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Probability-theoretical approach to the accuracy of the component assembly of multilink mechanisms

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Abstract. The technological process, ensuring the accuracy of mechanism assembly is formally a sequential summation of primary manufacturing errors and an analysis of obtained, generalized values to develop and implement specific technological methods to meet process requirements for the accuracy of the relative position of parts. To ensure the accuracy of crank mechanisms for misalignment of piston and cylinder axes, assembled from specific, randomly received component parts, production personnel deal with strictly defined crankshafts, connecting rods, pistons, cylinder blocks, and, consequently, with one or another manufacturing errors. To ensure the accuracy of these parts assembly in a mathematical sense, with such a sequence, will represent nothing more than an algebraic summation of primary errors and compilation of their numerical values with the maximum permissible values. A different situation arises in the implementation of assembly technological processes with automatic achievement of accuracy parameters by varying individual parts from their total number. As in this case fairly representative aggregates are analyzed, the final result of combining parts into assemblies will occur with a greater or lesser probability, especially when the sum of the maximum deviations of constituent links (parts) is not equal and not less than the maximum permissible total values.

1 Introduction

The release probability from the assembly site of high-quality products with automatic support and accuracy will be determined by modules of individual deviations and by distribution laws of actual deviations within the tolerance range.

The process of manufacturing or repairing parts is accompanied by primary production errors in the relative position of base surfaces. Due to the influence of a large number of random factors on machining, these errors are also random variables, the distribution laws

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of which approach the Gaussian law the more fully the Lyapunov A.M. conditions are met. when summing up any primary errors.

Theoretical or basic patterns of distribution of primary errors are in good agreement with practical distributions, as a rule, only with strict identification of technological processes designed and adopted as standard (basic) ones. When solving the problem of assembly optimization, all these provisions are not significant, as in the case of constructing its general solution, the choice of a specific distribution law of actual deviations within the tolerance range is not so fundamental that one does not try to look for such a solution that would hold various initial arguments. In this case, any analogies in the distribution patterns will transform the general solution into particular variants with obvious simplifications of computational operations.

Thus, for a component assembly of crank mechanisms for the misalignment of piston and cylinder axes with automatic provision of the required accuracy by a sequential selection of deviations of parts on a computer with the preliminary formation of arrays of high-quality assembly units by calculation, the meaning of the main task of the mathematical model of the assembly operation in this case is reduced to theoretical-probabilistic analysis of real mechanisms to:

— construct a general solution for probability of high-quality assemblies with distortions of pistons within specified limits for arbitrary but known tolerance fields and distribution laws of actual deviations in the relative position of the base surfaces of the constituent parts within these tolerances of the probabilities of the sum of random variables falling into some previously specified scattering interval

— assessment of expansion possibilities and boundaries of considered maximum deviations without noticeable deterioration in the quality indicators of assembled mechanisms.

2 Materials and Methods

We believe that all considered errors in the relative position of base surfaces of the parts of crank-connecting rod mechanisms are the essence of values, independent and random in nature, conditionally distributed within the tolerance range according to patterns, analytically described by the following dependencies:

a) the error in the relative position of cylinder liner and crankshaft axes according to Gauss's law (FIG. 1, a):

$$F_1(\psi_1) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[-\frac{(\psi_1 - a_1)^2}{2\sigma_1^2}\right], -\infty \le \psi_1 \le \infty; \quad (1)$$

b) the error in the relative position of the main and connecting rod journal axes of the crankshaft according to the triangle law (FIG. 1, b):

$$F_2(\psi_2) = K|\psi_2|, -\xi \le \psi_2 \le \xi; \quad (2)$$

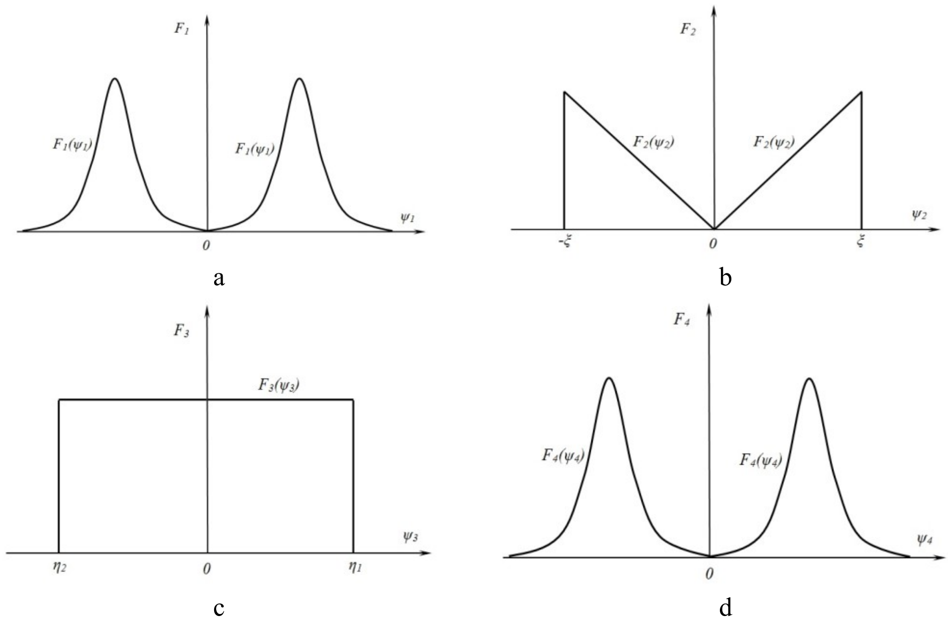

Fig. 1. Scheme of initial distributions of primary errors.

c) the error in the relative position of the connecting rod bearing axes according to the equal probability law (Fig. 1, c):

$$F_3(\psi_3) = \frac{1}{\eta_1 - \eta_2}, \eta_2 \le \psi_3 \le \eta_1; \tag{3}$$

d) the error in the relative position of piston generatrix piston and the piston pin hole according to the law (Fig. 1, d):

$$F_4(\psi_4) = \frac{1}{\sigma_4 \sqrt{2\pi}} \exp\left[-\frac{(\psi_4 - a_4)^2}{2\sigma_4^2}\right], -\infty \le \psi_4 \le \infty. \tag{4}$$

In the future, to simplify calculations, we assume $a_1 = a_4$ and $\sigma_1 = \sigma_4$, accordingly, the mathematical expectations and standard deviations of normally distributed random variables.

Mathematically, misalignment of a piston in the cylinder for four-link crank mechanisms is calculated for each individual unit as the sum of random variables:

$$\psi_{\sum} = \sum_1^4 \psi_i = \sum_1^4 x_i = x_1 + x_2 + x_3 + x_4. \tag{5}$$

Then, if the probability distribution densities of these values $P_i(x)$ are known, then, for the pairwise representation, for example, $q_1 = x_1 + x_2$ and $q_2 = x_3 + x_4$, accordingly, we can write the following expressions for the probability distribution densities of such sums:

$$\overline{P}_1(y) = \int_{-\infty}^{\infty} P_1(y - x) P_2(x) dx; \tag{6}$$

$$\overline{P}_2(y) = \int_{-\infty}^{\infty} P_3(y - x) P_4(x) dx, \tag{7}$$

where $y_1 = f_i(x_i)$,

or finally:

$$P(y) = \int_{-\infty}^{\infty} \overline{P_1}(y-x)\overline{P_2}(x)dx. \tag{8}$$

Expression (9), under certain conditions, solves the problem. However, if there are more than two random variables, for example, as in this case, then its practical implementation becomes rather difficult even with simple laws for determining the probability densities of specified values x_i.

3 Results and Discussion

The applied aspect of the probabilistic-theoretical analysis of the accuracy of the assembly of four-link crank mechanisms for the misalignment of the piston in the cylinder always puts forward the problem of finding a solution more accessible for practical use, which for analyzed conditions can be obtained by solution approximation (8) by the method of moments or asymptotic method based on Lyapunov's limit theorem. In the latter case, for given distributions of the sums of interdependent random variables x_i with mathematical expectations $M(x)$ variances σ_i^2 and central moments of the third and fourth orders of the sum x_i of the values, the probability distribution density $P_i(x)$ can be approximated by a Gram-Charlier series, if $P_i(x) = 0$ at $|x| \succ \alpha$ for certain α. Otherwise:

$$P_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x_0^2}{2}} \left[1 + \frac{1}{3!}\gamma_1\left(x_0^3 - 3x\right) + \frac{1}{4!}\gamma_2\left(x_0^4 - 6x_0^2 + 3\right) + \dots\right], \tag{9}$$

where $x_0 = \frac{x - M(x)}{\sigma}; \gamma_1 = \frac{M_3}{\sigma_3}; \gamma_2 = \frac{M_4}{\sigma_4} - 3$ are the coefficients of asymmetry and kurtosis for x; M_3, M_4 are the central moments, respectively, of the third and fourth orders for the probability density distribution.

With this problem formulation, the sought approximation is reduced to calculate the parameters of the total distribution using the well-known probability theory formulas:

$$M_{\sum}(x) = \sum_1^4 M(x); D(x) = \sum_1^4 \sigma_i^2 = M_2(x); \tag{10}$$

$$M_3(x) = \sum_1^4 M_3(x); M_4(x) = \sum_1^4 M_4(x). \tag{11}$$

For the initial distribution laws of considered primary production errors in the relative position of base surfaces of the parts of the crank mechanisms, we can write:

$$M(x_1) = M(x_4) = \frac{1}{\sigma\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} x e^{-\frac{(x-\alpha_1)^2}{2\sigma^2}} dx\right] = 0; \tag{12}$$

$$M(x_2) = K\int_{-\xi}^{\xi} x^2 dx = 0; \tag{13}$$

$$M(x_3) = \int_{-\eta_2}^{\eta_1} \frac{x}{\eta_1 - \eta_2} dx = \frac{\eta_1 - \eta_2}{2}. \tag{14}$$

Then:

$$M_{\sum}(x)=\sum_{1}^{4}M(x)=\frac{\eta_1-\eta_2}{2};M_{\sum}(x)=m. \tag{15}$$

By analogy, for variances or standard deviations, we have:

$$\sigma_1^2=\sigma_4^2=\left(\alpha_1^2+\sigma_1^2\right)\left[1+\Phi\left(\frac{\alpha_1}{\sigma_1\sqrt{2}}\right)\right]+\frac{\alpha_1\sigma_1\sqrt{2}}{\sqrt{\pi}}e^{-\frac{\alpha_1^2}{2\sigma_1^2}}; \tag{16}$$

$$\sigma_2^2=K\frac{\xi}{2};\sigma_3^2=\frac{\left(\eta_1-\eta_2\right)^2}{12}, \tag{17}$$

and consequently,

$$D(x)=\sigma^2=2\left(\alpha_1^2+\sigma_1^2\right)\left[1+\Phi\left(\frac{\alpha_1}{\sigma_1\sqrt{2}}\right)\right]+\frac{\alpha_1\sigma_1\sqrt{2}}{\sqrt{\pi}}e^{-\frac{\alpha_1^2}{2\sigma_1^2}}+ \tag{18}$$

$$+K\frac{\xi}{2}+\frac{\left(\eta_1-\eta_2\right)^2}{12}, \tag{19}$$

where $\Phi\left(\frac{\alpha_1}{\sigma_1\sqrt{2}}\right)$ is the probability integral, or error integral.

After calculation of corresponding integrals, the asymptotic representation can be rewritten as:

$$P_x(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x_0^2}{2}}\left[1+\frac{1}{4!}\gamma_2\left(x_0^4-6x_0^2+3\right)+\ldots\right], \tag{20}$$

as $M_3=0$ and $\gamma_1=0$. Then the probability of hitting a random variable as a sum of random primary production errors in a certain interval will be:

$$P\left(\alpha_0\le x\le\alpha_1\right)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x_0^2}{2}}\left[\left(1+\frac{3}{4!}\gamma_2\right)\int_{\alpha_0}^{\alpha_1}e^{-\frac{x_0^2}{2}}dx+\frac{\gamma_2}{4}\int_{\alpha_0}^{\alpha_1}e^{-\frac{x_0^2}{2}}\left(x_0^4-6x_0^2\right)dx+\ldots\right]. \tag{21}$$

4 Conclusions

Expression (21) was obtained, to solve the problem and, according to the initial laws of error distribution within certain tolerance fields, calculate the accuracy of the component assembly of crank mechanisms for the distortions of the piston in the cylinder, provided that the integrals included in this equality are represented in explicit form.

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