil has no internal heat source.
- (3) The airflow temperature of the same section is the same.

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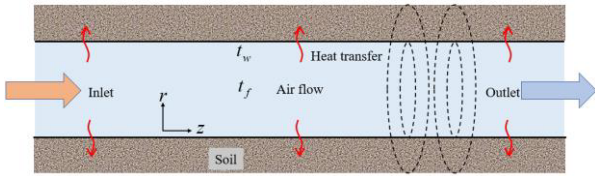


Fig. 1. Heat transfer process between airflow temperature and soil temperature in tunnel.

2.2 Differential equations

To solve the unsteady temperature distribution of airflow in the tunnel, it is necessary to solve the soil temperature field first. Based on the above assumptions, the differential equation of thermal conductivity in the cylindrical coordinate system of the soil temperature field around the tunnel is as follows:

$$\frac{\partial \theta(r, \tau)}{\partial \tau} = a \frac{\partial^2 \theta(r, \tau)}{\partial r^2} + \frac{a}{r} \frac{\partial \theta(r, \tau)}{\partial r} \quad (\tau > 0, r > 0) \quad (1)$$

The boundary conditions are as follows:

$$\begin{aligned} r = r_0, -\lambda \frac{\partial \theta(r, \tau)}{\partial r} \Big|_{r=r_0} &= h(t_f - t_0) - \theta(r, \tau) \Big|_{r=r_0} \\ r \rightarrow \infty, \theta(r, \tau) &= t_0 - t_0 = 0 \end{aligned} \quad (2)$$

The initial conditions are as follows:

$$\tau = 0, \theta(r, 0) = t_0 - t_0 = 0$$

The thermal conductivity differential equation can be solved by Laplace transform, the process is ignored here, and only the result is given. The soil temperature field is as follows:

$$\begin{aligned} \theta(r, \tau) &= t(r, \tau) - t_0 \\ &= (t_f - t_0) + \frac{2h(t_f - t_0)}{\pi\lambda} \int_0^\infty \frac{J_0(ur)H - Y_0(ur)W}{H^2 + W^2} \exp(-au^2\tau) \frac{1}{u} du \end{aligned} \quad (3)$$

The unsteady distribution of tunnel airflow temperature is derived from the soil temperature field. As shown in the figure, the micro-element segment is taken along the airflow direction, and the energy conservation analysis is performed on this micro-element segment. According to energy conservation, the convective heat exchange between air and soil in the micro-element section is equal to the sum of the internal energy increment of the air passing through the micro-element section and the internal energy increment per unit time of the air in the micro-element section. The equation is as follows:

$$h\pi d(t(r_0, \tau) - t_f(z, \tau)) = \frac{\partial t_f(z, \tau)}{\partial z} \dot{m}c_p + \frac{\partial t_f(z, \tau)}{\partial \tau} \rho\pi d c_p \quad (4)$$

The internal energy increment per unit time of the air in the micro-element segment (the second term on the right side of the equation) is smaller than the other two terms, so this term can be ignored, so the simplified mathematical expression is as follows:

$$h\pi d(t(r_0, \tau) - t_f(z, \tau)) = \frac{\partial t_f(z, \tau)}{\partial z} \dot{m}c_p \quad (5)$$

After solving this first-order linear inhomogeneous differential equation, the unsteady distribution of tunnel dimensionless airflow temperature is obtained as follows:

$$\Theta' = \frac{t_f(z, \tau) - t_0}{t_{f0} - t_0} = \exp\left(-\frac{h\pi d}{\dot{m}c_p}(1 - \Theta)z\right) \quad (6)$$

2.3 Validation

The innovative calculation formula for the unsteady distribution of the airflow temperature in the tunnel is derived based on previous research and is derived through rigorous theoretical derivation, so it is highly credible. To further improve its credibility, the experimental data is used to compare and verify the above innovative calculation results. The innovative calculation in this study was validated against experimental results carried out by Bansal et al.[9]. The detail of comparison was illustrated in Fig. 2.

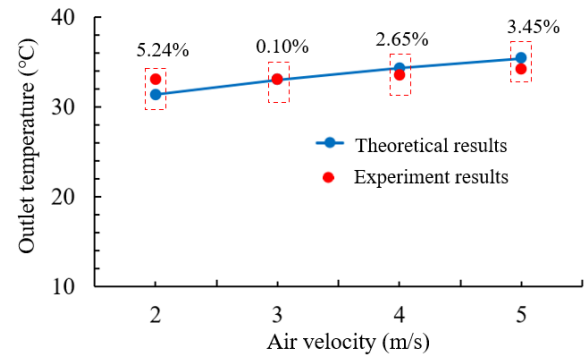


Fig. 2. The comparison between theoretical calculation results and experiments.

3 Results and discussion

3.1 General formula of heat regulating zone

It can be seen from equation (3) that the soil temperature field is related to the thermal conductivity of the soil, the convective heat transfer coefficient of the soil and the air, the air temperature, time, and location. It would be a lot of work to analyse these variables one by one. Therefore, the calculation formula needs to be dimensionless, and the dimensionless parameters are as follows:

$$R = \frac{r}{r_0}, \quad P = ur_0, \quad Fo = \frac{a\tau}{r_0^2}, \quad Bi = \frac{hr_0}{\lambda}, \quad \Theta(r, \tau) = \frac{t(r, \tau) - t_0}{t_f - t_0}$$

Bring these dimensionless parameters into equation (3):

$$\Theta(R, Bi, Fo) = 1 + \frac{2}{\pi} \int_0^\infty \frac{J_0(RP) \left[Y_0(P) + \frac{P}{Bi} Y_0(P) \right] - Y_0(RP) \left[J_0(P) + \frac{P}{Bi} J_0(P) \right]}{\left[Y_0(P) + \frac{P}{Bi} Y_0(P) \right]^2 + \left[J_0(P) + \frac{P}{Bi} J_0(P) \right]^2} \exp(-FoP^2) \frac{dP}{P} \quad (7)$$

According to formula (7), the dimensionless temperature of soil is related to Bi, Fo and dimensionless distance R. Assuming that Bi remains unchanged, formula (7) is used to calculate the soil temperature under different R and Fo conditions. The calculation conditions were illustrated in Table. 1:

The convective heat transfer coefficient between soil and air is calculated by the following formula:

$$h = 0.045 \frac{\lambda}{d} \text{Re}^{0.8} \quad (8)$$

Table 1. The calculation conditions

Parameters	
Inlet temperature t_f	30 °C

Initial soil temperature t_0	10 °C
Thermal conductivity of soil λ_s	1.41 W/(m·K)
Density of the soil ρ_s	1850 kg/m ³
Specific heat capacity of soil $c_{p,s}$	1840 J/(kg·K)
Thermal conductivity of air λ_a	2.6×10^{-2} W/(m·K)
Density of air ρ_a	1.205 kg/m ³
Specific heat capacity of air $c_{p,a}$	1005 J/(kg·K)
Air velocity u	4 m/s

According to formula (7), the soil temperature under different R and Fo conditions is calculated and drawn into a three-dimensional surface, as shown in Fig. 3.

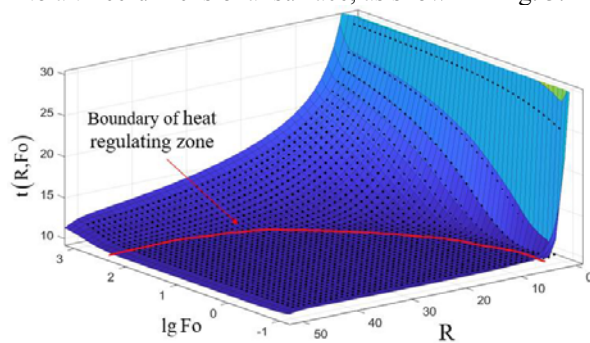


Fig. 3. The soil temperature for different R and Fo.

It can be seen from Fig. 3 that the soil temperature decreases as the dimensionless distance R increases, that is, the farther away from the tunnel wall, the lower the soil temperature. The reason is that the air temperature in the tunnel is as high as 30°C, and the initial soil temperature is as low as 10°C, which causes the air to transfer heat to the soil, and the temperature of the soil near the tunnel wall is heated. The soil further away from the tunnel wall receives less heat, resulting in lower temperatures. In addition, it can also be seen from the figure that the soil temperature increases as the dimensionless time Fo increases, that is, the longer the heat transfer time, the higher the soil temperature. The reason is that the air temperature in the tunnel is higher than the initial soil temperature, and with the increase of heat exchange time, the heat received by the soil increases, resulting in an increase in soil temperature.

The red line in Figure 3 is the boundary of the heat regulating zone. Below the red line is the soil area that is not affected by the air temperature inside the tunnel, and above the red line is the soil area that is affected by the air temperature inside the tunnel. The position where the dimensionless temperature is equal to 0.1 is defined as the boundary of the thermal regulation zone, represented by Rb, as follows:

$$\Theta(R, Bi, Fo) = 0.1$$

$$= 1 + \frac{2}{\pi} \int_0^{J_0(RP)} \frac{Y_0(P) + \frac{P}{Bi} Y_0(P)}{\left[Y_0(P) + \frac{P}{Bi} Y_0(P) \right]^2 + \left[J_0(P) + \frac{P}{Bi} J_0(P) \right]^2} \exp(-FoP^2) \frac{dP}{P} \quad (8)$$

The functional relationship between the heat regulating zone Rb and Bi and Fo is implicit in equation (8). Obviously, the heat regulating zone boundary Rb is not a constant value and should have different values for

different working conditions. If the working conditions are given, Bi and Fo can be calculated, and then the heat regulating zone boundary can be calculated from the functional relationship between Rb and Bi and Fo. Therefore, it is crucial to find the functional relationship $R_b = f(Bi, Fo)$. The functional relationship between Rb and Bi and Fo is implicit in equation (8), so it is necessary to transform this implicit function into an explicit functional relationship. At the same time, considering that Equation (8) is too complicated, a large amount of data is calculated by Equation (8) to fit the explicit function relationship. The fitting effect is shown in Figure 4.

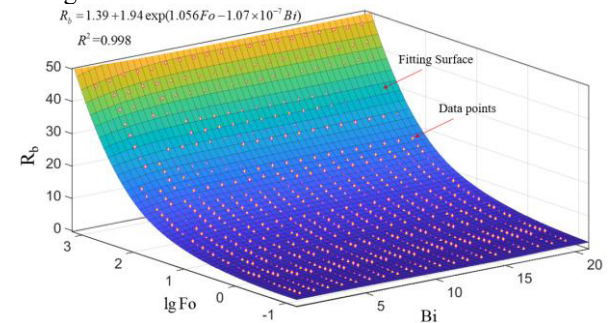


Fig. 4. The heat regulating zone for different Bi and Fo

The fitting calculation formula is as formula (9).

$$R_b = p1 + p2 \exp(p3Bi + p4Fo)$$

$$p1 = 1.38$$

$$p2 = 1.89$$

$$p3 = 2.47 \times 10^{-3}$$

$$p4 = 1.056$$

Goodness of fit R-Square = 0.999

Root Mean Square Error = 0.3914

Equation (9) is the basis for the value of the thermal regulation region Rb (outer boundary of the calculation region) when performing numerical simulations. Equation (9) can be used to calculate the thermal regulation zone under different working conditions where Fo is between 0.1 and 1000 and Bi is between 1 and 20. Since the form of formula (9) is relatively simple, no example is calculated here.

3.2 General formula of heat regulating zone

According to formula (6), when Bi is equal to 9, the tunnel airflow temperature under different z and Fo conditions is calculated and drawn into a three-dimensional surface, as shown in Fig. 5. The value of z ranges from 0 to 1000m. To show the temperature distribution of the long-term operation of the tunnel, the value of Fo ranges from 10^{-2} to 10^6 .

It can be seen from the Fig. 5 that when Fo is small, the temperature of the tunnel airflow gradually decreases along the flow direction, and the rate of decrease becomes slower and slower. The reason is that the airflow temperature at the tunnel entrance is high, and the temperature difference with the tunnel wall surface is large, resulting in a large heat exchange. However, as the distance z from the entrance increases, the airflow temperature decreases, the temperature difference between the airflow and the tunnel wall

decreases, and the heat exchange decreases, resulting in a slower rate of decrease in airflow temperature. Currently, the change of the tunnel airflow is approximately an exponential function

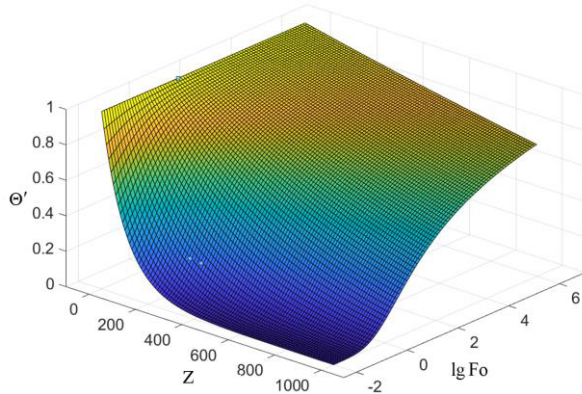


Fig. 5. The airflow temperature for different z and Fo

In addition, the tunnel airflow temperature is not only related to the distance z from the entrance, but also to the ventilation time, but it can be seen from the figure that when Fo is greater than 4, the effect of Fo on the tunnel airflow temperature can be ignored, that is, when Fo is greater than 4, the temperature distribution of the tunnel airflow is almost stable. The stabilized tunnel airflow temperature is very different from the initial tunnel airflow temperature, which means that when designing the EAHE or tunnel ventilation, it is necessary to consider the effect of ventilation time. Equation (6) can be used to calculate the effect of running time to achieve accurate and fast the design of EAHE. For example, there is a tunnel with a diameter of 1m and a total length of 400m. The tunnel inlet temperature is 30°C, the initial soil temperature is 10°C, and the air velocity is 4m/s. The soil and air properties are shown in Table 1. According to formula (6), it can be calculated that the temperature of the dimensionless airflow in the tunnel can be reduced to 0.29 after just 1 day of operation, that is, the airflow can be cooled to 15.8 °C, and the temperature difference between the inlet and outlet is 14.2 °C. But after one year of operation, the temperature of the dimensionless airflow in the tunnel can be reduced to 0.77, that is, the airflow can only be cooled to 25.4°C, and the temperature difference between the inlet and outlet is 4.6°C. After five years of operation, the temperature of the dimensionless airflow in the tunnel can be reduced to 0.82, that is, the airflow can only be cooled to 26.4°C, and the temperature difference between the inlet and outlet is 3.6°C.

4 Conclusion

In this paper, the calculation formula of the unsteady temperature field of the EAHE was derived and the calculation correlation of value of the heat regulating zone was also given. The research conclusions are as follows:

The calculation formula of the heat regulating zone R_b (outer boundary of the calculation domain) was derived as Equation (9). The applicable range of this formula is that Fo is between 0.1 and 1000 and Bi is

between 1 and 20. The calculation of this formula is accurate, the goodness of fit is 0.999, and the root mean square error is 0.3914.

The calculation formula of the unsteady the dimensionless airflow of EAHE was derived and agrees well with the experimental data. The cooling capacity of the EAHE is not only related to the tunnel length z , but also to dimensionless time. For EAHE design, the influence of operation time can be accurately calculated by formula (6).

Nomenclature

$t(r, \tau)$ $\Theta(r, \tau)$	Temperature, excess temperature and dimensionless temperature of soil.
$t_f(z, \tau)$ $\Theta'(z, \tau)$	Temperature and dimensionless temperature of airflow.
λ_s, λ_a	Thermal conductivity of soil and airflow (W/(m·K))
ρ_s, ρ_a	Density of soil and airflow (kg/m ³)
$c_{p,s}, c_{p,a}$	Specific heat capacity of soil and airflow (J/(kg·K))
r, r_0, z	Tunnel radius and length (m)
\dot{m}	Mass flow of air (kg/s)
τ	operation time (s)
h	Convective heat transfer coefficient (W/(m ² ·K))
Fo	Dimensionless time
Bi	Dimensionless thermal resistance ratio

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