Synthesis of the parameters of the elastic-damping device in the excavator's digging mechanism based on the use of the integral equation

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Abstract. The article is devoted to the issues of the synthesis of the parameters of an elastic-damping device built into the excavator's digging mechanism to reduce dynamic loads. The determination of the parameters of this device was carried out by solving the inverse problem of dynamics using the Volterra integral equation of the second kind, which allowed linking these parameters to the nature of the transition process without finding the frequencies of natural oscillations. The transition from the differential equation of the oscillatory motion of the digging mechanism to the corresponding integral equation has been carried out, and the parameters of its resolvent and the elastic-damping device that ensure a decrease in dynamic loads are determined. The results of numerical simulation of the elastic-damping device efficiency based on the use of the characteristics of the real excavator's digging mechanism are presented.

1 Introduction

The actual problem of creating mining machines (shovel, draglines, mine hoisting, drilling rigs, etc.) is the problem of reducing the dynamic loads caused by elastic oscillations in transient operating modes [1-3]. As it is known, the oscillatory component of movement reduces the accuracy of the movement of the operating mechanisms, accelerates the accumulation process of fatigue damage, reduces the durability and reliability of the executive mechanisms of mining machines [4-5]. An effective way to reduce dynamic loads in mining machines is to change elastic-dissipative properties of executive mechanisms by using the elastic-damping devices (EDD), which are built into design of the hoisting and travel mechanisms [6,7] of shovel and rotary excavators [8], draglines [9], mine hoist machines [10]. This makes it possible to shift the resonance frequencies to the region of lower values and to increase the intensity of oscillation energy dissipation. At the same time, the use of EDD in the mining machines mechanisms is associated with the search for the compromise solutions between the desired quality of transient process and the possibility of technical implementation of the proposed solution [8,11]. As noted by the authors [6,8], the advisability of installing the EDD in the design of single-bucket excavators is generally recognized, but effective operation of these devices in the "elastic" zone is difficult to implement due to the large value of the transmitted forces.

One of the main problems of using EDD in the mining machines mechanisms is the problem of choosing the optimal parameters of these devices

(stiffness and viscous friction coefficients) which is usually solved using frequency and root methods, including the method of normalized transfer functions, or direct mathematical modelling based on a dynamic model of the mechanism in which the place of inclusion of the EDD in the kinematic scheme, the value of the current loads and the conditions of its operation (for example, the maximum movement of the operating mechanism) [6-12] are taken into account. When using these methods, it is not possible to provide an explicit relationship between the design parameters of the EDD and the nature of the running transient process. In the work [13] the value of the use of the analytical methods of synthesis is noted, which makes it possible to establish the relationship between the nature of transient process and design parameters of controlled mechanical systems. Analytical methods of synthesis include methods based on the use of a concept of inverse dynamic problems, which consists in determining the required parameters from given finite or differential equations that ensure the desired nature of controlled motion. This concept was used in the works [14] and [15] to select the EDD parameters of dragline mechanisms based on setting the law of oscillatory motion of the operating mechanism and the form of its differential equation. At the same time, the task of the finite or differential equation of motion requires finding the natural oscillation frequency that limits the possibilities of using this approach. For this reason, it is of interest to develop the concept of inverse dynamic problems by setting the integral equations [16,17], which allow relating the parameters of the mechanical system

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to the nature of the transition process without finding the frequencies of natural oscillations.

In this paper, the synthesis of the parameters, using the example of the excavator's digging mechanism equipped with an elastic-damping device, is carried out based on solving the inverse dynamic problem using the Volterra integral equation of the second kind, and the results of numerical simulation of efficiency are presented.

2 Object and method of investigation

The calculation scheme of the excavator's digging mechanism equipped with the EDD located between the drum and the guide block is shown in fig. 1a [14]. In this figure the following designations are adopted: 1 is the two electric motors with the gearbox and drums; 2 and 3 are the guide blocks; 4 is the cable; 5 is the bucket; 6 is the EDD. Assuming that masses are concentrated, representing the cable in the form of a weightless thread with constant coefficients of stiffness and viscous friction and neglecting friction in the mating nodes of the mechanical part, we obtain the following system of differential equations of the electromechanical system:

$$\begin{cases} U_{SR} = (U_{ref} - K_{SS} \cdot \omega_1) \cdot K_{SR}, \\ U_{SR} \leq U_{\lim} = (K_1 - K_{SS} \cdot \omega_1) \cdot (\cos\alpha/\sin\alpha) + K_2, \\ U_{CR} = \dfrac{(U_{SR} - (K_{CS} \cdot I_a))(T_a s + 1)}{T_{CR} s}, \\ E_p = \dfrac{U_{CR} \cdot K_b}{T_b s + 1}, E_{dv} = C_e \cdot \omega_1, \\ I_a = \dfrac{E_p - E_{dv}}{T_a s + 1} K_a, \\ M_{dv} = C_e \cdot I_a, \\ J_1 s \omega_1 = M_{dv} - \dfrac{c_{12}\Delta\omega_{13}}{s} - b_{12}\Delta\omega_{13}, \\ J_2 s \omega_2 = \dfrac{c_{12}\Delta\omega_{13}}{s} + b_{12}\Delta\omega_{13} - M_c, \\ J_3 s \omega_3 = M_{12} - \dfrac{c\Delta\omega_3}{s} - b\Delta\omega_3, \\ M_{12} = \dfrac{c_{12}\Delta\omega_{13}}{s} - b_{12}\Delta\omega_{13}; \\ \Delta\omega_{13} = \omega_1 - \omega_2 - \omega_3. \end{cases} \qquad (1)$$

where U_{SR} is the output voltage of the speed controller; U_{ref} is the reference voltage; $U_{\lim}$ is the clamping voltage for forming the steeply falling section of the mechanical characteristic; U_{CR} is the output voltage of the current controller; E_p is the converter voltage; E_{dv} is the DC-motor voltage; I_a is the armature current; K_1 is the tuning coefficient of mechanical characteristics; K_2 is the coefficient of stop point adjustment; K_{SS} is the gain of the speed sensor; K_{SR} is the coefficient of

the speed controller; K_{CS} is the gain of the current sensor; K_a is the coefficient of the armature circuit; K_b is the converter gain; α is the angle of inclination of the mechanical characteristic; T_a is the time constant of the armature circuit; T_{CR} is the time constant of the current controller; T_b is the time constant of the converter; C_e is the voltage constant; c_{12} is the stiffness of the cable; b_{12} is the coefficient of viscous friction in the cable; c is the stiffness of the damper, b is the coefficient of viscous friction of the damper, J_1 is the inertia moment of the two motors, and equivalent inertia moment of gearbox and drums; J_2 is the inertia moment of the bucket filled with rock; J_3 is the inertia moment of the damper; ω_1, ω_2, ω_3 are the angular velocities; s is the Laplace operator.

Since the control system of the electric drive makes it possible to exclude oscillations from the electrical part of the system, we will consider only free oscillations of the mechanical component without taking into account the damping properties of the electric drive [12]. The quality of the transition process occurring in a mechanical system will be determined by the parameters of the characteristic polynomial obtained on the system basis (1) (see fig. 1b).

a)

b)
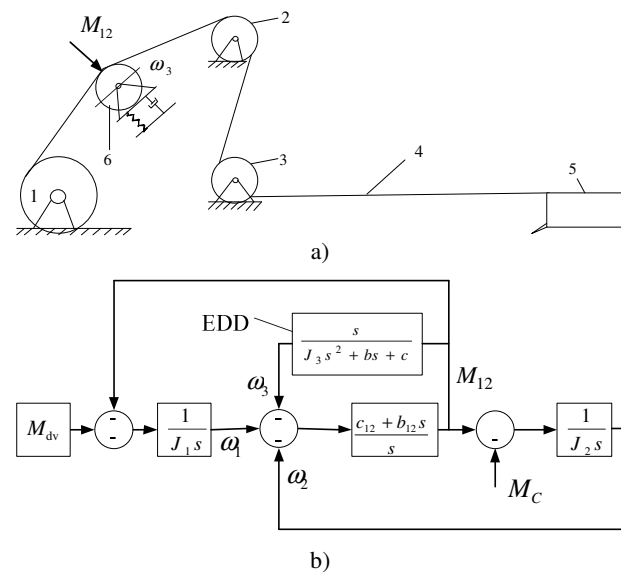
Fig. 1. Kinematic (a) and structural (b) schemes of digging mechanism

Assuming that the input actions from the electric motor M_{dv} and the load M_c are equal to zero and neglecting the moment of inertia of the EDD, we solve the expression (1) with respect to the elastic force M_{12}:

$$\dddot{M}_{12} + a_0\ddot{M}_{12} + a_1\dot{M}_{12} + a_2 M_{12} = 0. \qquad (2)$$

Here $\quad a_0 = \dfrac{bJ_{12}+c+c_{12}}{b_{12}+b}$; $\qquad a_1 = \dfrac{b\omega_{12}^2+cb_{12}J_{12}}{b_{12}+b}$;

$a_2 = \dfrac{\omega_{12}^2 c}{b_{12}+b}$; $J_{12} = \dfrac{J_1+J_2}{J_1 J_2}$ is the ratio of the moments

of inertia; $\omega_{12} = \sqrt{\dfrac{c_{12}(J_1+J_2)}{J_1 J_2}}$ is the natural frequency.

Let's set the initial conditions for expression (2) corresponding to the single step change in the elastic force:

$$\dddot{M}_{12}(0) = 0 \; ; \; \dot{M}_{12}(0) = 0 \; ; \; M_{12}(0) = 1 \; . \qquad (3)$$

Let us reduce the differential equation (2) to the integral form [16, 17] by introducing the notation $\ddot{M}_{12} = u(t)$ and integrating both sides of this equality taking into account the initial conditions (3). Finding sequentially the values of all derivatives and substituting them into (2), after some transformations we obtain:

$$u(t) + \int_0^t \left[a_0 + \frac{a_1(t-y)}{1!} + \frac{a_2(t-y)^2}{2!} \right] u(y) dy = -a_2 \; . \quad (4)$$

The equality (4) is the integral Volterra equation of the second kind with respect to an unknown function $u(t)$, which is included in this equation as a summand and as a dependence $u(y)$ under the integral sign. The kernel of the integral equation is the known function:

$$K(t, y) = -\left[a_0 + \frac{a_1(t-y)}{1!} + \frac{a_2(t-y)^2}{2!} \right].$$

To solve the integral equation (4) it is necessary to find the resolvent of this kernel. Using the method of successive approximations and taking the kernel itself as a first approximation, we determine all iterated kernels that can be represented as the following recurrent dependence:

$$K_m(t, y) = (-1)^m \sum_{k=0}^{m} \binom{m}{k} \sum_{i=0}^{m-k} \binom{m-k}{i} a_0^{m-k-i} a_1^i a_2^k \frac{(t-y)^{2k+i+m-1}}{(2k+i+m-1)!}.$$

Summing up all the kernels from the unity to infinity, we find the general expression for finding the resolvent:

$$R(t, y) = \sum_{m=1}^{\infty} (-1)^m \sum_{k=0}^{m} \binom{m}{k} \sum_{i=0}^{m-k} \binom{m-k}{i} a_0^{m-k-i} a_1^i a_2^k \frac{(t-y)^{2k+i+m-1}}{(2k+i+m-1)!}.$$

The resulting expression for the resolvent contains the system parameters, presented as coefficients a_0, a_1 and a_2, which can vary in a fairly wide range that complicates the analysis. In order to simplify this

expression, we will use the transition to the new value of the argument $t - y = \dfrac{\tau}{a_0}$ proposed in [17], as a result of which we obtain:

$$R(t, y) = a_0 \sum_{m=1}^{\infty} (-1)^m \sum_{k=0}^{m} \binom{m}{k} \sum_{i=0}^{m-k} \binom{m-k}{i} c_1^i c_2^k \frac{\tau^{2k+i+m-1}}{(2k+i+m-1)!} . (5)$$

Here $c_1 = \dfrac{a_1}{a_0^2}$, $c_2 = \dfrac{a_2}{a_0^3}$ and the transition to real time will be determined by the scale a_0.

Taking into account (5), the solution of the integral equation (4) is written in the following form:

$$u(t) = -a_2 - a_2 \int_0^t R(t, y) dy \; . \qquad (6)$$

From expression (6) it follows that the form of the unknown function will directly depend on the parameters of the resolvent (5). Thus, it becomes possible to solve the inverse dynamic problem to realize the desired nature of motion by establishing a relationship between the parameters of the mechanical system and the parameters of the resolvent (5).

Let us find the relationship between the parameters of the resolvent c_1 and c_2 with the quality indicators of the transition process. It is known that the duration of the transition process at different roots of the characteristic polynomial is determined by one of them that provides the greatest constant of the attenuation time. Therefore, the transition process with multiple roots of the characteristic polynomial will be minimal in the duration [12]. A variant of the polynomial with real roots was studied in detail in [17]. We consider the case when the roots of the characteristic polynomial obtained on the basis of (2) are multiples and complex conjugates:

$$s^3 + a_0 s^2 + a_1 s + a_2 = 0 \; .$$

Assuming that the roots of the polynomial are $s = x a_0$, we pass to the parameters of the resolvent c_1 and c_2:

$$x^3 + x^2 + c_1 x + c_2 = 0 \; . \qquad (7)$$

The polynomial (7) has three roots: x_1, x_2 and x_3, one of which is always real negative, and the other two roots during oscillatory motion have a complex conjugate form. The further the roots x_1, x_2 and x_3 will be from the imaginary axis, the faster the transient process in the mechanical system will proceed. Let us assume that the root x_1 is real negative, x_2 and x_3 are complex conjugate. If the real part of the complex conjugate roots is modulo greater than the root x_1, then the duration of the transient process will be determined by its value.

Therefore, it makes sense to introduce the following dependencies between the roots:

$$x_1 = -a \; ; \; x_{2,3} = -ka \pm j\sqrt{\frac{1-\xi^2}{\xi^2}}ka,$$

where a is the real root; ξ is the parameter of relative damping; k is the coefficient that characterizes the distance of the roots from each other.

With a sufficient distance of the complex conjugate roots from the real one and with the condition that the real root is located closer to the imaginary axis, we obtain a form of the transient process close to the aperiodic one, since the component of the oscillatory motion, determined by the complex conjugate roots, will damp much faster and not have a significant effect to its curve. It should be noted that with $k > 1$ the form of the transient process is determined by the real root (aperiodic process), and with $k < 1$ - by a pair of complex conjugate roots (damping oscillatory process).

Let us establish the relationship of the roots of the polynomial with the parameters of mechanical system c_1 and c_2, which must have positive values, determined by the physical realizability of the system.

$k > 1$ and the closest root to the imaginary axis will be real. Using Vietta's formulas, reflecting the relationship between the roots and the coefficients of the polynomial (7), we compose the following system of equations:

$$\begin{cases} x_1 + x_2 + x_3 = -1 \\ x_1x_2 + x_1x_3 + x_2x_3 = c_1 \\ x_1x_2x_3 = -c_2 \end{cases}.$$

(8)

Substituting x_1 and $x_{2,3}$ into (8), we obtain the following relations between the parameters of the polynomial and the coefficients of the roots:

$$a = \frac{1}{1+2k} \; ; \qquad (9)$$

$$c_1 = a^2(2k + \frac{k^2}{\xi^2}) \; ; \qquad (10)$$

$$c_2 = \frac{k^2a^3}{\xi^2} \; . \qquad (11)$$

Substituting the expression (9) in (10) and (11), we establish the relationships between the parameters c_1 and c_2 with the coefficients k and ξ:

$$c_1 = \frac{1}{(1+2k)^2}(2k + \frac{k^2}{\xi^2}) \; ; \qquad (12)$$

$$c_2 = \frac{k^2}{\xi^2(1+2k)^3} \; . \qquad (13)$$

Putting forward an additional condition - $\xi > 0.5$, which will ensure the rapid damping of the oscillation component from a pair of complex conjugate roots, we will determine the region of existence of the parameters c_1 and c_2 on the basis of the expressions (12) and (13). Choice of the coefficient k is determined from the condition for ensuring maximum response time and k varies in the range of 0...2. As follows from the expression (9), the parameter a decreases rapidly with increasing k; therefore, the response time decreases, so the value of the coefficient k should be close to 1.

The recommended ranges for the optimal combination of parameters of the polynomial (7) for the condition of aperiodic motion ($1 < k < 2$ and $\xi = 0.5$) will be located in the ranges: $c_1 = 0.68...0.8$ and $c_2 = 0.148...0.128$. It should be noted that the maximum value c_1 corresponds to the minimum value c_2, and the maximum response time will be observed at a value k close to 1.

Let's investigate the change of the coefficients of the characteristic polynomial (7) at fixed values of the parameters of the digging mechanism [15]: $J_1 = 572 kg \cdot m^2$; $J_2 = 60 kg \cdot m^2$; $c_{12} = 7500 N \cdot m / rad$; $b_{12} = 150 N \cdot m \cdot s / rad$; and variable values of the EDD parameters c and b. Using the dependences $c_1 = \frac{a_1}{a_0^2}$ and $c_2 = \frac{a_2}{a_0^3}$ we construct the regions of existence of the parameters of the polynomial (7) during changing the parameters c and b, and select the regions for the optimal combination of these parameters (see fig. 2).
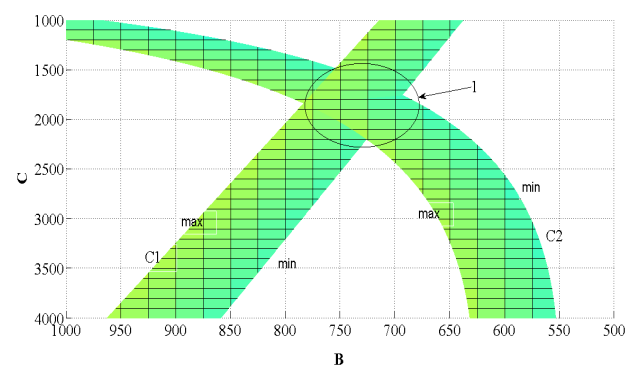
Fig. 2. Regions of optimal combination of parameters c and b

In fig. 2, the region 1 defines the boundaries of the damper parameters changing c and b, which correspond to the optimal combination of the parameters of the polynomial c_1 and c_2 for the given relative damping equal to $\xi = 0.5$. The lower point of intersection of the regions of existence of the parameters

c_1 and c_2 determines the condition for obtaining the maximum response time for the given quality indicators of transient processes. The calculated optimal values of the EDD parameters for this point were $c = 2200 N \cdot m / rad$ and $b = 725 N \cdot m \cdot s / rad$.

3 Research and discussion

To check the efficiency of the EDD with selected parameters, a numerical simulation of transient processes of the excavator's digging mechanism was carried out based on the system of equations (1), with the following parameters close to the real mechanism: $U_{ref} = 0...10$; $U_{scor} = -10 - 1.3$; $K_{SS} = 0.151$; $K_1 = 10$; $K_2 = 8$; $K_{SR} = 8$; $K_{CS} = 0.00313$; $T_{CR} = 0.864$ sec; $K_b = 120$; $T_b = 0.01$ sec; $C_e = 17.37$; $K_a = 33$; $T_a = 0.082$ sec; $J_1 = 572$ and $J_2 = 60$ $kg \cdot m^2$; $c_{12} = 7500$ $\frac{N \cdot m}{rad}$; $b_{12} = 150$ $\frac{N \cdot m \cdot \sec}{rad}$; $c = 2200 N \cdot m / rad$; $b = 725 N \cdot m \cdot s / rad$.

The studies were carried out for the start-up mode at the nominal speed, changing the load on the bucket and the locking mode. During simulation, the values of elastic force in the cable M_{12} and the drive torque M_{dv}, the drive speed ω_1 and the bucket speed ω_2 were recorded. The electromechanical system with synthesized parameters of the EDD was compared with the mechanism equipped with the EDD ($c = 3000 N \cdot m / rad$; $b = 800 N \cdot m \cdot s / rad$), the parameters of which were determined by the dependences proposed in [14]. The curves of the transient process are shown in figures 3a and 3b, where curves obtained with $c = 3000 N \cdot m / rad$; $b = 800 N \cdot m \cdot s / rad$ are designated by number 1, and curves with parameters of the EDD obtained on the basic of resolvent (5) are designated by number 2.

a)

b)
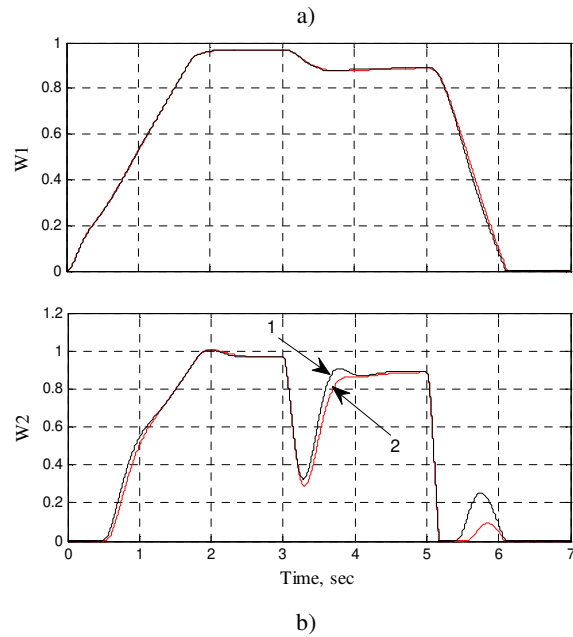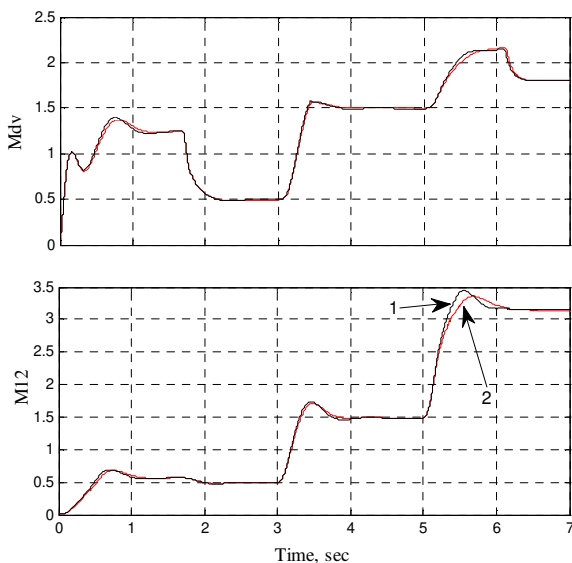
Fig. 3. The transient processes of the torques M_{12}, M_{dv} and angular velocities ω_1, ω_2

Analysis of the oscillograms from this figure shows that the synthesis of the EDD parameters, based on the choice of the resolvent coefficients (5) according to the specified quality indicators, allows reducing dynamic loads while maintaining the response time. At the same time, some deviations of the elastic force curve M_{12} from the adopted aperiodic law was observed, which could be explained by the accepted assumptions in equation (2).

This approach is essentially close to the method of normalized transfer functions [13], in which the normalized polynomial is represented by the sum of elementary units, and their parameters are determined by time constants and relative damping, depending on natural frequencies. However, the proposed method provides greater clarity and simplicity, since the problem of finding the optimal parameters is reduced to simple algorithms containing elementary algebraic operations.

4 Conclusion

The proposed method for parameters synthesis of the excavator's elastic-damping device, based on the use of the Volterra integral equation of the second kind, allowed one to explicitly link those parameters with the quality indicators (oscillatory and response time) of the transient process, determined by the resolvent coefficients. They do not contain natural frequencies and directly depend only on the physical parameters of the mechanical system. The obtained analytical dependences reflect a direct relationship between the system parameters and the quality indicators of the transient process and allow us to formalize the procedure for parametric synthesis of such devices. They can be used when creating new EDD to evaluate the effectiveness of known technical solutions.

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