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Multivariate Grey PredictionModel with Priority Accumulation of New Information

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Abstract. According to the new information priority principle of grey system, this paper tries to optimize the traditional multivariate grey prediction model. Firstly, the basic theory of the traditional grey prediction model is put forward. Based on this, the background value is improved by using the new information priority principle, and the cumulative generation with parameters is defined. Taking the settlement trend of A4# building of an engineering project in Anhui province as an example, the model is applied to the settlement analysis, and the proposed model is compared with the existing grey prediction model, the average percentage absolute error between the predicted value and the observed value is calculated, and the regression graphs of each model are drawn. Through the analysis, we can see that the established model has achieved a good effect, and then verified the practicability and reliability of the proposed model.

1 Introduction

With the continuous development of China, China's industrial and civil construction industry has also made great progress, a variety of complex and large engineering buildings are increasing. The construction of the engineering building changes the original state of the ground and exerts a certain pressure on the foundation of the building, which causes the deformation of the foundation and surrounding stratum. In order to ensure the safe use of the building and provide data for the future reasonable design, therefore, the building settlement should be observed during the construction process and after the building is put into operation. The settlement observation of buildings is a complex process, which is affected by many factors, and each monitoring point also affects and restricts each other. With the attention and research on the observation of building subsidence, a variety of prediction methods have emerged at home and abroad, such as time series, linear regression analysis, support vector machine, artificial neural network model, grey system model. The scientific and reasonable settlement prediction model is not only beneficial to the safety of buildings, but also important to the decision making. [1-3]

Established by Chinese scholar professor Deng Julong in 1982, grey system theory is a method specially used to study the problem of uncertain systems in which some information is known and some information is unknown.4 The grey prediction model can be divided into univariate grey prediction model and multivariate grey prediction model according to the number of modeling variables. The univariate grey prediction model is represented by GM(1,1), whose modeling object is only one time series data. It mainly mines the system operation law contained in the time series data through the grey generation method, and then realizes the prediction of the system development trend. The multivariable grey prediction model is represented by GM(1,N). The modeling object of this model is composed of a system feature sequence and n-1 correlation factor sequence. The modeling process fully considers the influence of correlation factors on the change trend of the system. However, the model has many shortcomings in modeling mechanism and structure. So many scholars have improved GM(1,1), such as Railway passenger volume prediction based on GA-GM(1,N,α) power model by Xu Kun[8], Prediction of corrosion rate of carbon steel based on GM(1,N) model by Zheng Ruyan[9], Real estate price forecast of Zhengzhou based on GM (1, N) model by Zhang Rongyan[10], Study on groundwater salinity in Linzhang county based on GM (1, N) model by Zhang Ziyue[11], Research on the accuracy of optimized grey GM(1,N) -weighted Markov model in road traffic noise prediction by Huang Chaoqiang[12]. At present, grey prediction model has been widely used in fields such as urban environment, traffic management, energy analysis.[5-7]

It can be seen from the above literatures that there are still defects in multivariate gray prediction. Therefore, this paper combines the traditional multivariate gray prediction with the new information priority in the axiom of gray system, defines the accumulation generation with parameter form, and establishes a multivariate gray prediction model with new information priority accumulation.

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2 Traditional multivariate grey prediction model

Definition 1. Set $X_1^{(0)}$ as a sequence of dependent variables13：

$$X_1^{(0)}=\left(X_1^{(0)}(1),X_1^{(0)}(2),...,X_1^{(0)}(m)\right) \quad (1)$$

The sequence $X_i^{(0)}(i=1,2,...,N)$ is the sequence of independent variables with high correlation with sequence $X_1^{(0)}$：

$$X_i^{(0)}=\left(X_i^{(0)}(1),\ X_i^{(0)}(2),...,X_i^{(0)}(m)\right) \quad (2)$$

$X_i^{(1)}$ is the first order cumulative sequence of $X_i^{(0)}$ $(i=1,2,...,N)$：

$$X_i^{(1)}=\left(X_i^{(1)}(1),X_i^{(1)}(2),...,X_i^{(1)}(m)\right) \quad (3)$$

Where

$$X_i^{(1)}(k)=\sum_{j=1}^{k}X_i^{(0)}(k) \quad (4)$$

$$k=1,2,...,m$$

$Z_i^{(1)}$ is the sequence generated next to the mean of $X_i^{(1)}$：

$$Z_i^{(1)}=\left(Z_i^{(1)}(2),Z_i^{(1)}(3),...,Z_i^{(1)}(m)\right) \quad (5)$$

Where

$$Z_j^{(1)}(k)=0.5\times\left(X_j^{(1)}(k)+X_j^{(1)}(k-1)\right)$$
$$k=1,2,...,m \quad (6)$$

So

$$X_1^{(0)}(k)+aZ_1^{(1)}(k)$$
$$=b_2X_2^{(1)}(k)+b_3X_3^{(1)}(k)+...+b_NX_N^{(1)}(k) \quad (7)$$

is called traditional multivariate gray prediction model

Theorem 1 $X_i^{(0)},X_i^{(1)}(i=1,2,...,N)$ and $Z_1^{(1)}$ refer to definition 1， Then, the least square estimation of the parameter $\hat{a}=[a,b_2,...,b_N]^T$ of the traditional multivariate gray prediction model is satisfied

1) If m=N+1 and $|B|\neq 0$ then $\hat{a}=B^{-1}Y$;
2) If m>N+1 and $|B^TB|\neq 0$ then $\hat{a}=\left(B^TB\right)^{-1}B^TY$;
3) If m<N+1 and $|B^TB|\neq 0$ then $\hat{a}=B^T\left(B^TB\right)^{-1}Y$; (8)

Where

$$B=\begin{pmatrix} -Z_1^{(1)}(2) & X_2^{(1)}(2) & L & X_2^{(1)}(2) \\ -Z_1^{(1)}(3) & X_2^{(1)}(3) & L & X_2^{(1)}(2) \\ M & M & & M \\ -Z_1^{(1)}(\mathrm{m}) & X_2^{(1)}(2) & K & X_2^{(1)}(2) \end{pmatrix}$$

$$\mathrm{Y}=\begin{pmatrix} X_1^{(0)}(2) \\ X_1^{(0)}(3) \\ M \\ X_1^{(0)}(m) \end{pmatrix} \quad (9)$$

Proof 1 When k=2, 3, …, m, from Eq. (7), we have

$$\begin{cases} X_1^{(0)}(2)=-aZ_1^{(1)}(2)+b_2X_2^{(1)}(2)+b_3X_3^{(1)}(2)+...+b_NX_N^{(1)}(2) \\ X_1^{(0)}(3)=-aZ_1^{(1)}(3)+b_2X_2^{(1)}(3)+b_3X_3^{(1)}(3)+...+b_NX_N^{(1)}(3) \\ \mathrm{K} \\ X_1^{(0)}(m)=-aZ_1^{(1)}(m)+b_2X_2^{(1)}(m)+b_3X_3^{(1)}(m)+...+b_NX_N^{(1)}(m) \end{cases}$$

In matrix form:

$$Y=\mathrm{B}\hat{a}$$

If m=N+1 and $|B|\neq 0$ then $\hat{a}=B^{-1}Y$;

If m>N+1 and $|B^TB|\neq 0$ then $\hat{a}=\left(B^TB\right)^{-1}B^TY$;

When m<N+1, B is a column full rank matrix, and $|B^TB|\neq 0$,the column full rank decomposition of B is:

（3） B=DC

Therefore, the generalized inverse matrix B^+ of B is as follows:

$$B^+=C^T\left(CC^T\right)^{-1}\left(D^TD\right)^{-1}D^T$$

And we have

$$\hat{a}=C^T\left(CC^T\right)^{-1}\left(D^TD\right)^{-1}D^TY$$

Let's take D as the identity matrix I_{n-1} and we have

$$\hat{a}=B^T\left(B^TB\right)^{-1}Y$$

End of proof.16

Definition 2 Linear differential equation

$$\frac{dX_1^{(1)}}{dt}+aX_1^{(1)}(t)$$
$$=b_2X_2^{(1)}(t)+b_3X_3^{(1)}(t)+...+b_NX_N^{(1)}(t) \quad (10)$$

is called the whitening equation of the traditional multivariate gray prediction model.

Theorem 2 When the change range of $X_j^{(1)}(j=2,3,...,N)$ is small, $\sum_{j=2}^{N}b_jX_j^{(1)}(t)$ can be regarded as the grey constant, and the time response equation14 of the traditional multivariate grey prediction model is：

$$\hat{X}_1^{(1)}(k)=[X_1^{(0)}(1)-\frac{1}{a}\sum_{j=2}^{N}b_jX_j^{(1)}(1)]e^{-a(k-1)}+\frac{1}{a}\sum_{j=2}^{N}b_jX_j^{(1)}(k) \quad (11)$$

Reduction type：

$$\hat{X}_1^{(0)}(1)=X_1^{(1)}(1)$$
$$\hat{X}_1^{(0)}(k)=\hat{X}_1^{(1)}(k)-\hat{X}_1^{(1)}(k-1) \quad (12)$$
$$k=1,2,...,m$$

Proof 2 The solution of equation (10) is：

$$X_1^{(1)}(t)=X_1^{(1)}(1)e^{-a(t-1)}+\int_1^t e^{-a(t-\tau)}\sum_{j=2}^{N}b_jX_j^{(1)}(\tau)d\tau \quad (13)$$

When the change range of $X_j^{(1)}(j=2,3,...,N)$ is small, $\sum_{j=2}^{N}b_jX_j^{(1)}(t)$ can be regarded as the grey constant. Then the convolution in equation (13) can be written as

$$\int_1^t e^{-a(t-\tau)}\sum_{j=2}^{N}b_jX_j^{(1)}(\tau)d\tau$$

$$=\sum_{j=2}^{N} b_j X_j^{(1)}(t) \int_1^t e^{-a(t-\tau)} d\tau \tag{14}$$
$$=\frac{1}{a}\left(1-e^{-a(t-1)}\right)\sum_{j=2}^{N} b_j X_j^{(1)}(t)$$

Substitute (14) into (13), take t=k, then

$$X_1^{(1)}(k)=X_1^{(1)}(1)e^{-a(k-1)}+\frac{1}{a}\left(1-e^{-a(k-1)}\right)\sum_{j=2}^{N} b_j X_j^{(1)}(k)$$
$$=[X_1^{(0)}(1)-\frac{1}{a}\sum_{j=2}^{N} b_j X_j^{(1)}(1)]e^{-a(k-1)}+\frac{1}{a}\sum_{j=2}^{N} b_j X_j^{(1)}(k) \tag{15}$$

End of proof.

3 Traditional multivariate gray prediction model based on new interest priority

In the grey system principle, it can be known from the principle of new information priority that the role of new information cognition is greater than the role of old information, which directly affects the future trend of the system and plays a major role in the future development of realistic information. Therefore, this paper defines cumulative generation with parameter form15.

Definition 2 Set $X_1^{(0)}$ as a sequence of dependent variables:

$$X_1^{(0)}=\left(X_1^{(0)}(1), X_1^{(0)}(2), \ldots, X_1^{(0)}(m)\right) \tag{16}$$

The sequence $X_i^{(0)}(i=1,2,...,N)$ is the sequence of independent variables with high correlation with sequence:

$$X_i^{(0)}=\left(X_i^{(0)}(1),\ X_i^{(0)}(2),\ \ldots, X_i^{(0)}(m)\right) \tag{17}$$

$X_i^{(1)}$ is the first order cumulative sequence of $X_i^{(0)}$ $(i=1,2,...,N)$:

$$X_i^{(1)}=\left(X_i^{(1)}(1), X_i^{(1)}(2) \ldots, X_i^{(1)}(m)\right) \tag{18}$$

where $\lambda \in (0,1)$, so

$$X_i^{(1)}(k)=\sum_{j=1}^{k} \lambda^{m-j} X_i^{(0)}(k)$$
$$k=1,2,...,m \tag{19}$$

$Z_i^{(1)}$ is the sequence generated next to the mean of $X_i^{(1)}$:

$$Z_i^{(1)}=(Z_i^{(1)}(2), Z_i^{(1)}(3),..., Z_i^{(1)}(m)) \tag{20}$$

where

$$Z_j^{(1)}(k)=0.5\left(X_j^{(1)}(k)+X_j^{(1)}(k-1)\right) \tag{21}$$
$$k=1,2,...,m$$

so

$$X_1^{(0)}(k)+aZ_1^{(1)}(k)=b_2X_2^{(1)}(k)+b_3X_3^{(1)}(k)+...+b_NX_N^{(1)}(k) \tag{22}$$

is the traditional multivariate gray prediction model based on new information priority, NIPGM(1,N) model for short.

Table 1 Original settlement observation data

Cumulative settlement /mm							
cycle	The total number of days	load	A410	A403	A405	A410	Overall trend settlement
1	0	3	0.67	0.82	0.71	1.03	0.76
2	17	6	1.48	1.57	1.35	1.9	1.5
3	36	9	2.22	2.23	1.96	2.72	2.2
4	54	12	2.9	2.8	2.46	3.5	2.83
5	70	15	3.49	3.56	2.85	4.31	3.47
6	91	18	4.12	4.15	3.13	5.25	4.14
7	108	21	4.9	4.81	3.36	6.04	4.74
8	127	24	5.42	5.52	3.54	6.67	5.27
9	143	24	5.73	5.96	3.79	7.27	5.66
10	162	24	6.01	6.25	3.96	7.69	5.93
11	182	24	6.23	6.42	4.09	8.01	6.11

Table 2 Model data accuracy comparison (unit: mm)

cycle	observations	NIPGM(1,N) model		GM(1,1) model		DGM(1,1) model	
		Predictive value	Relative error	Predictive value	Relative error	Predictive value	Relative error
1	0.76	0.7600	0	0.7600	0	0.7600	0
2	1.50	1.2093	0.2907	2.3697	0.8697	2.3850	0.885
3	2.20	2.2624	0.0624	2.6666	0.4666	2.6822	0.4822
4	2.83	2.8355	0.0055	3.0008	0.1708	3.0164	0.1864

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5	3.47	3.4093	0.0607	3.3768	0.0932	3.3923	0.0777
6	4.14	4.0199	0.1201	3.8000	0.34	3.8150	0.325
7	4.74	4.7802	0.0402	4.2762	0.4638	4.2904	0.4496
8	5.27	5.2874	0.0174	4.8121	0.4579	4.8251	0.4449
9	5.66	5.5898	0.0702	5.4151	0.2449	5.4263	0.2337
10	5.93	5.8629	0.0671	6.0937	0.1637	6.1025	0.1725
11	6.11	6.0775	0.0325	6.8573	0.7473	6.8629	0.7529
MAPE		2.8959%		12.1739%		12.2622%	

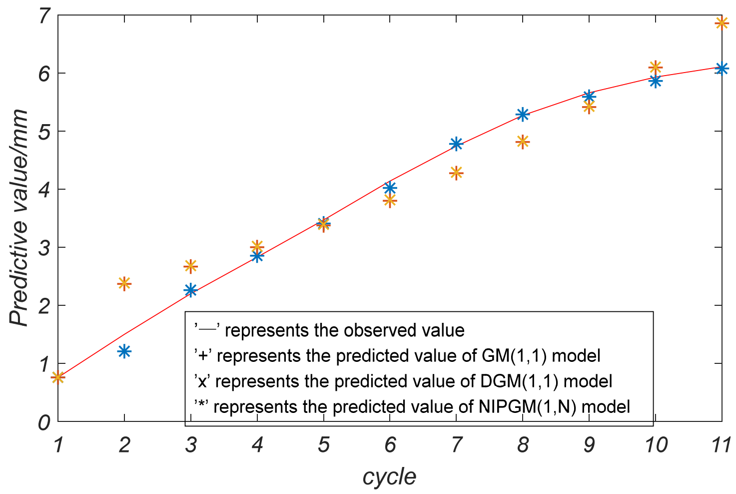

Fig. 1. Model data comparison

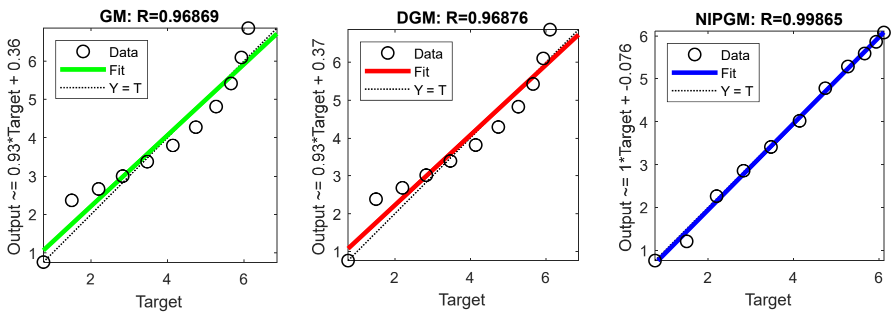

Fig. 2. Return to figure

4 The example analysis

This paper, taking a A4 building project in Anhui province as an example[16], sedimentation analysis application NIPGM(1,N) model, the dependent variable of the trend of settlement as a whole, before 4 issue of the settlement observation data as the known data, on the basis of settlement observation data of 5 to 7 as a search for the parameters of the accumulation generation sequence, and thereby to 8 and the basis of settlement observation data to predict.

4.1 The observed data

A total of 10 observation points are set up in building A4, from which observation points were randomly selected for data analysis.

4.2 Settlement analysis bases on the traditional multivariable grey prediction model of new interest priority

The observation point A401 is taken as the sequence of independent variables, and the overall trend settlement is the sequence of dependent variables. NIPGM(1,N) model is used to predict the settlement. For the overall trend settlement, the predicted value of the three models and the relative error comparison between the predicted value and the observed value are shown in table 2. The data of the three models and the observed value are shown in figure 1. The regression diagram of the three models is shown in figure 2.

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As can be seen from figure 1, the predicted value obtained by the NIPGM(1,N) model is closer to the observed value curve with a small error. However, the predicted values of GM(1,1) model and DGM(1,1) model are far from the observed values, and the errors are large.

From figure 2, we can also see that the NIPGM(1,N) model can be better fitted by the regression equation. The correlation coefficient of NIPGM(1,N) model regression graph R=0.99865, the correlation coefficient of GM(1,N) model regression graph R=0.96869, the correlation coefficient of DGM(1,N) model regression graph R=0.96876. The correlation coefficient of NIPGM(1,N) model is closer to 1 than that of GM(1,1) model and DGM(1,1) model. In summary, the predicted value of NIPGM(1,N) model is consistent with the observed value, and the relative error is small. The prediction effect is better than that of GM(1,1) model and DGM(1,1) model.

5 Conclusion

The observation of building settlement is affected by many factors, and each monitoring station also affects and restricts each other. If only the influence of individual factors is considered, there may be a large error. Therefore, this paper considers the multivariate grey prediction model, and uses the new information first principle to improve the background value of the traditional multivariate grey model. Therefore, NIPGM (1, N) model was proposed to reduce the error and obtain more accurate results. Taking A4 floor of an engineering project in Anhui Province as an example, the settlement analysis is carried out, and compared with GM (1,1) model and DGM (1,1) model, it can be seen that the established model has a better effect. The prediction effect and higher reliability provide an effective method for future prediction analysis.

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