## $H_{\infty}$ fault estimation of robot joint in finite frequency domain

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**Abstract.** The fault estimation of robot joint is of great significance to improve the reliability and stability of robot joint. Based on the design of fault diagnosis observer and the finite frequency theory, a method of H infinite fault estimation with frequency domain is proposed. The design method combines H infinite filter wave with finite frequency technology effectively, and has strong anti-interference performance, Compared with other design methods, the method proposed in this paper can improve the accuracy of fault estimation

### **1** Introduction

.Since its invention in the last century, industrial robots have been widely used in many industries, such as industrial production, national defense, military industry, transportation, food and medical treatment, etc, It simulates human joints, and its joint structure is driven by controller and servo system to complete complex work,. From the control point of view, the multi joint manipulator is a multi input multi output nonlinear control system, At the same time, in order to meet the needs of continuous, stable and reliable operation in the harsh environment of industrial production site, strict requirements are put forward for the design of its core controller and control algorithm, and the robot joint is required to have the function of fault estimation to ensure the continuous production of the production line and reduce the loss caused by joint failure[1, 2].

As a new hotspot of control theory, fault diagnosis has always been concerned by many researchers. In recent years, many research methods have emerged in this field, such as fault diagnosis based on sliding mode, fault diagnosis based on fault diagnosis observer. At the same time, the research in intelligent field has also been applied in the field of fault diagnosis, such as the method based on neural network observer, A method of deep learning.

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# 1.1. Mathematical model of plane two joint manipulator

For the convenience of calculation and analysis, Through dynamic modeling, the state equation of planar two joint rigid manipulator can be expressed as:

$$\begin{bmatrix} a & b\cos(\theta_2 - \theta_1) \\ b\cos(\theta_2 - \theta_1) & c \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$
(1)
$$+ \begin{bmatrix} -b\dot{\theta}_2^2\sin(\theta_2 - \theta_1) \\ -b\dot{\theta}_1^2\sin(\theta_2 - \theta_1) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

with,



l.

 $q_1$ 

Fig. 1.Schematic diagram of two joint rigid manipulator

x

In order to establish the mathematical model of the manipulator to meet the design requirements of the fault estimator, this paper makes the following deformation of the planar two joint manipulator model [3].

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$$\begin{bmatrix} a & b\cos(\theta_2 - \theta_1) \\ b\cos(\theta_2 - \theta_1) & c \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}^{-1}$$
(2) with:

$$m_{11} = m_{22} = a / [ac - b^2 \cos^2(\theta_2 - \theta_1)]$$
  

$$m_{12} = m_{21} = -b \cos(\theta_2 - \theta_1) / [ac - b^2 \cos^2(\theta_2 - \theta_1)]$$
(3)

The model of the deformed manipulator is obtained as follows:

$$\dot{X} = A(\theta)X + B(\theta)u \tag{4}$$

with

$$A(\theta) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -m_{12}bv_{1}\sin(\theta_{2} - \theta_{1}) & -m_{11}bv_{1}\sin(\theta_{2} - \theta_{1}) \\ 0 & 0 & -m_{22}bv_{1}\sin(\theta_{2} - \theta_{1}) & -m_{12}bv_{1}\sin(\theta_{2} - \theta_{1}) \end{bmatrix}$$
(5)  
$$B(\theta) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

After the above deformation, the nonlinear strong coupling planar two joint rigid manipulator model can be transformed into an easy to handle LPV model

# 2 The finite frequency domain index and generalized KYP theorem

The generalized KYP lemma is a generalization of the standard KYP lemma. Compared with other finite frequency domain methods, such as frequency weighting method and frequency axis gridding method, the generalized KYP lemma effectively establishes the relationship between the frequency domain condition and the time domain condition in the system angle, which is easy to understand in form and easy to apply in engineering

Lemma 1 (generalized KYP lemma): consider system 1 and transfer function matrix 2, there is a symmetric matrix  $\Pi$ , and the following two conditions are equivalent:

(i) Inequalities in finite frequency domain

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^{I} \prod \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} < 0, \quad \forall \, \omega \in \Omega$$
 (6)

(ii) there exist  $n \times n$  Hermitian matrices P and Q satisfying Q > 0, and

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^{T} \Xi \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^{T} \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} < 0$$
(7)

Lemma 2: considers system 1 and transfer function 2, and makes symmetric matrix have the following conditions equivalent:

(i) Finite frequency inequality:

$$\sigma_{\min}(G(j\omega)) > \beta, \quad \forall \omega \in [0, \omega_l]$$
(8)

Among them,  $\beta$  it is a positive scalar

(ii) there are matrix variables P, Q, which meet Q > 0, and

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^{T} \Xi \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^{T} \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} < 0$$
(9)

with,

$$\Xi = \begin{bmatrix} -Q & P + j\frac{\omega_l}{2}Q \\ P + j\frac{\omega_l}{2}Q & 0 \end{bmatrix}$$
(10)

Proof: because

$$\Pi = \begin{bmatrix} -I & 0\\ 0 & -\beta^2 I \end{bmatrix}, \quad \omega \in [0, \overline{\omega}]$$
(11)

Formula (6) becomes

$$G(j\omega)^{T}G(j\omega) > \beta^{2}I$$
(12)

$$\sigma_{\min}(G(j\omega)) > \beta, \quad \forall \omega \in [0, \omega_l]$$
(13)

Quoting lemma 1, the theorem is proved.

Lemma 3 (Finsler lemma) (i) Matrix inequality

$$\vec{U}L(\vec{U})^T < 0 \tag{14}$$

Set up, where  $\vec{U}$  is the satisfied  $\vec{U}U = 0$  matrix (ii) in a matrix such that

$$L + UY + Y^T U^T < 0 \tag{15}$$

In this paper, the joint fault estimator based on finite frequency domain technology can improve the accuracy of fault estimation.

(1) The finite frequency method can improve the sensitivity of fault estimator to frequent low frequency faults

<sup>(2)</sup> At the same time, considering the antiinterference performance of the designed fault estimator, the influence of external interference on the fault estimator is reduced, and the probability of false alarm and missing alarm is reduced

③ Compared with other traditional finite frequency methods, this method reduces the amount of calculation and is easy to be applied in engineering practice

### 3 Design of joint fault estimator for robot

Next, the design method of finite frequency domain fault estimator for robot shutdown is given by combining observer theory and finite frequency theory [4, 5].

#### 3.1. Mathematical model of fault estimator

Consider the following linear time invariant systems

$$\dot{x} = Ax + Bu(t) + B_f f(t) + B_d d(t)$$

$$v(t) = Cx$$
(16)

In the above formula, x(t) represents the system state, f(t) represents the fault signal, d(t) represents the external interference of the system, f(t) represents the system output, and the finger joint position in the manipulator system.  $A,B,B_f,B_d,C$  are known constant matrices

The general method to estimate fault information is to design an observer for the target system

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H(y(t) - \hat{y}(t)) \\ \dot{\hat{y}}(t) = C\hat{x}(t) \\ \hat{f}(t) = V(y(t) - \hat{y}(t)) \end{cases}$$
(17)

In order to reduce the deviation between the fault estimation and the actual value as much as possible, the state estimation error e(t) and the fault estimation error

f(t) are defined.

$$e(t) = x(t) - \hat{x}(t)$$

$$\tilde{f}(t) = f(t) - \hat{f}(t)$$
(18)

According to equation (17) and equation (18), the following error system can be obtained

$$\begin{cases} \dot{e}(t) = (\mathbf{A} - HC)e(t) + B_d d(t) + B_f f(t) \\ \tilde{f}(t) = f(t) - VCe(t) \end{cases}$$
(19)

Therefore, the design problem of the finite frequency domain fault estimator for robot joints can now be transformed into solving the matrix H, V, so that the error system of the above equation (19) satisfies the following performance conditions:

$$\|G_{\tilde{f}f}(s)\|_{\infty}^{[-\omega,\omega]} < \gamma_f \tag{20}$$

#### 3.2. Fault sensitivity condition

In this part, we consider that d(t) is equal to 0 in (19), and we get

$$\tilde{\tilde{x}}(t) = \overline{A\tilde{x}}(t) + B_f f(t)$$

$$\tilde{f}(t) = -VC\tilde{x}(t) + f(t)$$
(21)

with,  $\overline{A} = A - HC$ 

In order to improve the sensitivity of the observer to the fault signal, the following theorem is proposed

Theorem 1: consider the system (21), consider the transfer function as

$$G(s) = C(sI - A)^{-1}B + D, \Pi = \begin{bmatrix} -I & 0\\ 0 & -\beta^2 I \end{bmatrix}$$
(22)

With known positive real number, when the following inequality conditions are satisfied.

$$\sigma_{\min}(G(j\omega)) > \beta, \quad \forall \omega \in [0, \omega_l]$$
(23)

The sufficient condition is that there exists a symmetric matrix, P, Q such that the following matrix inequalities hold

$$\begin{bmatrix} \Theta_l + \overline{M}\overline{A} + \overline{A}^T M^T & -\overline{M}^T + \overline{P}_l^T + \Gamma^T \overline{A}^T \\ * & -Q - \Gamma - \Gamma^T \end{bmatrix} < 0 \quad (24)$$

with

$$\overline{M} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \ \overline{A} = \begin{bmatrix} A & B \end{bmatrix}, \ \overline{P}_l = \begin{bmatrix} P \\ 0 \end{bmatrix}$$
(25)  
$$\Theta_l = \begin{bmatrix} -\omega_l^2 Q & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}$$

It is proved that the frequency inequality (23) is equivalent to that in the low frequency range according to lemma 1 (GKYP).

$$\begin{bmatrix} A^{T} & I \\ B^{T} & 0 \end{bmatrix} \begin{bmatrix} -Q & P \\ P & a_{I}^{2}Q \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C^{T} & 0 \\ D^{T} & I \end{bmatrix} \prod \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} < 0 \quad (26)$$

It is easy to see that equation (26) can be written in the following form

$$\begin{bmatrix} I & \overline{A}^T \end{bmatrix} \begin{bmatrix} \Theta_l & P_l \\ \overline{P}_l^T & -Q \end{bmatrix} \begin{bmatrix} I \\ \overline{A} \end{bmatrix} < 0$$
(27)

from

$$U = \begin{bmatrix} I & \overline{A}^T \end{bmatrix}$$
(28)

According to Lemma 3, the sufficient and necessary condition of equation (28) is that there exists a matrix Y such that

$$\begin{bmatrix} \Theta_l & \overline{P}_l \\ \overline{P}_l^T & -Q \end{bmatrix} + UY + Y^T U^T < 0$$
(29)

from

$$Y = \begin{bmatrix} M_1^T & M_2^T & \Gamma^T \end{bmatrix}$$
(30)

By substituting formula (23) and formula (29) into formula (26), formula (24) can be obtained

#### 3.3. Design of joint fault observer for LPV robot

Let the motion range of planar two joint robot be  $\theta_{1\min} \le \theta_1 \le \theta_{1\max}, \theta_{2\min} \le \theta_2 \le \theta_{2\max}$  According to the convex decomposition theory, the state equation of the robot can be described as

$$\dot{X} = \left[\sum_{i=1}^{4} \partial_i(\theta) A(\theta_{mi})\right] X + \left[\sum_{i=1}^{4} \partial_1(\theta) B(\theta_{mi})\right] u \quad (31)$$
with,  

$$\partial_1(\theta) = (\theta_{2\max} - \theta_2)(\theta_{1\max} - \theta_1)/(\theta_{2\max} - \theta_{2\min})(\theta_{1\max} - \theta_{1\min})$$

$$\partial_2(\theta) = (\theta_{2\max} - \theta_2)(\theta_1 - \theta_{1\min})/(\theta_{2\max} - \theta_{2\min})(\theta_{1\max} - \theta_{1\min})$$

$$\partial_3(\theta) = (\theta_2 - \theta_{2\min})(\theta_1 - \theta_{1\min})/(\theta_{2\max} - \theta_{2\min})(\theta_{1\max} - \theta_{1\min})$$

$$\partial_4(\theta) = (\theta_2 - \theta_{2\min})(\theta_{1\max} - \theta_1)/(\theta_{2\max} - \theta_{2\min})(\theta_{1\max} - \theta_{1\min})$$

$$\sum_{i=1}^{4} \partial_i(\theta) = 1$$

Aiming at the vertex of robot joint convex polyhedron model, the joint fault observer parameters  $H_1, H_2, H_3, H_4, V_1, V_2, V_3, V_4$  are solved

$$L = \partial_1(\theta)L_1 + \partial_2(\theta)L_2 + \partial_3(\theta)L_3 + \partial_4(\theta)L_4$$

$$V = \partial_1(\theta)V_1 + \partial_2(\theta)V_2 + \partial_3(\theta)V_3 + \partial_4(\theta)V_4$$
(33)

# 4 Simulation and experimental verification

The robot system of formula (1) is simulated by MATLAB, and the parameters of fault observer are calculated by LMI toolbox

$$L_{1} = L_{3} = \begin{bmatrix} 1.67 & -0.03 \\ -0.03 & 1.67 \\ 0.41 & 0 \\ 0 & 0.41 \end{bmatrix} \\ L_{2} = L_{4} = \begin{bmatrix} 1.67 & 0.03 \\ 0.03 & 1.67 \\ 0.41 & 0 \\ 0 & 0.41 \end{bmatrix}$$
(34)  
$$V_{1} = V_{3} = \begin{bmatrix} -2.14 & 0 \\ 0 & -2.14 \end{bmatrix} \\ V_{2} = V_{4} = \begin{bmatrix} 2.14 & 0 \\ 0 & 2.14 \end{bmatrix}$$
(35)

The corresponding four closed-loop poles of the fault observer are:

 $\{-1.9843 \pm 1.4569i, -1.9843 \pm 1.4569i\}$ 

It can be seen that the designed finite frequency domain H-infinity fault observer meets the stability requirements. In the simulation, the fault signal is given as sine wave and step signal respectively, and the noise signal is white noise. The observation effect of the designed finite frequency domain fault observer is compared with that of the general full frequency domain fault observer



Fig. 2 fault estimation effect of finite frequency domain fault estimator on step migration



**Fig. 3** estimation effect of finite frequency domain fault estimator on sinusoidal fault in design frequency domain



**Fig. 4** estimation effect of finite frequency domain fault estimator on sinusoidal fault beyond design frequency domain

It can be seen from Fig. 2 and Fig. 3 that the designed finite frequency domain fault observer has high sensitivity to fault, and has good fault tracking accuracy for sine wave and triangular wave time-varying fault signals, which achieves good fault estimation effect

It can be seen from Figure 4 that when the fault frequency exceeds the designed frequency of the fault observer, the output fault waveform accuracy of the fault observer will decrease, and the design index cannot be reached.

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