Numerical study of nonlinear problems in the dynamics of thin-walled structural elements

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Abstract. Mathematical model of the problem of vibration of thin-walled structural elements has been constructed based on Kirchhoff-Love theory. The problem is reduced, using the Bubnov-Galerkin method, to the solution of a set of nonlinear integro-differential Volterra type equations with weakly-singular kernels of relaxation. A numerical method based on the use of quadrature formulae being used for their solution. The influence of rheological parameters of the material on the values of critical velocity and amplitude-frequency characteristics of viscoelastic thin-walled structural elements is analyzed. It is shown that tacking account viscoelastic properties of the material of thin-walled structures lead to a decrease in the critical rate of gas flow.

1 Introduction

The theory of viscoelasticity is attracting more and more interest from researchers due to the widespread use of new materials in technology and traditional materials in specific conditions. Evidence of this is the publication of several articles [1-5]. Recently, much attention has been paid to studying the dynamics of essentially nonlinear viscoelastic mechanical systems [6-12], [26, 27].

The basic research trend consisted of preliminary reduction of problems using variational methods of a continuous structure to a system with one-or-two degrees of freedom, which was then analyzed either numerically or using analytical methods of nonlinear mechanics. The main attention was paid to determining the qualitative effects caused by the impact of nonlinear forces.

The problems of bending, strength and dynamics of viscoelastic thin-walled structural elements were studied by B.A. Khudayarov and his students [13-25], [28-43].

This work is devoted to mathematical modelling and creating an algorithm for the numerical solution of dynamic problems of hereditary deformable systems.

Consider the dynamics of a thin-walled structure accounting for hereditary properties of the material according to the generalized Timoshenko theory in a geometrically nonlinear statement.

Under accepted assumptions, the mathematical model of this problem relative to the functions $w=w(x, y, t)$, $\psi_x = \psi_x(x, y, t)$, $\psi_y = \psi_y(x, y, t)$ and $\Phi=\Phi(x, y, t)$ is described by the equation under corresponding boundary and initial conditions:

$$\frac{K^2 E}{2(1+\mu)}\left(1-R^*\right)\left(\nabla^2 w + \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}\right) + L(w,\Phi) - \frac{\ae p_\infty}{hV_\infty}\left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right) - \rho\frac{\partial^2 w}{\partial t^2} = 0,$$

$$\frac{D}{h}\left(1-R^*\right)\left[\frac{\partial^2 \psi_x}{\partial x^2} + \frac{1}{2}(1+\mu)\frac{\partial^2 \psi_y}{\partial x \partial y} + \frac{1}{2}(1-\mu)\frac{\partial^2 \psi_x}{\partial y^2}\right] -$$

$$\frac{K^2 E}{2(1+\mu)}\left(1-R^*\right)\left(\frac{\partial w}{\partial x} + \psi_x\right) - \rho\frac{h^2}{12}\frac{\partial^2 \psi_x}{\partial t^2} = 0, \left(x \leftrightarrow y\right)$$

$$\frac{1}{E}\nabla^4\Phi = -\frac{1}{2}\left(1-R^*\right)L(w,w) \tag{1}$$

2 Methods

Let the thin-walled structure be supported by hinges on all edges. Satisfying the boundary conditions of the problem, we choose expressions for functions $w=w(x, y, t)$, $\psi_x = \psi_x(x,y,t)$, $\psi_y = \psi_y(x,y,t)$ based on the polynomial approximation in the following form:

$$w(x, y, t) = \sum_{n=1}^{N}\sum_{m=1}^{M} w_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$

$$\psi_x(x, y, t) = \sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{xnm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$

$$\psi_y(x, y, t) = \sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{ynm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b} \tag{2}$$

Substituting the first expression (2) into the fourth equation of system (1) and equating in both sides of this equation the coefficients for the same harmonics of trigonometric functions, we find the force function:

$$\Phi(x,y,t) = E\sum_{i,j=1}^{N}\sum_{r,s=1}^{M}\left(1-R^*\right)w_{ir}w_{js}[C_{irjs}\cos\frac{(i+j)\pi x}{a}\cos\frac{(r+s)\pi y}{b} +$$

$$+A_{irjs}\cos\frac{(i+j)\pi x}{a}\cos\frac{(r-s)\pi y}{b} + D_{irjs}\cos\frac{(i-j)\pi x}{a}\cos\frac{(r+s)\pi y}{b} +$$

$$+B_{irjs}\cos\frac{(i-j)\pi x}{a}\cos\frac{(r-s)\pi y}{b}] \tag{3}$$

where

$$C_{irjs} = -\frac{\lambda^2 ir(ir\text{-}js)}{4\left[\left(i+j\right)^2 + \lambda^2\left(r+s\right)^2\right]^2}, \quad A_{irjs} = \frac{\lambda^2 ir(ir+js)}{4\left[\left(i+j\right)^2 + \lambda^2\left(r-s\right)^2\right]^2},$$

$$D_{irjs} = \frac{\lambda^2 ir(ir+js)}{4\left[\left(i-j\right)^2 + \lambda^2\left(r+s\right)^2\right]^2}, \quad B_{irjs} = -\frac{\lambda^2 ir(ir-js)}{4\left[\left(i-j\right)^2 + \lambda^2\left(r-s\right)^2\right]^2}, \quad \lambda = \frac{a}{b}$$

Substituting (2) and (3) into the first three equations of (1) and performing the Bubnov-Galerkin procedure with respect to w_{kl}, $\psi_{xkl\ l}$, ψ_{ykl}, we obtain

$$\rho\,\ddot{w}_{kl} + \frac{\text{ӕ}\varphi_\infty}{V_\infty h}\,\dot{w}_{kl} + \frac{K^2 E\pi^2}{2(1+\mu)b^2}\left[\left(\frac{k}{\lambda}\right)^2 + l^2\right]\left(1-R^*\right)w_{kl} +$$

$$+\frac{2\text{ӕ}\varphi_\infty V}{ahV_\infty}\sum_{n=1}^{N} n\left(\gamma_{n+k} - \gamma_{n-k}\right)w_{nl} +$$

$$+\frac{K^2 E\pi}{2(1+\mu)b}\left(1-R^*\right)\left[\left(\frac{k}{\lambda}\right)\psi_{xkl} + l\psi_{ykl}\right] +$$

$$+\frac{E\pi^2}{b^4}\sum_{n,i,j=1}^{N}\sum_{m,r,s=1}^{M} a_{klnmirjs}w_{nm}\left(1-R^*\right)w_{ir}w_{js} = 0,$$

$$\rho h^2\,\ddot{\psi}_{xkl} + \frac{Eh^2}{2(1-\mu^2)}\left[\frac{2\pi^2 k^2}{a^2} + \frac{\pi^2 l^2}{b^2}(1-\mu) + \frac{12K^2}{h^2}(1-\mu)\right]\left(1-R^*\right)\psi_{xkl} +$$

$$+\frac{E\pi^2 h^2 kl}{2ab(1-\mu)}\left(1-R\ \right)\psi_{ykl} + \frac{6K^2 E\pi k}{a(1+\mu)}\left(1-R\ \right)w_{kl} = 0,$$

$$\rho h^2\,\ddot{\psi}_{xkl} + \frac{Eh^2}{2(1-\mu^2)}\left[\frac{2\pi^2 k^2}{a^2} + \frac{\pi^2 l^2}{b^2}(1-\mu) + \frac{12K^2}{h^2}(1-\mu)\right]\left(1-R^*\right)\psi_{xkl} +$$

$$+\frac{E\pi^2 h^2 kl}{2ab(1-\mu)}\left(1-R^*\right)\psi_{ykl} + \frac{6K^2 E\pi k}{a(1+\mu)}\left(1-R^*\right)w_{kl} = 0, \tag{4}$$

$$\rho h^2\,\ddot{\psi}_{ykl} + \frac{Eh^2}{2(1-\mu^2)}\left[\frac{2\pi^2 l^2}{b^2} + \frac{\pi^2 k^2}{a^2}(1-\mu) + \frac{12K^2}{bh}(1-\mu)\right]\left(1-R^*\right)\psi_{ykl} +$$

$$+\frac{E\pi^2 h^2 kl}{2ab(1-\mu)}\left(1-R^*\right)\psi_{xkl} + \frac{6K^2 E\pi l}{b(1+\mu)}\left(1-R^*\right)w_{kl} = 0$$

$$w_{kl}(0) = w_{0kl},\ \dot{w}_{kl}(0) = \tilde{w}_{0kl}, \psi_{xkl}(0) = \psi_{x0kl},\ \dot{\psi}_{xkl}(0) = \tilde{\psi}_{x0kl},$$

$$\psi_{ykl}(0) = \psi_{y0kl}, \ \dot{\psi}_{ykl}(0) = \tilde{\psi}_{y0kl}, k = 1, 2, ..., N; l = 1, 2, ..., M \ ,$$

where $\gamma_k = \dfrac{\alpha_k}{k}$; coefficient $\alpha_k = 1$, if k is odd, if k is even or equal to 0, then $\alpha_k = 0$; $a_{klnmirjs}$ are the dimensionless coefficients.

Introducing into the system (4) dimensionless quantities

$$\frac{w_{kl}}{h}, \frac{V_\infty t}{a}, \frac{aR(t)}{V_\infty}$$

while maintaining the previous notation relative to the dimensionless unknowns $w_{kl} = w_{kl}(t)$, $\psi_{xkl} = \psi_{xkl}(t)$ and $\psi_{ykl} = \psi_{ykl}(t)$ we obtain

$$\ddot{w}_{kl} + M_p \varpi \lambda \delta \left\{ \dot{w}_{kl} + \frac{K^2 M_E \pi^2 \lambda^2}{2(1+\mu)} \left[\left(\frac{k}{\lambda}\right)^2 + l^2 \right] (1 - R^*) w_{kl} + \right.$$

$$+ 2M \ M_p \varpi \lambda \delta \sum_{n=1}^{N} n(\gamma_{n+k} - \gamma_{n-k}) w_{nl} +$$

$$+ \frac{K^2 M_E \pi \lambda^2 \delta}{2(1+\mu)} (1 - R^*) \left[\left(\frac{k}{\lambda}\right) \psi_{xkl} + l \psi_{ykl} \right] +$$

$$\frac{p^2 M_E \lambda^2}{\delta^2} \sum_{n,i,j=1}^{N} \sum_{m,r,s=1}^{M} a_{klnmirjs} w_{nm} (1 - R^*) w_{ir} w_{js} = 0,$$

$$\ddot{\psi}_{xkl} + \frac{M_E}{2(1-\mu^2)} \left[2\pi^2 k^2 + \pi^2 \lambda^2 (1-\mu) l^2 + 12 K^2 \lambda^2 \delta^2 (1-\mu) \right] (1 - R^*) \psi_{xkl} +$$

$$+ \frac{M_E \pi^2 \lambda}{2(1-\mu)} kl (1 - R^*) \psi_{ykl} + \frac{6 K^2 M_E \pi \lambda \delta}{1+\mu} k (1 - R^*) w_{kl} = 0, \qquad (5)$$

$$\ddot{\psi}_{ykl} + \frac{M_E}{2(1-\mu^2)} \left[2\pi^2 \lambda^2 l^2 + \pi^2 (1-\mu) k^2 + 12 K^2 \lambda^2 \delta^2 (1-\mu) \right] (1 - R^*) \psi_{ykl} +$$

$$+ \frac{M_E \pi^2 \lambda}{2(1-\mu)} kl (1 - R^*) \psi_{xkl} + \frac{6 K^2 M_E \pi \lambda^2 \delta}{1+\mu} l (1 - R^*) w_{kl} = 0,$$

$$w_{kl}(0) = w_{0kl}, \ \dot{w}_{kl}(0) = \tilde{w}_{0kl}, \psi_{xkl}(0) = \psi_{x0kl}, \ \dot{\psi}_{xkl}(0) = \tilde{\psi}_{x0kl},$$

$$\psi_{ykl}(0) = \psi_{y0kl}, \ \dot{\psi}_{ykl}(0) = \tilde{\psi}_{y0kl}, k = 1, 2, ..., N; l = 1, 2, ..., M \ ,$$

where $\delta = \dfrac{b}{h}$, $M_p = \dfrac{p_\infty}{\rho V_\infty^2}$, $M_E = \dfrac{E}{\rho V_\infty^2}$, $M^* = \dfrac{V}{V_\infty}$.

4 Results and discussion

Here, the upper exact limit of the velocity set {V} is chosen as a criterion for determining the critical rate of gas flow; it ensures the convergence of the Bubnov-Galerkin expansion (2) for all $t=0$ (Fig. 1), i.e. the following condition is satisfied

$$w\left(x,y,t\right) = \sum_{n=1}^{N}\sum_{m=1}^{M} w_{nm}\left(t\right)\varphi_{nm}\left(x,y\right) < 1$$

At $V>V_{cr}$, the oscillatory motion occurs at rapidly increasing amplitudes and can lead to structure destruction (Fig. 2); at $V<V_{cr}$, the vibration amplitude damps (Fig. 3). Note that at $V>V_{cr}$, the expansion (2) diverges.

Here, the initial data are taken as: $A=0.0104$, $\alpha=0.1$, $\beta=0.0166$ (material KAST-B 90^0), $M_E=4.71$, $M_P=0.003$, $V=1900$ m/s (Fig.1), 2300 m/s (Fig.2), 600 m/s (Fig.3).

The results of calculating a thin-walled structure with the hereditary properties of a material streamlined by a supersonic gas flow at $V<V_{cr}$ are presented in Figs. 4 - 6. For this purpose, the gas flow velocity is assumed to be 800 m/s when studying the behaviour of a thin-walled structure with various physical and geometric parameters. Unless other data are specified, the following values are taken as the initial ones: $A=0.0104$, $\alpha=0.1$, $\beta=0.0166$ (material KAST-B 90^0), $M_E=4.71$, $M_P=0.003$.

Figs. 4 - 6 show the dependences of functions w, u and v, respectively, on time in the midpoint of a thin-walled structure not considering ($A = 0$ - curve 1) and considering the hereditary properties of the material ($A = 0.0099$, $\alpha = 0.1$, $\beta = 0$ .001 (material) KAST-B 0^0 - curve 2; $A = 0.0208$, $\alpha = 0.1$, $\beta = 0.0166$ (material KAST-B 45^0) - curve 3).

As seen from Fig. 4, an account for hereditary properties of the material leads to oscillatory process attenuation. Although the solutions to problems with and without account for hereditary properties of the material in the initial period of time differ little from each other, over time, the hereditary properties of the material have a significant impact.
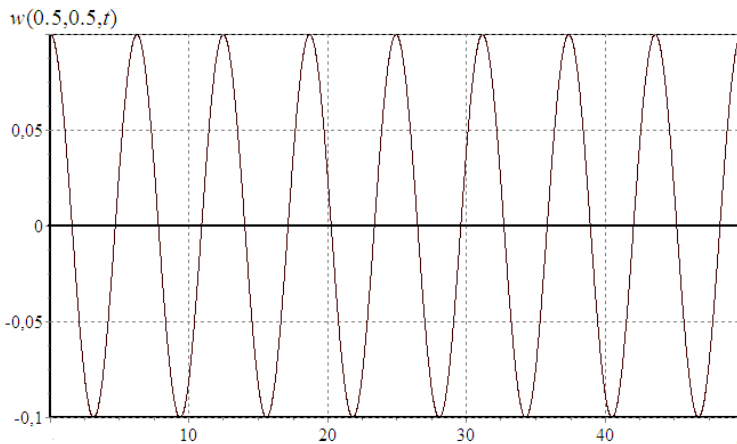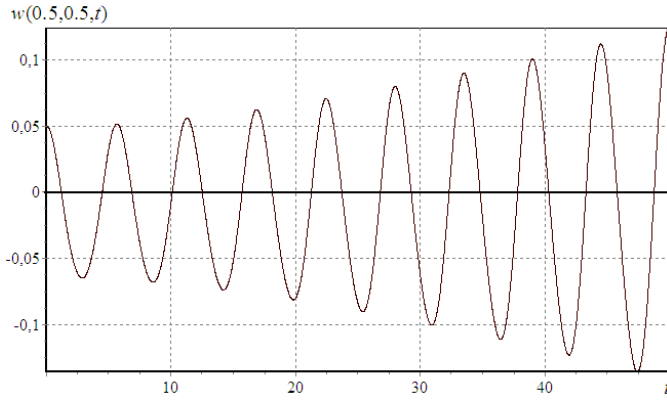

Fig.1. Deflection dependence on time
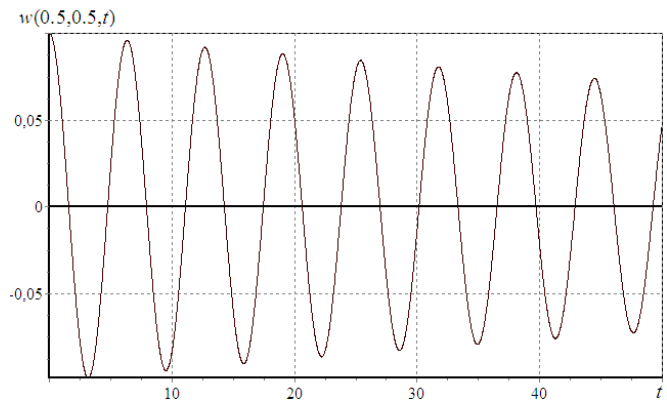
Fig.2. Deflection dependence on time
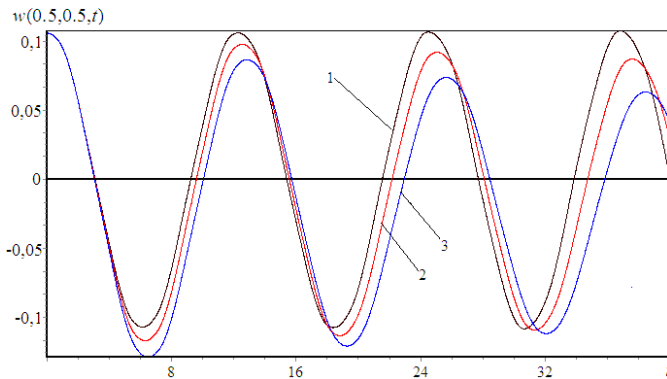

Fig.3. Deflection dependence on time

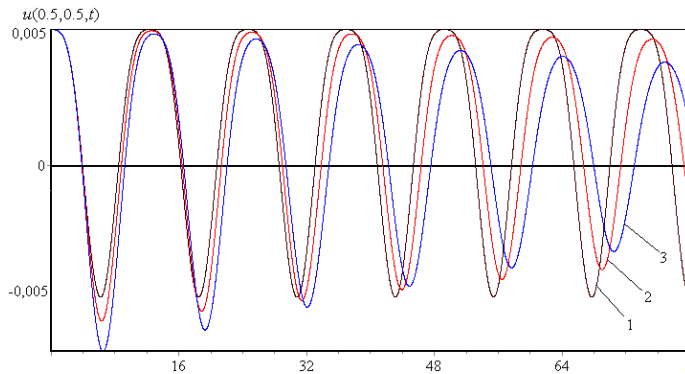**Fig.4.** Function *w* dependence on time: *A*=0 (1); KAST-B 0^0 (2); KAST-B 45^0 (3)

Fig.5. Function *u* dependence on time: *A*=0 (1); KAST-B 0^0 (2); KAST-B 45^0 (3)
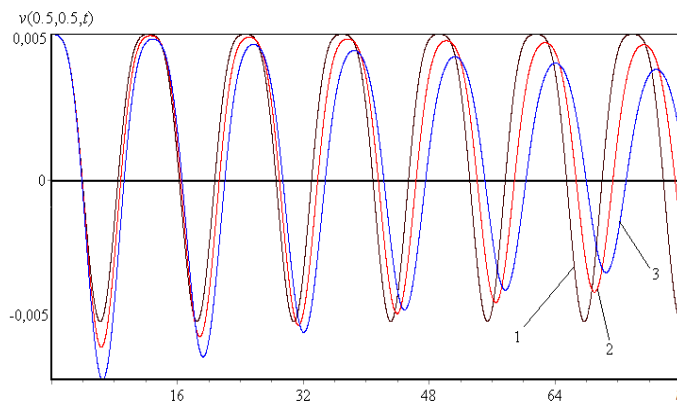

Fig.6. Function *v* dependence on time: *A*=0 (1); KAST-B 0^0 (2); KAST-B 45^0 (3)
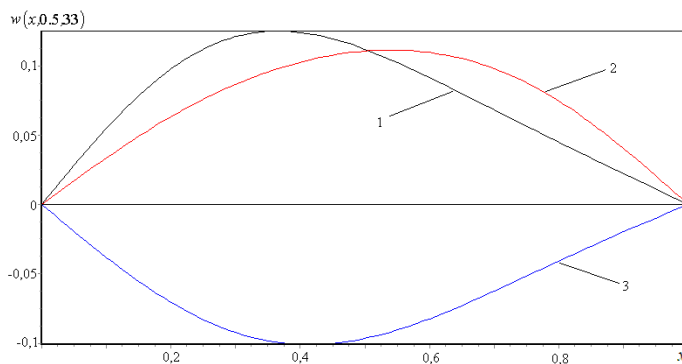

Fig.7. Comparison of the modes of vibrations at *t*=33: *A*=0 (1); KAST-B 0^0 (2); KAST-B 45^0 (3)

Figure 7 shows the change in the function *w* along the length of a thin-walled structure at $t = 33$. Consideration of hereditary properties shows a decrease in the values of maximum deflections.

4 Conclusions

Mathematical models of the dynamics problems of thin-walled structural elements are built considering hereditary properties of the material. The basic resolving integro-differential

equations of dynamic problems of the hereditary theory are obtained using the Bubnov-Galerkin method.

The effect of various properties of structure material on the values of critical velocity and amplitude-frequency characteristics is analyzed.

An analysis of the results revealed some new effects:

- an account for hereditary properties of the material of thin-walled structures leads to a decrease in the critical rate of gas flow;

- an account for geometrical nonlinearity leads to an increase in critical velocity;

- an account for aerodynamic nonlinearity does not significantly change the value of critical velocity.

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