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Evaluation of Manufacturing Process based on the Geometric Variation Model in Multi-station Machining Processes

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Abstract. The objective of this paper is to explore the evaluation method of manufacturing process to verify its effectiveness based on the limitation of the variations which occur in multi-station machining processes. Firstly, the manufacturing process of a mechanical part is considered as a mechanism mainly consisted of machine-tool, part-holders, machined part, and cutting tools; And small displacement torsors (SDTs) are applied to describe all deviations in the manufacturing process, including the variation deviations of the machined surfaces of a part with regards to their nominal positions, the gap deviations associated to each joint between two contact surfaces, etc; Then, the 3D manufacturing variation model is established based on the relations between the machining feature variations and the functional tolerance requirements to realize the evaluation of manufacturing process. Finally, an application example is given to illustrate the proposed method.

1 Introduction

Multi-station machining process is the most common, most important and most difficult to control in the manufacturing industry. The research on evaluation method of manufacturing process based on 3D manufacturing variation model for multi-station machining processes plays an important role in estimating the geometrical and dimensional quality of manufactured parts, optimizing the process route of products, generating robust process plans, and eliminating downstream manufacturing problems. In order to consider the influence of geometric process variations, Bourdet and Ballot [1] proposed a three-dimensional variations model by using the small displacement torsor (SDT) to model geometric deviations in manufacturing process. Based on the concept of SDT, Legoff et al. [2] put forward a method for performing tridimensional analysis and synthesis of machining tolerances. Villeneuve et al. [3] have proposed a three-dimensional model on manufacturing tolerancing for mechanical parts in which the SDT concept is used to model the machined parts, part-holders, and machining operations. Louati et al. [4] proposed a machining tolerancing method using SDT theory to optimize a manufactured part setting. Abellán-Nebot et al. [5] analyzed two 3D manufacturing variation models, the stream of variation model (SoV) and model of the manufactured part (MMP), in multi-station machining systems and compared their main characteristics and applications. Furthermore, Laifa et al. [6] presented a 3D formalization of manufacturing tolerancing which associates the concept of SDTs, the functional constraints, and manufacturing process capability. This approach enables the evaluation of manufacturing process by limiting the variations which occur at the various production setups. Despite some research achievements have been made in this related field, there are still many open issues to be explored. This paper focuses on 3D modeling of the geometrical variations in the processes of part machining and exploring the evaluation of manufacturing process by using the SDT and its transfer formula.

2 Small displacement torsor and its transfer formula

It is generally known that any variation of a geometrical feature from its nominal position can be characterized by a SDT with three rotation components (α, β, γ) around x, y, z axes and three translation components (u, v, w) along x, y, z axes, respectively [7]. The SDT $\{\boldsymbol{\tau}_{P_i/P}\}_{(O_i,R_i)} = \{\boldsymbol{\varphi} \quad \boldsymbol{\varepsilon}_{O_i}\}$, which synthesizes the position and orientation of an associated feature Pi relative to its nominal feature at a given point O_i in a local reference frame R_i, can be expressed as:

$$\{\boldsymbol{\tau}_{P_i/P}\}_{(O_i,R_i)} = \{\boldsymbol{\varphi} \quad \boldsymbol{\varepsilon}_{O_i}\} = \begin{Bmatrix} \alpha & u \\ \beta & v \\ \gamma & w \end{Bmatrix}_{(O_i,R_i)} \tag{1}$$

For a special case, the variation torsor of a plane feature Pi with regard to its nominal plane can be expressed as:

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$$\{\boldsymbol{\tau}_{P_i/P}\}_{(O_i,R_i)} = \{\boldsymbol{\varphi} \quad \boldsymbol{\varepsilon}_{O_i}\} = \begin{Bmatrix} \alpha & U \\ \beta & U \\ U & w \end{Bmatrix}_{(O_i,R_i)} \qquad (2)$$

where point O_i belongs to the plane and R_i is a local reference frame whose axis z_i is the normal of the plane, capital U represents the undetermined component in the expression of a torsor.

To calculate operations on these torsors, the following two properties are defined [3]:

$$\forall a \in R, \quad a + U = U \qquad (3)$$

$$\forall a,b \in R^2, \quad a \cdot U + b \cdot U = U \qquad (4)$$

Considering $\{\boldsymbol{\tau}_{\mathrm{P_i/P}}\}_{(\mathrm{O_i,R_i})} = \{\boldsymbol{\varphi} \quad \boldsymbol{\varepsilon}_{\mathrm{O_i}}\}$ a SDT at point O_i in a local reference frame R_i, this SDT at a given point O expressed in the global reference frame R_0, will become:

$$\{\boldsymbol{\tau}_{P_i/P}\}_{(O,R_0)} = \left\{\mathbf{R}_{0,i} \cdot \varphi \quad \mathbf{R}_{0,i} \cdot (\boldsymbol{\varepsilon}_{O_i} + (\mathbf{R}_{0,i}^T \cdot \overrightarrow{OO_i}) \times \varphi)\right\} \quad (5)$$

where $\mathbf{R}_{0,i}$ is the rotation matrix from R_0 to R_i, $\mathbf{R}_{0,i}^{\mathrm{T}}$ is the transposed matrix of $\mathbf{R}_{0,i}$, and $\overrightarrow{\mathrm{OO_i}}$ is the translation vector from R_0 to R_i expressed in R_0.

If there is only translation transformation between R_0 and R_i, the above transfer formula will be simplified as:

$$\{\boldsymbol{\tau}_{P_i/P}\}_{(O,R_0)} = \left\{\boldsymbol{\varphi} \quad \boldsymbol{\varepsilon}_{O_i} + \overrightarrow{OO_i} \times \boldsymbol{\varphi}\right\} \qquad (6)$$

3 Modelling of manufacturing variations

The manufacturing process of a part is generally composed of different set-ups, each set-up is regarded as a mechanism mainly consisted of machine-tool, part-holders, machined part, and cutting tools. We suppose that the geometrical variations in the machining process are small enough to be modeled with SDT.

3.1 The SDT chain in the machining process

In order to model the whole manufacturing process, three types of torsors need to be defined: the global variation SDT; the deviation SDT; the gap SDT.

For set-up S_k, the global varation SDTs associated with the machined part P, the part-holder H, and the machining operation M are defined respectively: $\boldsymbol{\tau}_{P/R}^{S_k}$, $\boldsymbol{\tau}_{H/R}^{S_k}$, and $\boldsymbol{\tau}_{M/R}^{S_k}$. For set-up S_k, the deviation SDTs associated with the machined surface P_i, the part-holder surface H_i, and the machining operation surface M_i are defined respectively: $\boldsymbol{\tau}_{P_i/P}$, $\boldsymbol{\tau}_{H_i/H}^{S_k}$, $\boldsymbol{\tau}_{M_i/M}^{S_k}$. For set-up S_k, the gap SDT $\boldsymbol{\tau}_{P_i/H_i}^{S_k}$ expresses the variations of the interface between surface P_i of the machined part and the corresponding surface H_i of the part-holder. Given that the parts do not interpenetrate at the contacts, each fixed component of the torsors is regarded as nil. So, for any set of two interacting surfaces (P_i, H_i), we can obtain the SDT chain as follows:

$$\boldsymbol{\tau}_{P_i/H_i} = \boldsymbol{\tau}_{P_i/P} + \boldsymbol{\tau}_{P/R} - \boldsymbol{\tau}_{H/R} - \boldsymbol{\tau}_{H_i/H} \qquad (7)$$

3.2 Modelling of the geometrical variations

The manufacturing process of a part generally consists of several different set-ups, and we will discuss how these geometrical variations are transferred between set-ups.

It is assumed that torsor $\boldsymbol{\tau}_{P_b/P_a}$ can represent the functional tolerance between two machined surfaces P_a and P_b of part P, and its expression is as follows:

$$\boldsymbol{\tau}_{P_b/P_a} = \boldsymbol{\tau}_{P_b/P} + \boldsymbol{\tau}_{P/P_a} = \boldsymbol{\tau}_{P_b/P} - \boldsymbol{\tau}_{P_a/P} \qquad (8)$$

Suppose surfaces P_a and P_b are machined in set-ups S_1 and S_2 respectively, Eq. (8) becomes:

$$\begin{aligned}\boldsymbol{\tau}_{P_b/P_a} = &(\boldsymbol{\tau}_{P_b/R}^{S_2} + \boldsymbol{\tau}_{P_i/P} - \boldsymbol{\tau}_{P_i/H_i}^{S_2} - \boldsymbol{\tau}_{H_i/H}^{S_2} - \boldsymbol{\tau}_{H/R}^{S_2}) \\ &- (\boldsymbol{\tau}_{P_a/R}^{S_1} + \boldsymbol{\tau}_{P_i/P} - \boldsymbol{\tau}_{P_i/H_i}^{S_1} - \boldsymbol{\tau}_{H_i/H}^{S_1} - \boldsymbol{\tau}_{H/R}^{S_1})\end{aligned} \qquad (9)$$

Thus, the relationship between functional tolerances and various variations in each set-up can be established to reveal how geometric variations are transferred between set-ups. This relationship can further guide the process engineers to evaluate the manufacturing process.

4 Evaluation of the manufacturing processes

In order to verify the effectiveness of the manufacturing process, we will discuss the manufacturing process evaluation method that takes into account the limitations on the manufacturing variations by the tolerances related to the functional requirements.

4.1 Parallelism tolerance requirement

As shown in Fig. 1, the parallelism tolerance of surface P_7 related to datum ***A*** (P_1) indicates that the actual plane P_7 shall be contained between two parallel planes $\boldsymbol{T}_{pa}$ apart which are parallel to datum plane A.

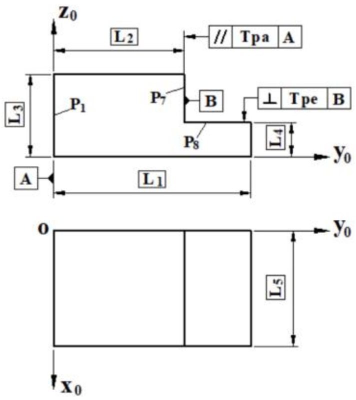

Fig. 1. Tolerance requirements

One uses the torsors to model the variations of machined plane P_7 and datum surface ***A*** (P_1) with regard to their nominal surface in the global reference frame R_0. And according to Eq. (8), one gets:

$$\{\boldsymbol{\tau}_{P_7/P_1}\}_{(O,R_0)}=\{\boldsymbol{\tau}_{P_7/P}\}_{(O,R_0)}-\{\boldsymbol{\tau}_{P_1/P}\}_{(O,R_0)} \\ =\begin{Bmatrix}\alpha_{P_7/P}-\alpha_{P_1/P} & U \\ U & v_{P_7/P}-v_{P_1/P} \\ \gamma_{P_7/P}-\gamma_{P_1/P} & U\end{Bmatrix} \qquad (10)$$

As shown in Fig.2, the variations of toleranced surface P_7 relative to datum surface P_1 is defined by the displacement of any point of the tolerance surface M_{P7} compared to the point corresponding M_{P1}. Since the displacement depends only on the rotation variations, it can be calculated as:

$$\overrightarrow{M_{P_1}M_{P_7}}=\begin{pmatrix}\alpha_{P_7/P}-\alpha_{P_1/P} \\ U \\ \gamma_{P_7/P}-\gamma_{P_1/P}\end{pmatrix}\times\begin{pmatrix}x_i \\ y_i \\ z_i\end{pmatrix} \\ =\begin{pmatrix}-y_i(\gamma_{P_7/P}-\gamma_{P_1/P}) \\ x_i(\gamma_{P_7/P}-\gamma_{P_1/P})-z_i(\alpha_{P_7/P}-\alpha_{P_1/P}) \\ y_i(\alpha_{P_7/P}-\alpha_{P_1/P})\end{pmatrix} \qquad (11)$$

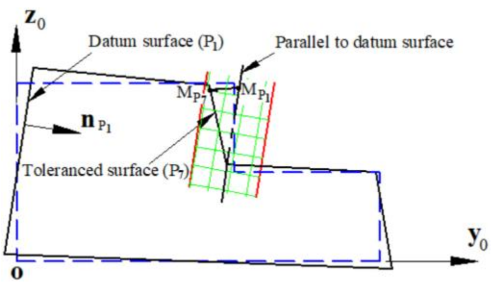

Fig. 2. Variations between toleranced surface and datum surface

To satisfy the parallelism tolerance requirement, one gets:

$$\overrightarrow{M_{P_1}M_{P_7}}\cdot\overrightarrow{\mathbf{n}_{P_1}}\le T_{\mathrm{pa}} \qquad (12)$$

where $\overrightarrow{\mathbf{n}_{P_1}}=\begin{pmatrix}\gamma_{P_1/P} \\ 1 \\ -\alpha_{P_1/P}\end{pmatrix}$ is the normal vector to the datum plane ***A***. By neglecting the higher-order terms beyond the first order, inequality (12) becomes:

$$x_i(\gamma_{P_7/P}-\gamma_{P_1/P})-\mathrm{z_i}(\alpha_{P_7/P}-\alpha_{P_1/P})\le T_{\mathrm{pa}} \qquad (13)$$

4.2 Perpendicularity tolerance requirement

As shown in Fig.1, the perpendicularity tolerance of surface P_8 related to datum surface ***B*** (P_7) indicates that the actual surface P_8 shall be contained between two parallel planes $\boldsymbol{T}_{\mathrm{pe}}$ apart that are perpendicular to datum plane ***B***.

One uses the torsors to model the variations of machined surface P_8 related to datum surface ***B*** (P_7) in the global reference frame R_0:

$$\{\boldsymbol{\tau}_{P_8/P_7}\}_{(O,R_0)}=\{\boldsymbol{\tau}_{P_8/P}\}_{(O,R_0)}-\{\boldsymbol{\tau}_{P_7/P}\}_{(O,R_0)} \\ =\begin{Bmatrix}\alpha_{P_8/P}-\alpha_{P_7/P} & U \\ U & U \\ U & U\end{Bmatrix} \qquad (14)$$

As shown in Fig.3, the variations of toleranced surface P_8 relative to datum surface P_7 is defined by the displacement of any point of the tolerance surface M_{P8} compared to the corresponding point M_{P7} of the situation surface perpendicular to the datum surface P_7. Since the displacement also depends only on the rotation variations, it can be calculated as:

$$\overrightarrow{M_{P_7}M_{P_8}}=\begin{pmatrix}\alpha_{P_8/P}-\alpha_{P_7/P} \\ U \\ U\end{pmatrix}\times\begin{pmatrix}x_i \\ y_i \\ z_i\end{pmatrix}=\begin{pmatrix}U \\ -z_i(\alpha_{P_8/P}-\alpha_{P_7/P}) \\ y_i(\alpha_{P_8/P}-\alpha_{P_7/P})\end{pmatrix} \qquad (15)$$

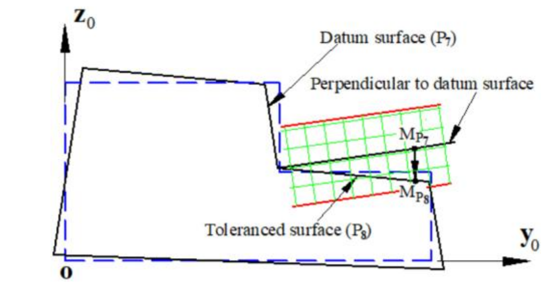

Fig. 3. Variations between toleranced surface and situation surface

Similarly, to satisfy the perpendicularity tolerance requirement, one has:

$$\overrightarrow{M_{P_7}M_{P_8}}\cdot\overrightarrow{\mathbf{n}_{P_7}}\le T_{\mathrm{pe}} \qquad (16)$$

where $\overrightarrow{\mathbf{n}_{P_7}}=\begin{pmatrix}\alpha_{P_7/P} \\ -\beta_{P_7/P} \\ 1\end{pmatrix}$ is the normal vector to the situation plane perpendicular to datum surface P_7. By neglecting the higher-order terms beyond the first order, inequality (16) becomes:

$$y_{\mathrm{i}}(\alpha_{P_8/P}-\alpha_{P_7/P})\le T_{\mathrm{pe}} \qquad (17)$$

5 Application Example

This section presents a mechanical part (see Fig. 4) to demonstrate how the proposed method can be used for evaluating the manufacturing process. The functional requirements which are transferred are the parallelism of plane P_7 with respect to datum ***A*** on plane P_1, the perpendicularity of plane P_8 with respect to datum *B* on plane P_7 as described in Fig. 4. And Figs. 5-7 show the machining process of this part, which consists of three set-ups performed on a numerical control (NC) machine-tool.

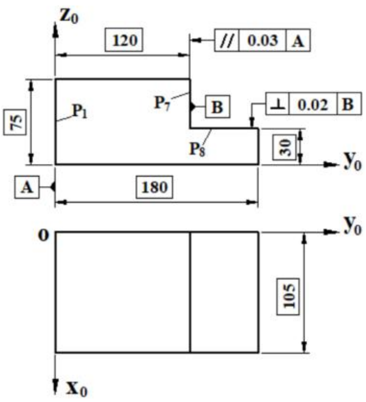

Fig. 4. Machined part geometry

5.1 Set-up 10

In set-up 10, there are no positioning surfaces. As shown in Fig. 5, the machined surfaces are marked as P_i, the local reference frame R_i (O_i, x_i, y_i, z_i) for each machined surface is defined as: Axis z_i is normal to surface P_i pointing towards the outside of the entity; Origin O_i, axes x_i and y_i of the reference frame belong to surface P_i. In the global reference frame R_0 (O, x_0, y_0, z_0) of Fig. 5, we have:

$$\{\boldsymbol{\tau}_{P_i/P}\}_{(O_i,R_i)} = \begin{Bmatrix} \alpha_{P_i} & U_P \\ \beta_{P_i} & U_P \\ U_P & w_{P_i} \end{Bmatrix}, \quad i \in \{1, 2, \cdots 6\}$$

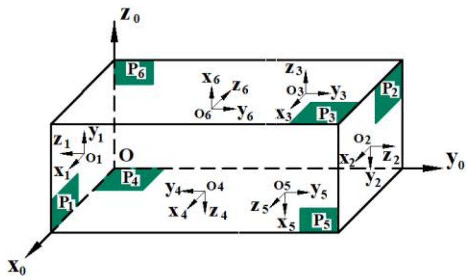

Fig. 5. The primarily machined part and reference frames

5.2 Set-up 20

In this set-up, surface 1 of the part is machined. As shown in Fig. 6 (a), the machined part is positioned on a plane (main positioning surface H_4/P_4), a cylindrical surface with radius r_c (2nd positioning surface H_2/P_2) and a spherical surface whose radius is r_s (3rd positioning surface H_6/P_6) in an isostatic set-up. The global SDT $\boldsymbol{\tau}^{20}_{\mathrm{P/R}}$ of the machined part can be obtained by combining the torsors associated with joints between the part and the part-holder. Here, the part-holder support points are also marked as O_i in a local frame R_i (O_i, x_i, y_i, z_i), $i \in \{2, 4, 6\}$.

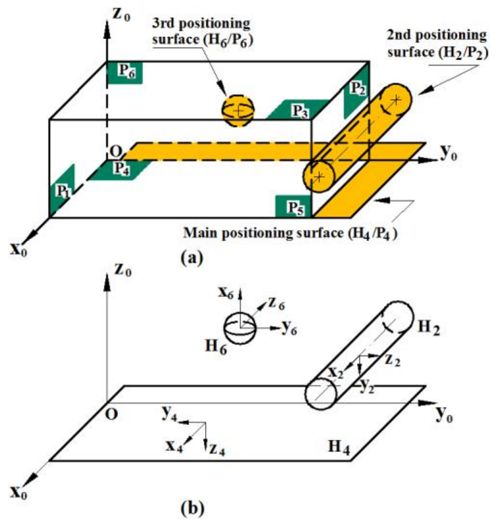

Fig. 6. Set-up20: Milling of surface P_1

As shown in Fig. 6(b), in a local frame R_i (O_i, x_i, y_i, z_i), $i \in \{2, 4, 6\}$, suppose the global variations of part-holder H are integrated within the deviation torsor of surface H_4 relative to its nominal position in this set up for the main positioning surface H_4, and combine with the expression of the torsor matrices in Table 1, we have:

$$\{\boldsymbol{\tau}^{20}_{H_4/H}\}_{(\mathrm{H_4,R_4})} = \begin{Bmatrix} \alpha^{20}_{H_4} & U_H \\ \beta^{20}_{H_4} & U_H \\ U_H & w^{20}_{H_4} \end{Bmatrix}$$

$$\{\boldsymbol{\tau}^{20}_{H_2/H}\}_{(\mathrm{H_2,R_2})} = \begin{Bmatrix} U_H & U_H \\ \beta^{20}_{H_2} & v^{20}_{H_2} \\ \gamma^{20}_{H_2} & w^{20}_{H_2} \end{Bmatrix}$$

$$\{\boldsymbol{\tau}^{20}_{H_6/H}\}_{(\mathrm{H_6,R_6})} = \begin{Bmatrix} U_H & u^{20}_{H_6} \\ U_H & v^{20}_{H_6} \\ U_H & w^{20}_{H_6} \end{Bmatrix}$$

We use Eq. (6) to calculate torsors $\{\boldsymbol{\tau}^{20}_{\mathrm{H_i/H}}\}_{(\mathrm{O_i,R_i})}, i \in \{2, 4, 6\}$, in which the translation vectors from $\{\mathrm{R_{H_i}}\}$ to $\{\mathrm{R_{O_i}}\}$ are $\overrightarrow{H_2O_2} = (0, 0, -r_c)$, $\overrightarrow{H_4O_4} = (0, 0, 0)$ and $\overrightarrow{H_6O_6} = (0, 0, -r_s)$, respectively. So for these three positioning surfaces, one will have:

$$\{\boldsymbol{\tau}^{20}_{H_4/H}\}_{(\mathrm{O_4,R_4})} = \{\boldsymbol{\tau}^{20}_{H_4/H}\}_{(\mathrm{H_4,R_4})} = \begin{Bmatrix} \alpha^{20}_{H_4} & U_H \\ \beta^{20}_{H_4} & U_H \\ U_H & w^{20}_{H_4} \end{Bmatrix}$$

$$\{\boldsymbol{\tau}^{20}_{H_2/H}\}_{(\mathrm{O_2,R_2})} = \begin{Bmatrix} U_H & U_H\text{-}r_c\beta^{20}_{H_2} \\ \beta^{20}_{H_2} & v^{20}_{H_2} + r_c U_H \\ \gamma^{20}_{H_2} & w^{20}_{H_2} \end{Bmatrix}$$

$$\{\boldsymbol{\tau}^{20}_{H_6/H}\}_{(\mathrm{O_6,R_6})} = \begin{Bmatrix} U_H & u^{20}_{H_6}\text{-}r_s U_H \\ U_H & v^{20}_{H_6} + r_s U_H \\ U_H & w^{20}_{H_6} \end{Bmatrix}$$

Then we can calculate the torsors $\{\boldsymbol{\tau}^{20}_{\mathrm{P_i/P}}\}_{(\mathrm{O,R_0})}$ and

$\{\boldsymbol{\tau}^{20}_{\mathrm{H_i/H}}\}_{(\mathrm{O,R_0})}$ $i \in \{2,4,6\}$ at point O expressed in the global reference frame R_0 (O, x_0, y_0, z_0) by using Eq. (5), suppose the origin coordinate of the local reference frame R_i is O_i (a_i, b_i, c_i), $i \in \{2,4,6\}$ (see Fig.5), the rotation matrices $\mathbf{R}_{0,\mathrm{i}}$ from R_0 to R_i respectively are:

$$\mathbf{R}_{0,2}=\begin{Bmatrix}1&0&0\\0&0&1\\0&-1&0\end{Bmatrix},\ \mathbf{R}_{0,4}=\begin{Bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{Bmatrix},\ \mathbf{R}_{0,6}=\begin{Bmatrix}0&0&-1\\0&1&0\\1&0&0\end{Bmatrix}$$

We get the expressions of $\{\boldsymbol{\tau}_{P_4/P}\}_{(\mathrm{O,R_0})}$, $\{\boldsymbol{\tau}_{P_2/P}\}_{(\mathrm{O,R_0})}$, $\{\boldsymbol{\tau}_{P_6/P}\}_{(\mathrm{O,R_0})}$, $\{\boldsymbol{\tau}^{20}_{H_4/H}\}_{(\mathrm{O,R_0})}$, $\{\boldsymbol{\tau}^{20}_{H_2/H}\}_{(\mathrm{O,R_0})}$, $\{\boldsymbol{\tau}^{20}_{H_6/H}\}_{(\mathrm{O,R_0})}$.

According to Eq. (7), we can get the gap SDT $\{\boldsymbol{\tau}^{20}_{P_i/H_i}\}_{(\mathrm{O,R_0})}$ at point O expressed in global reference frame R_0 (O, x_0, y_0, z_0). Suppose $\{\boldsymbol{\tau}_{P_i/H_i}\}_{(\mathrm{O,R_0})}=\{\boldsymbol{\varphi}\quad \boldsymbol{\varepsilon}_O\}$ then,

$$\{\boldsymbol{\tau}_{P_i/H_i}\}_{(\mathrm{O_i,R_i})}=\left\{\mathbf{R}^T_{0,i}\cdot\boldsymbol{\varphi}\quad \mathbf{R}^T_{0,i}\cdot(\boldsymbol{\varepsilon}_\mathrm{O}+\boldsymbol{\varphi}\times\overrightarrow{\mathrm{OO_i}})\right\} \quad (18)$$

Using Eq. (18) to calculate the gap SDT $\{\boldsymbol{\tau}^{20}_{P_i/H_i}\}_{(\mathrm{O_i,R_i})}$ at point O_i expressed in local reference frame R_i (O_i, x_i, y_i, z_i). And considering the properties (3) and (4), one gets:

$$\{\boldsymbol{\tau}^{20}_{P_4/H_4}\}_{(\mathrm{O_4,R_4})}=\begin{Bmatrix}\alpha_{P_4}+\alpha_P^{20}-\alpha_H^{20}-\alpha_{H_4}^{20} & U\\ \beta_{P_4}-\beta_P^{20}+\beta_H^{20}-\beta_{H_4}^{20} & U\\ U & w_{P_4}-w_P^{20}+w_H^{20}-w_{H_4}^{20}-b_4\alpha_P^{20}+b_4\alpha_H^{20}+a_4\beta_P^{20}-a_4\beta_H^{20}\end{Bmatrix}$$

$$\{\boldsymbol{\tau}^{20}_{P_2/H_2}\}_{(\mathrm{O_2,R_2})}=\begin{Bmatrix}U & U\\ \beta_{P_2}-\gamma_P^{20}+\gamma_H^{20}-\beta_{H_2}^{20} & U\\ U & w_{P_2}+v_P^{20}-v_H^{20}-w_{H_2}^{20}-c_2\alpha_P^{20}+c_2\alpha_H^{20}+a_2\gamma_P^{20}-a_2\gamma_H^{20}\end{Bmatrix}$$

$$\{\boldsymbol{\tau}^{20}_{P_6/H_6}\}_{(\mathrm{O_6,R_6})}=\begin{Bmatrix}U & U\\ U & U\\ U & w_{P_6}-u_P^{20}+u_H^{20}-w_{H_6}^{20}-c_6\beta_P^{20}+c_6\beta_H^{20}+b_6\gamma_P^{20}-b_6\gamma_H^{20}\end{Bmatrix}$$

Considering the hierarchy of the part/part-holder positioning in an isostatic set-up, the components of $\{\boldsymbol{\tau}^{20}_{\mathrm{P_4/H_4}}\}_{(\mathrm{O_4,R_4})}$ are nil because the contact between the two main positioning surfaces (H_4/P_4) has no interpenetrating parts. One gets the following equations:

$$\begin{cases}\alpha_{P_4}+\alpha_P^{20}-\alpha_H^{20}-\alpha_{H_4}^{20}=0\\ \beta_{P_4}-\beta_P^{20}+\beta_H^{20}-\beta_{H_4}^{20}=0\\ w_{P_4}-w_P^{20}+w_H^{20}-w_{H_4}^{20}-b_4\alpha_P^{20}+b_4\alpha_H^{20}+a_4\beta_P^{20}-a_4\beta_H^{20}=0\end{cases}$$

Similarly, for the second and third positioning surfaces, we have:

$$\begin{cases}\beta_{P_2}-\gamma_P^{20}+\gamma_H^{20}-\beta_{H_2}^{20}=0\\ w_{P_2}+v_P^{20}-v_H^{20}-w_{H_2}^{20}-c_2\alpha_P^{20}+c_2\alpha_H^{20}+a_2\gamma_P^{20}-a_2\gamma_H^{20}=0\end{cases}$$

$$w_{P_6}-u_P^{20}+u_H^{20}-w_{H_6}^{20}-c_6\beta_P^{20}+c_6\beta_H^{20}+b_6\gamma_P^{20}-b_6\gamma_H^{20}=0$$

Then, $\{\boldsymbol{\tau}^{20}_{P/R}\}_{(\mathrm{O,R_0})}$ can be derived as:

$$\{\boldsymbol{\tau}^{20}_{P/R}\}_{(\mathrm{O,R_0})}=\begin{Bmatrix}-\alpha_{P_4}+\alpha_H^{20}+\alpha_{H_4}^{20} & w_{P_6}+u_H^{20}-w_{H_6}^{20}-c_6\beta_{P_4}+c_6\beta_{H_4}^{20}+b_6\beta_{P_2}-b_6\beta_{H_2}^{20}\\ \beta_{P_4}+\beta_H^{20}-\beta_{H_4}^{20} & -w_{P_2}+v_H^{20}+w_{H_2}^{20}-c_2\alpha_{P_4}+c_2\alpha_{H_4}^{20}-a_2\beta_{P_2}+a_2\beta_{H_2}^{20}\\ \beta_{P_2}+\gamma_H^{20}-\beta_{H_2}^{20} & w_{P_4}+w_H^{20}-w_{H_4}^{20}+b_4\alpha_{P_4}-b_4\alpha_{H_4}^{20}+a_4\beta_{P_4}-a_4\beta_{H_4}^{20}\end{Bmatrix}$$

Using Eq. (18), we get the expressions of $\{\boldsymbol{\tau}^{20}_{P/R}\}_{(\mathrm{O_1,R_1})}$ and $\{\boldsymbol{\tau}_{P_1/P}\}_{(\mathrm{O_1,R_1})}$. According to Eq. (5), for the machined surface P_1 in this set-up we have:

$$\{\boldsymbol{\tau}_{P_1/P}\}_{(\mathrm{O,R_0})}=\begin{Bmatrix}\alpha_{P_1}+\alpha_{P_4}-\alpha_H^{20}-\alpha_{H_4}^{20} & U\\ U & -w_{P_1}+w_{P_2}-v_H^{20}-w_{H_2}^{20}+c_2\alpha_{P_4}-c_2\alpha_{H_4}^{20}+a_2\beta_{P_2}-a_2\beta_{H_2}^{20}-a_1\beta_{P_1}+c_1\alpha_{P_1}\\ \beta_{P_1}-\beta_{P_2}-\gamma_H^{20}+\beta_{H_2}^{20} & U\end{Bmatrix}$$

5.3 Set-up 30

In set-up 30, surfaces 7 and 8 are machined. As shown in Fig.7 (a), the part is positioned on a plane (main positioning surface H_4/P_4), a cylindrical surface with radius r_c (2nd positioning surface H_1/P_1) and a spherical surface with radius r_s (3rd positioning surface H_6/P_6) in an isostatic set-up.

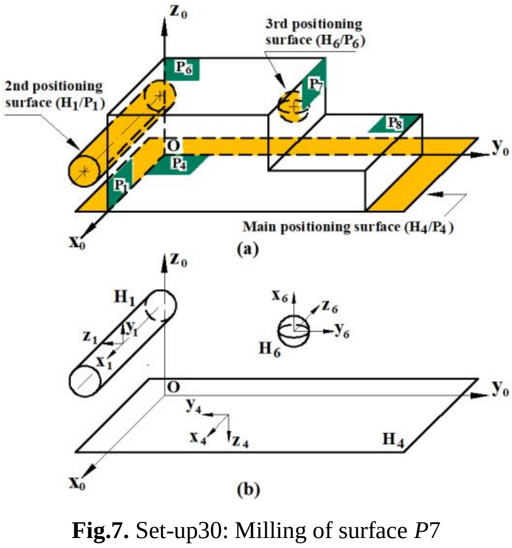

Fig.7. Set-up30: Milling of surface *P7*

Fig. 7 (b) shows the positioning device H_i of the part-holder in a local frame R_i (O_i, x_i, y_i, z_i), $i \in \{1, 4, 6\}$. Consider the main positioning surface H_4, and suppose the global variations of part-holder H are integrated within the deviation torsor of surface H_4 relative to its nominal position in set up 30. Similarly, according to Eq. (6) we have the expressions of $\{\boldsymbol{\tau}^{30}_{H_4/H}\}_{(\mathrm{O_4,R_4})}$, $\{\boldsymbol{\tau}^{30}_{H_1/H}\}_{(\mathrm{O_1,R_1})}$ and $\{\boldsymbol{\tau}^{30}_{H_6/H}\}_{(\mathrm{O_6,R_6})}$.

Suppose the origin coordinate of the local reference frame R_i is O_i (a_i, b_i, c_i), $i \in \{1, 4, 6\}$ (see Fig. 5), we can calculate the torsors $\{\tau_{P_i/P}\}_{(O,R_0)}$ and $\{\tau_{H_i/H}\}_{(O,R_0)}$ at point O expressed in the global reference frame R_0 (O, x_0, y_0, z_0) by using Eq. (5), one gets the expressions of $\{\boldsymbol{\tau}_{P_4/P}\}_{(O,R_0)}$, $\{\boldsymbol{\tau}_{P_1/P}\}_{(O,R_0)}$, $\{\boldsymbol{\tau}_{P_6/P}\}_{(O,R_0)}$, $\{\boldsymbol{\tau}^{30}_{H_4/H}\}_{(O,R_0)}$, $\{\boldsymbol{\tau}^{30}_{H_1/H}\}_{(O,R_0)}$ and $\{\boldsymbol{\tau}^{30}_{H_6/H}\}_{(O,R_0)}$. According to Eq. (7), we can get the gap SDT $\{\boldsymbol{\tau}^{30}_{P_i/H_i}\}_{(O,R_0)}$ at point O expressed in global reference frame R_0 (O, x_0, y_0, z_0). Using Eq. (18) to calculate the gap SDT $\{\boldsymbol{\tau}^{30}_{P_i/H_i}\}_{(O_i,R_i)}$ at point O_i expressed in reference frame R_i (O_i, x_i, y_i, z_i). And considering the properties (3) and (4), one gets the expressions of $\{\boldsymbol{\tau}^{30}_{P_4/H_4}\}_{(O_4,R_4)}$, $\{\boldsymbol{\tau}^{30}_{P_1/H_1}\}_{(O_1,R_1)}$ and $\{\boldsymbol{\tau}^{30}_{P_6/H_6}\}_{(O_6,R_6)}$.

So $\{\boldsymbol{\tau}^{30}_{P/R}\}_{(O,R_0)}$, $\{\boldsymbol{\tau}_{P_7/P}\}_{(O,R_0)}$ and $\{\boldsymbol{\tau}_{P_8/P}\}_{(O,R_0)}$ can be derived as:

$$\{\boldsymbol{\tau}^{30}_{P/R}\}_{(O,R_0)} = \begin{Bmatrix} -\alpha_{P_4} + \alpha^{30}_H + \alpha^{20}_{H_4} & w_{P_6} + u^{30}_H - w^{30}_{H_6} - c_6\beta_{P_4} + c_6\beta^{20}_{H_4} - b_6\beta_{P_1} + b_6\beta_{P_2} + b_6\gamma^{20}_H - b_6\beta^{20}_{H_2} + b_6\beta^{30}_{H_1} \\ \beta_{P_4} + \beta^{30}_H - \beta^{20}_{H_4} & w_{P_1} - w_{P_2} + v^{20}_H + w^{20}_{H_2} - c_2\alpha_{P_4} + c_2\alpha^{20}_{H_4} - a_2\beta_{P_2} + a_2\beta^{20}_{H_2} + v^{30}_H - w^{30}_{H_1} - c_1\alpha^{20}_H + a_1\beta_{P_1} - a_1\beta^{30}_{H_1} \\ -\beta_{P_1} + \beta_{P_2} + \gamma^{20}_H - \beta^{20}_{H_2} + \gamma^{30}_H + \beta^{30}_{H_1} & w_{P_4} + w^{30}_H - w^{30}_{H_4} + b_4\alpha_{P_4} - b_4\alpha^{20}_{H_4} + a_4\beta_{P_4} - a_4\beta^{20}_{H_4} \end{Bmatrix}$$

$$\{\boldsymbol{\tau}_{P_7/P}\}_{(O,R_0)} = \begin{Bmatrix} \alpha^{30}_{P_7} + \alpha_{P_4} - \alpha^{30}_H - \alpha^{20}_{H_4} & U \\ U & w^{30}_{P_7} + a_7\beta^{30}_{P_7} + c_7\alpha^{30}_{P_7} - w_{P_1} + w_{P_2} - v^{20}_H - w^{20}_{H_2} + c_2\alpha_{P_4} - c_2\alpha^{20}_{H_4} + a_2\beta_{P_2} - a_2\beta^{20}_{H_2} - v^{30}_H + w^{30}_{H_1} + c_1\alpha^{20}_H - a_1\beta_{P_1} + a_1\beta^{30}_{H_1} \\ -\beta^{30}_{P_7} + \beta_{P_1} - \beta_{P_2} - \gamma^{20}_H + \beta^{20}_{H_2} - \gamma^{30}_H - \beta^{30}_{H_1} & U \end{Bmatrix}$$

$$\{\boldsymbol{\tau}_{P_8/P}\}_{(O,R_0)} = \begin{Bmatrix} \alpha^{30}_{P_8} + \alpha_{P_4} - \alpha^{30}_H - \alpha^{20}_{H_4} & U \\ \beta^{30}_{P_8} - \beta_{P_4} - \beta^{30}_H + \beta^{20}_{H_4} & U \\ U & w^{30}_{P_8} + a_8\beta^{30}_{P_8} - b_8\alpha^{30}_{P_8} - w_{P_4} - w^{30}_H + w^{30}_{H_4} - b_4\alpha_{P_4} + b_4\alpha^{20}_{H_4} - a_4\beta_{P_4} + a_4\beta^{20}_{H_4} \end{Bmatrix}$$

5.4 Respect of the functional tolerances

According to Section 3, we establish the relationship between the functional tolerances and the geometrical variations in the manufacturing process to evaluate the validity of the manufacturing process.

- Consider the parallelism tolerance requirement between surface P_1 and P_7. The variation torsor of surface P_7 relative to surface P_1 (datum surface) is:

$$\{\boldsymbol{\tau}_{P_7/P_1}\}_{(O,R_0)} = \begin{Bmatrix} \alpha^{30}_{P_7} - \alpha^{30}_H - \alpha_{P_1} + \alpha^{20}_H & U \\ U & w^{30}_{P_7} + a_7\beta^{30}_{P_7} + c_7\alpha^{30}_{P_7} - v^{30}_H + w^{30}_{H_1} + c_1\alpha^{20}_H + a_1\beta^{30}_{H_1} - c_1\alpha_{P_1} \\ -\beta^{30}_{P_7} - \gamma^{30}_H - \beta^{30}_{H_1} & U \end{Bmatrix}$$

The geometrical variation between tolerance surface P_7 and its nominal position related to the datum plane P_1 is defined by the displacement of any point of P_7 compared to the corresponding point of P_1. And this variation only depends on rotation variations, which can be calculated as:

$$45(-\alpha^{30}_{P_7} + \alpha^{30}_H + \alpha_{P_1} - \alpha^{20}_H) - 105(\beta^{30}_{P_7} + \gamma^{30}_H + \beta^{30}_{H_1}) \le 0.03$$

- Consider the perpendicularity tolerance requirement between surface P_7 and P_8. And this variation also depends only on rotation variations. It needs to check all the points of the toleranced surface are located in the spatial region between two parallel planes spaced with a distance of the perpendicularity tolerance (theoretically perpendicular to the specified datum plane). The variation of toleranced surface P_8 relative to datum surface P_7 can be written as:

$$\{\boldsymbol{\tau}_{P_8/P_7}\}_{(O,R_0)} = \{\boldsymbol{\tau}_{P_8/P}\}_{(O,R_0)} - \{\boldsymbol{\tau}_{P_7/P}\}_{(O,R_0)}$$
$$= \begin{Bmatrix} \alpha^{30}_{P_8} - \alpha^{30}_{P_7} & b_8U_P - c_8\beta^{30}_{P_8} + c_7U_P + b_7\beta^{30}_{P_7} \\ \beta^{30}_{P_8} - U_P & U_P - a_8U_P + c_8\alpha^{30}_{P_8} - w^{30}_{P_7} - a_7\beta^{30}_{P_7} - c_7\alpha^{30}_{P_7} \\ \beta^{30}_{P_7} + U_P & w^{30}_{P_8} + a_8\beta^{30}_{P_8} - b_8\alpha^{30}_{P_8} + U_P - a_7U_P + b_7\alpha^{30}_{P_7} \end{Bmatrix}$$

In order to respect the parallelism tolerance requirement, the following inequality should be satisfied:

$$\left| 60(\alpha^{30}_{P_8} - \alpha^{30}_{P_7}) \right| \le 0.02$$

6 Conclusions

Our research focuses on the development of 3D manufacturing variation model and verifying the validity of manufacturing process in multi-station machining processes. In multi-station machining process, each machining set-up is regarded as a single mechanism, and SDTs are employed to express the geometrical variations of parts caused by the machining operations and positioning dispersions during the successive machining set-ups. The SDT chain can be established based on the link between the process planning and functional tolerances, which makes it possible to evaluate the manufacturing process. The proposed method is illustrated by an example.

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