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Nonlinear properties of hybrid construction of coatings of buildings and structures

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Abstract. Stress and strain state of hybrid (combined) systems including flexible and rigid elements is studied in the article. Theoretical approach is presented. The feature of the systems studied is described, i.e. structural nonlinearity. Numerical analysis is presented. It is pointed out that vibrations of such structures upon conditions of resonance differ from those of classical bar structures, i.e. if for rigid bar systems the amplitudes of vibration at resonant disturbance increase monotonously, in combined (hybrid) system alternate switching off tie-bars stabilizes the amplitude of vibration at a certain value and transfers vibrations in the beating mode that can be considered as an internal vibration absorber.

1 Introduction

Combined systems are the assemblage of rigid and flexible elements, at that, flexible elements due to their features take only stretching forces (Fig. 1).

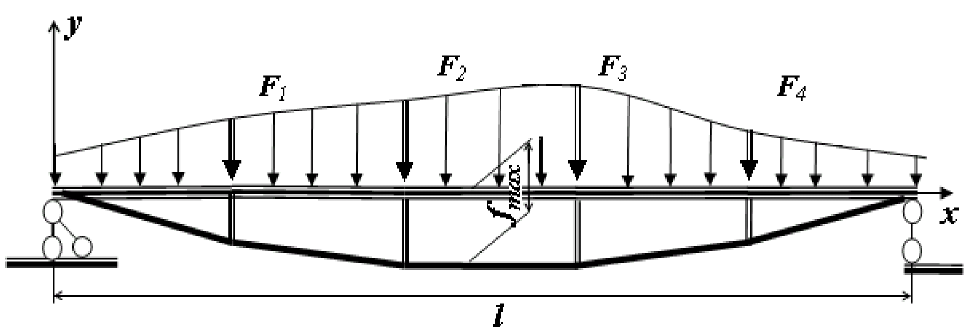

Fig. 1. The general view of strut frame. Where: 1 – stiffening beam, 2 – tie-bar, 3 – supports.

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Such structures are used as bearing elements of structure and installation roofs. Strength and stability calculation of rigid element systems is given in a number of publications [1,2,3,4,5,6], and combined systems and ideal cable systems, in [8,9,10,11,12,13,14,15].

Recently, new structural concepts for bearing structures of transport works and building roofs have been proposed. In particular, RF patented (Nos 2169243, 2169242) technical solutions have been proposed and put into practice. The basis for the solutions offered is a hybrid system including stiffening beam reinforced with preliminarily stressed tie-bars.

Various dynamic loads may have impact on the systems considered: wind, industrial seismic loads caused for example by a passing train, etc. This defines the necessity of hybrid system dynamic calculation.

Vibrations of such combined systems have nonlinear behaviour resulting from variability of elastic repulsion of preliminarily stressed tie-bars, longitudinal and transverse strains of beam, capability of tie-bars to take stretching forces only, etc.

For evaluation of impact of the above factors, necessity arises to develop mechanical and mathematical model of preliminarily stressed strut-frame vibrations.

2 Methods, methodology

To create mathematical model for calculation of hybrid systems, finite element scheme of dynamic analysis which takes into account geometrical and structural nonlinearity has been developed. The model has been developed on the basis of earlier conducted studies [16].

Expressions of full potential and kinetic energy for the system with i elements have the following form:

$$V=\sum_{i-1}^{m}V_i;\quad T=\sum_{i=1}^{m}T_i; \tag{1}$$

Here, m is the number of system elements.

After putting the obtained expressions for V and T into Lagrange equations of the II kind, the system of equations of system balance is obtained for determination of vector of nodal displacements $\{\,\overline{q}\,\}$ in global coordinates at impact of random external loads:

$$[\overline{K}]\{\overline{q}(t)\}+[\overline{M}]\{\ddot{\overline{q}}(t)\}=\{\overline{P}(t)\}, \tag{2}$$

where $[\,\overline{K}\,]$ is rigidity matrix of system in global coordinates obtained by formula, $[\,\overline{M}\,]$ is system mass matrix in global coordinates.

$$[\,\overline{M}\,]=[\,H\,]^T\,[\,M_G\,][\,H\,]\text{, here }[\,M_G\,]=\left|[\,M'\,]_1\ldots[\,M'\,]_m\right|, \tag{3}$$

where $[\,M'\,]_i$ is mass matrix of i -th finite element in global coordinates.

Generally, rigidity matrix and mass matrix in the local coordinate system look like as follows:

$$[U] = \left[\begin{array}{ccc|ccc} A & & & B & & \\ & C & D & & E & F \\ & D & G & & -F & H \\ \hline B & & & A & & \\ & E & -F & & C & -D \\ & F & H & & -D & G \end{array}\right], \quad (4)$$

where for rigidity matrix and mass matrix

$$\left.\begin{array}{l} A=\dfrac{EF}{a},\ B=\dfrac{-EF}{a},\ C=\dfrac{12EI}{a^3}, \\ D=\dfrac{6EI}{a^2},\ E=-\dfrac{12EI}{a^3},\ F=\dfrac{6EI}{a^2}, \\ G=\dfrac{4EI}{a},\ H=\dfrac{2EI}{a}, \end{array}\right\} \qquad \left.\begin{array}{l} A=\dfrac{ma}{3},\ B=\dfrac{ma}{6},\ C=\dfrac{13ma}{35}, \\ D=\dfrac{11ma^2}{210},\ E=\dfrac{9ma}{70},\ F=-\dfrac{13ma^2}{420}, \\ G=\dfrac{ma^3}{105},\ H=-\dfrac{ma^3}{140}, \end{array}\right\}, \quad (5)$$

Here, m is element mass per unit length, whereas a is element length.

Similarly, $[U'] = [T]^T [U][T]$ is obtained.

After placing the expressions obtained for matrix of transformation [T] and expression for [U], the result will be as follows:

$$[U'] = \left[\begin{array}{ccc|ccc} l^2A+m^2C & lm(A-C) & -mD & l^2B+m^2E & lm(B-E) & -mF \\ lm(A-C) & l^2C+m^2A & lD & lm(B-E) & l^2E+m^2B & lF \\ -mD & lD & G & mF & -lF & H \\ \hline l^2B+m^2E & lm(B-E) & mF & l^2A+m^2C & lm(A-C) & mD \\ lm(B-E) & l^2E+m^2B & -lF & lm(A-C) & l^2C+m^2A & -lD \\ -mF & lF & H & mD & -lD & G \end{array}\right]. \quad (6)$$

Here, l and m are direction cosines of axes of global coordinates with respect to local coordinate system.

In some cases it is necessary to consider elements for which the vector of internal forces or its separate components turn out to be zero in a certain shift range: $R_i = 0$.

Such a necessity occurs, for example, at modelling of a stretched rope which at certain compression starts to sag and stops having impact on the system. Besides, such an element, as a rule, is already strained by the initial moment of time (for example, tie-bar is pulled with a certain force), and, thus, influences the system that shall be also considered.

This element will be considered at its own, local coordinate system. Then, {P}, {R}, {q} are vectors of external nodal load, element nodal forces and element nodal shifts, correspondingly, and [K] and [M] are element mass and rigidity matrices. Besides, it is necessary to consider vector $\{R^0\}$, which is the vector of initial nodal forces that are tie-bar pre-stressing forces.

To consider impact of initial nodal forces on system, one can modify the vector of external nodal load as follows:

$$\{P'\} = \{P\} - \{R^0\}. \quad (7)$$

Here, sign “–“ occurs due to the fact that one shall consider element impact on system and not vice versa. This approach allows considering element unstrained by the first moment of time.

Further, let it be that at some moment of time there is strained state of element defined by vector of nodal shifts {q}. Then, nodal forces are determined by formula: {R} = [K]{q}. Let i be an index of component for which "switching off" internal force for this element is permitted. In this case, one should check the following two conditions: the value of i-th component of current vector of nodal forces shall differ by sign from the value of the corresponding component of the vector of initial nodal forces and shall exceed it by module,

$$sign(R_i) \neq sign(R_i^0),\ |R_i| > |R_i^0|.$$

At that, elastic and inertia attributes of the element are "switched off" for this component, in other words, rigidity and element mass matrices are transformed as follows:

$$[K'] = [D][K][D],\ [M'] = [D][M][D], \quad (8)$$

where D is modified single matrix whose i-th element on the main diagonal is equal to zero. Rigidity and mass matrices remain transformed in such a way so long as the presented above conditions of tie-bars "switching off" are met simultaneously.

Thus, equation of dynamic balance of the system in global coordinates is modified as follows:

$$[\overline{K}']\{\overline{q}(t)\} + [\overline{M}']\{\ddot{\overline{q}}(t)\} = \{\overline{P}'(t)\}, \quad (9)$$

where $[\overline{K}']$ and $[\overline{M}']$ are system mass and rigidity matrices in global coordinates, transformed taking into account "switched off" elements, and $\{\overline{P}'(t)\}$ is vector of external nodal forces modified taking into account the initial nodal forces of system elements.

For solution of this equation system, explicit scheme of Runge-Kutta-Merson of the fourth order of time accuracy was used with automatic monitoring of solution accuracy. Result processing is taking place.

To determine the vibration frequencies of the system, solution is resolved in Fourier row.

3 Conclusion

Numerical analysis of work of considered structures has proved that the behaviour of vibrations of preliminarily stressed combined systems differs substantially from the similar bar ones. Thus, the frequency of own vibrations of rigid bar systems ω_0 does not depend on initial disturbance, and for combined systems this statement is just only upon condition of not switching off tie-bars.

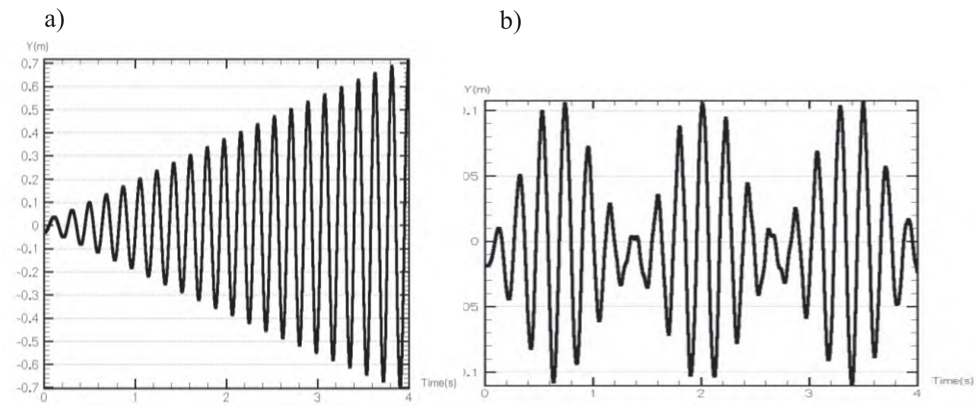
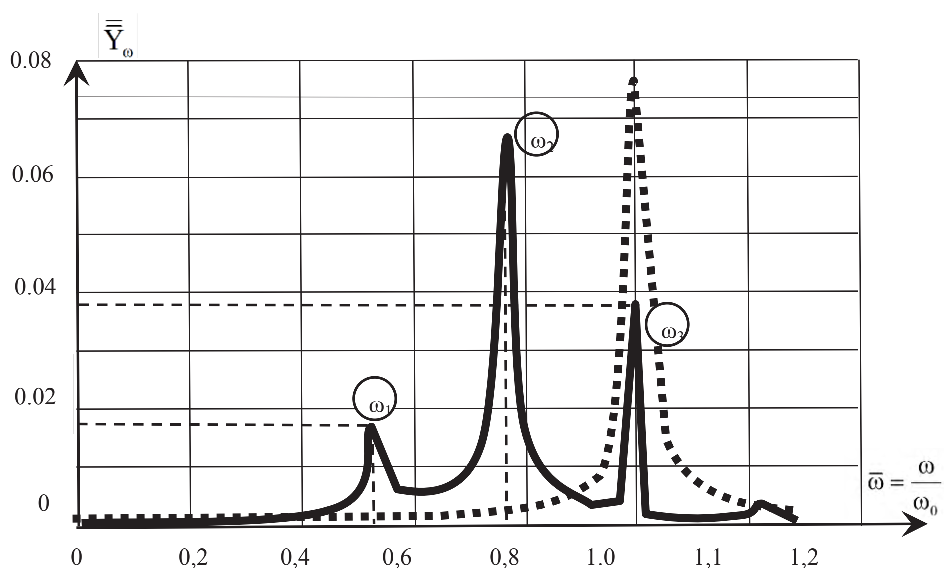


Fig. 2. Oscillograms of vibrations at resonant frequency of disturbing force: a) beam, b) the same as strut frame with switched off tie-bars.

In Fig. 2 one can see oscillogram of forced vibrations of combined system (Fig. 1) at resonant frequency of external disturbing force.

From Fig. 2 one can see that at ordinary beam vibrations, the amplitudes at resonant disturbance increase monotonously, and in combined system alternate switching off tie-bars stabilizes the amplitude at a certain value and transfers vibrations in beating mode. This feature of combined systems may be regarded as the internal vibration absorber. In Fig. 3 one can see spectrograms of beam and strut frame vibrations.



$\bar{\bar{Y}}_\omega$ - the value of spectral amplitude.

Fig. 3. Results of spectral analysis of forced vibrations by one semi-wave of strut frame and equivalent beam.

The spectrum of strut frame vibration amplitudes differs from beam vibrations spectrogram: instead of resonant increase of vibration amplitudes one can observe the beating mode characterised by periodic increase and decrease of vibration amplitudes. In vibration spectrogram there are three frequencies of vibrations, correspondingly, ω_2 has the largest spectral amplitude. The value of strut-framed beam vibration amplitude value is less than that of the tested one. Thus, if an ordinary beam is featured with continuous increase of vibration amplitudes in time, for strut frame the vibration amplitude has quite a specific stable value.

By results of conducted studies technical solutions of strut-framed structures have been developed and patented, which allow using the factor of structural nonlinearity to decrease forces and strains in them [17, 18, 19, 20, 21].

4 Summary

1. Design model for evaluation of stress and strain state of hybrid systems consisting of flexible and rigid elements taking into account the factors of geometrical and structural nonlinearities is proposed.
2. Revealed features of considered systems to be taken into account at their real designing.

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