

Velocity distribution in a turbulent flow

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Abstract. The article considers the issue of velocity distribution in a turbulent flow. The logarithmic velocity profile obtained on the basis of semiempirical theory of turbulence by L. Prandtl is given. A comparison of the calculations of the velocity distribution for various dependencies with the measurement data is performed. It is shown that the calculated dependencies show discrepancy with each other and experimental data. The study of the hydraulic characteristics of water flows is required to solve problems of water supply and water disposal, engineering problems of hydraulic construction, regulation of riverbed processes and environmental monitoring of water bodies. The characteristics of the turbulence of a uniform and uneven flow need to be clarified, that will be possible due to the study of the theoretical background of hydraulics and hydrodynamics. Despite the usage of modern computer technologies, the creation of an actual theory of turbulence is still not complete. The separation of the fluid motion into molar and basic motion led to a non-closed system of equations of turbulent motion, that slowed down further progress in the search for a solution for turbulent flow. The most widely used theory is the semi-empirical theory of turbulence by L. Prandtl, based on a phenomenological approach that links turbulence with the characteristics of the averaged flow. The use of modern technology has allowed us to obtain new experimental data that require analysis and generalization. The calculated dependences for the velocity distribution developed on the basis of various phenomenological models, including the well-known logarithmic profiles of L. Prandtl, are compared with the measurement data. The calculations of the velocity distribution for various dependencies are compared with the measurement data. A significant quantitative, and sometimes qualitative, discrepancy of the calculated dependences with each other and with the experimental data was revealed. The discrepancy between the results of calculating the velocity distribution for different dependencies requires an additional critical analysis of the accuracy and reliability of the experimental data of other authors and performing experimental studies under different conditions using a mutually agreed method. The low effectiveness of the considered statistical approaches leads to the need to develop new approaches that represent a combination of some theoretical concepts and experimental data.

1 Introduction

The study of the hydraulic characteristics of water flows is required to solve problems of water-supply and water disposal, engineering problems of hydraulic construction, regulation of riverbed processes and environmental monitoring of water bodies. The characteristics of

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the turbulence of a uniform and nonuniform flow need to be clarified. It will be possible due to the study of the theoretical background of hydraulics and hydrodynamics.

Existing methods for determining flow rates using various experimental means, such as hydrometric propeller, surface and depth floats, various devices that measure the physical and chemical properties of water, as well as methods based on the use of ultrasound and Doppler effect, ignore the features of the flow in the wall area, that are important in evaluation cavitation erosion and predicting washouts in the areas, where the influence of the flow rate is great.

Nowadays, it is known that friction forces acting between the layers of the liquid make these layers move at different speeds. With distance from the dynamic axis of motion the velocity decreases and in the wall area of the flow it considered equal to zero: the liquid "sticks" to the wall of the pipeline or channel.

The velocity distribution in the laminar flow of a liquid represents a parabolic dependence. In the turbulent flow, the velocity distribution has a wider top due to the alignment of the velocities in the main mass of the flow caused by the mixing of liquid masses. It also stands to mention that it is theoretically not possible to obtain a velocity profile in a turbulent flow. In a turbulent flow, the true velocity at each point does not remain constant due to the chaotic motion of the liquid particles.

Active and increasingly complex in their methodology studies of turbulent flows have continued after the work of O. Reynolds was published in 1898 [1]. Reynolds applied the principle of superposition and divided the fluid motion into the main (time-averaged) and molar (pulsation) motions. The latter is summed up with the averaged motion at every moment of time, forming the current turbulent fluid motion. Despite the usage of modern computer technologies, the creation of an actual theory of turbulence is still incomplete. The separation of the fluid motion into molar and basic motions has created a non-closed system of equations of turbulent motions that slowed down further progress in the search for a solution for the turbulent flow.

To date, the problem of velocity distribution in a turbulent flow is being studied by both foreign and national specialists [2-5]. For example, the staff of the HaHE (Hydraulics and Hydrotechnical engineering) department of MSUCE has established a universal correlation between the coefficient of hydraulic resistance, the Karman parameter, and the indicator of the velocity distribution degree [6].

2 Methods

The most widely used theory is the semi-empirical theory of turbulence by L. Prandtl, based on a phenomenological approach that connects turbulence with the characteristics of the averaged flow [7]. This link is implemented in the form of a simple relation:

$$u'_x = l \frac{du}{dz} \quad (1)$$

(l – length of the mixing path),

That has the following physical meaning: the turbulent pulsations of the lengthwise direction arise due to the chaotic exchange of mass and the amount of motion between the neighboring layers of the turbulent "shear" flow. This physical meaning was not used in further studies of turbulence. Using of approximate thermodynamic Lorentz relation:

$$\tau = \overline{u'_x u'_z} \quad (2)$$

and a number of assumptions, L. Prandtl managed to obtain a logarithmic velocity profile, that was confirmed by Nikuradze in his experimental data:

$$u = \frac{u_*}{\kappa} \ln z + C \quad (3)$$

Many scientists, such as J. V. Boussinesq, A. A. Satkevich, L. D. Landau, and a number of others, have tried to clarify or refute the assumptions of Prandtl that were used to create the logarithmic velocity profile [8-10]. The results of the research are of great importance for practice, but they do not answer the main question: what are the physical causes of turbulence in the near-wall area, where the flow was considered to be laminar. The answer to this question has only begun coming to light in recent decades due to the development of measurement techniques that can show the state of motions in the wall layers. The study of the flow in the boundary layers made with ultra-microscope, using the technique of hydrogen bubbles, the thermoanemometrical measurements, and, finally, laser Doppler measurements revealed the appearance of instability spots of the flow near the wall and the fluid mixing in the boundary layers of flow [11,12]. These phenomena need further research and theoretical description. The measurements revealed a clear periodicity of the viscous wall flow destruction. Consequently, the masses of liquid from the near-wall layer penetrate into the main flow, transferring the momentum of the friction force. Such an intermittent flow was theoretically described on the basis of a simplified equation of motion by H. A. Einstein and H. Lee [13], who managed to obtain a velocity profile for a nonstationary flow in a viscous sublayer. This calculation was transformed by V. S. Borovkov and F. G. Mairanovsky [14], as well as Y. V. Bryanskaya [15]. As a result, it was possible to obtain characteristics of the periodicity of the growth and destruction of the sublayer that are consistent with the measurement data of S. G. Nichas [16] and other researchers. It should be noted that the compensatory movements of water masses that occurred as a result of the destruction of the sublayer were ignored in the calculations

The logarithmic velocity distribution currently used in various literature and in practice is based on the semiempirical theory of turbulence and is experimentally confirmed by the measurements of I. Nikuradze [17, 18]. The logarithmic velocity profile is determined by the Karman parameter (the first constant of turbulence) and the value C , that is sometimes called the second constant of turbulence, their values are determined on the basis of experimental data of I. Nikuradze and are on average equal for smooth pipes $\kappa=0,4$; $C=5,5$; for rough pipes – $\kappa=0,4$; $C=8,48$.

Subsequent analysis of I. Nikuradze's measurements and new experimental data from other authors indicate that the parameters of the logarithmic velocity profile are not constant. However, the relation between the parameters of the logarithmic velocity profile and the influence of the hydraulic characteristics of the flow on their value were not revealed.

3 Results and discussions

The comparison of the known logarithmic profiles with I. Nikuradze's measurement data is shown in Fig. 1 and Fig. 2:

– for smooth pipes:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{u_* z}{\nu} + 5,5 \quad (4)$$

– for rough pipes:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{z}{k_s} + 8,48 \quad (5)$$

detects an increasing discrepancy between the logarithmic profiles and the measured velocities, which coincide only in the near-wall region of the flow with a thickness of $0,15r$ [19].

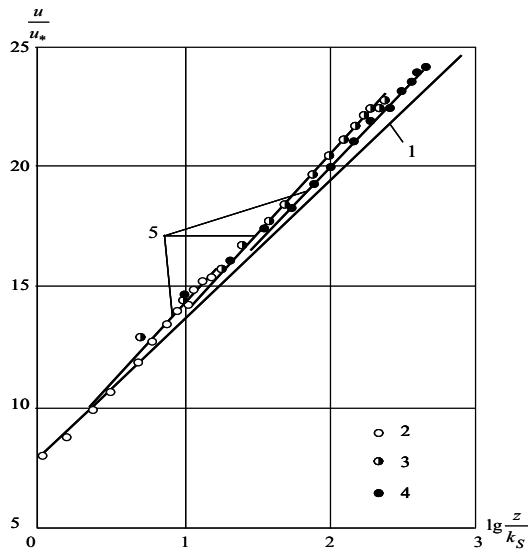


Fig. 1. Comparison of speed profiles with measurement data

1 — profile I. Nikuradze; 2-4 — measurement data of I. Nikuradze (2 — $Re = 108 \cdot 10^3$; $r/k_S = 15$; 3 — $Re = 344 \cdot 10^3$; $r/k_S = 252$; 4 — $Re = 970 \cdot 10^3$; $r/k_S = 507$); 5 — calculation according to (4) with correction (7)

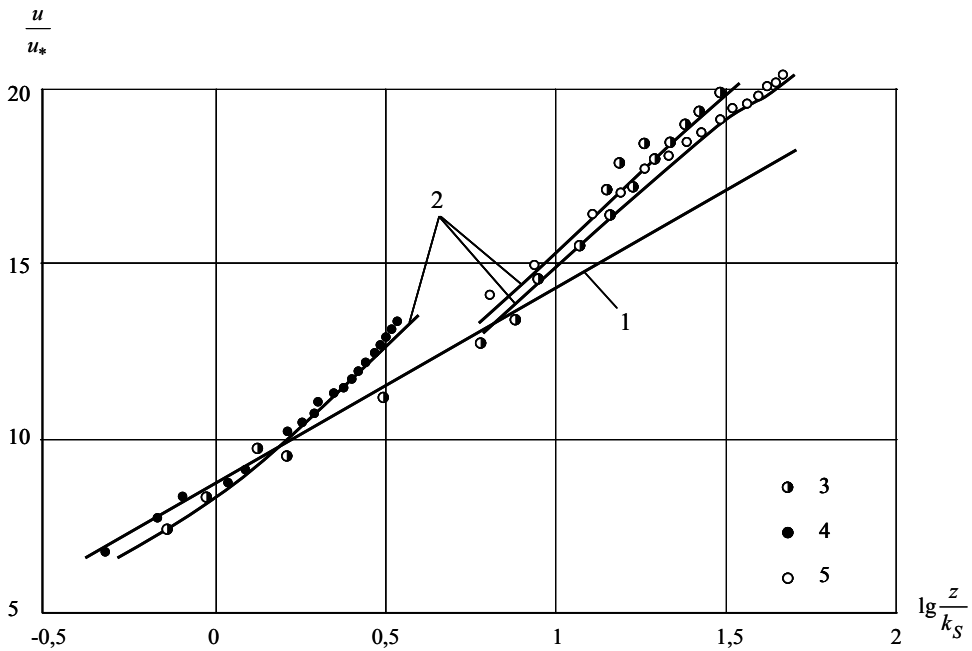


Fig. 2. Comparison of speed profiles with measurement data

1 — calculation by the ratio (4) at $\kappa = 0,4$ and $u\kappa / u_* = 8,48$; 2 — calculation for (5) adjusted for (8) [20]; 3 — measurement data of S. Korrsina and A. Kistler, $\delta / k_S = 30$ (corrugated surface); 4 — measurement data of V.S. Borovkov, $h / k_S = 4$ (cylindrical protrusions arranged in a staggered pattern); 5 — the same, $h / k_S = 50$ (wooden canal)

The usage of the following expression, that approximates the data of many dimensions, proved to be quite effective [14]:

$$\frac{v_T}{\kappa u_* h} = \frac{z}{h} \frac{1 - \frac{z}{h}}{1 + \frac{z}{h} - \frac{1}{4} \left(\frac{z}{h} \right)^2}, \quad (6)$$

by which it was possible to obtain the following correction functions for the logarithmic profile:

- for flow in pipes

$$\frac{\Delta u}{u_*} = \frac{1}{2\kappa} \frac{z}{r} \left(1 - \frac{1}{2} \frac{z}{r} \right); \quad (7)$$

- for flow in canals

$$\frac{\Delta u}{u_*} = \frac{1}{\kappa} \frac{z}{h} \left(1 - \frac{1}{8} \frac{z}{h} \right). \quad (8)$$

The comparison of profiles (4) and (5), including the correction functions (7) and (8), with the measurement data in channels and pipes, shown in Fig. 1 and Fig. 2, indicates good convergence. As shown in [14], the correction of the velocity profiles also corresponds to the well-known experimental laws of resistance.

It should be noted that such approaches are basically empirical and dependences (7) and (8) contain an indeterminate parameter and, most importantly, do not find any clear physical explanations.

A comparison of the various dependencies for the velocity distribution over the flow depth with the experimental data is shown in Fig. 3

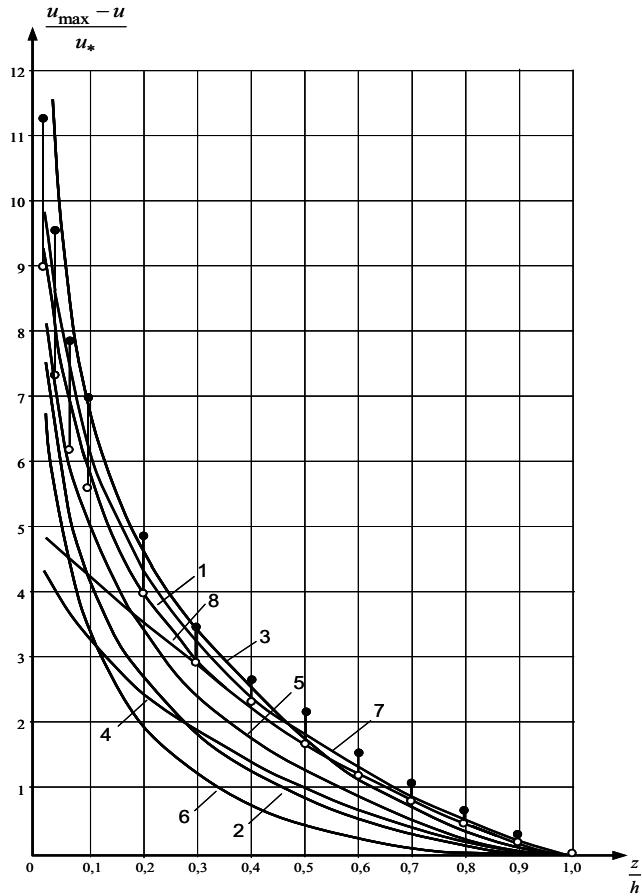


Fig. 3. Comparison of various dependencies with an experimental data:

$$\left. \begin{array}{l} \text{Re} = 3,2 \cdot 10^6 \\ \text{Re} = 4,1 \cdot 10^3 \end{array} \right\} \begin{array}{l} \bullet \\ \circ \end{array} \text{ — experiments of I. Nikuradze;}$$

1 — L. Prandtl; 2 — G. Taylor–Velikanov; 3 — A. D. Altshul; 4 — D. Taylor; 5 — T. Karman; 6 — I. P. Ginzburg; 7 — V. S. Borovkov; 8 — A. Darcy

4 Conclusions

The calculated dependences for the velocity distribution developed on the basis of various phenomenological models reveal a significant quantitative, and sometimes qualitative, discrepancy with each other and with the experimental data (Fig. 3).

This circumstance requires an additional critical analysis of the accuracy and certainty of the experimental data of other authors and the execution of experimental studies under different conditions according to a mutually agreed methodology.

The low effectiveness of the considered statistical approaches leads to the need to develop new approaches that represent a combination of some theoretical concepts and experimental data.

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