# Modelling and Performance Analysis of Electric Car-Sharing Systems Using Petri Nets

A. Hamroun<sup>4</sup>, K. Labadi<sup>\*1,2,3</sup>, and M. Lazri<sup>4</sup>

<sup>1</sup>ECAM-EPMI, 13 Bld de l'Hautil, 95092 Cergy-Pontoise, France

<sup>2</sup>LR2E Laboratoire de Recherche en Energétique et Eco-Innovation Industrielle

<sup>3</sup>Quartz-Lab (EA 7393) - <sup>4</sup>Université de Tizi-Ouzou, LAMPA, Algérie

**Abstract.** Car sharing systems emerged as a new answer to mobility challenges in smart and sustainable cities. Despite their apparent success, design and exploitation of such systems raise crucial strategic and operational challenges. To help planners and decision makers, simulation, analysis and optimization models are unavoidable. Based on the formal modelling and analysis power of stochastic Petri nets, this paper proposes a discrete event simulation model for electric car sharing systems for performance and analysis purposes, taking into account their complex dynamic behaviour, organization and parameters including capacities of the stations, battery and energy availability, locations of charging stations and also their car maintenance activities, not negligible compared to the case of bike-sharing systems.

## 1. General introduction

With ecological and environmental transition, electric car-sharing systems emerged as new sustainable mobility in many cities around the world. They are introduced to face some crucial problems related to climate change, oil dependency, air pollution, and traffic congestion. Historically speaking, the original concept of car-sharing system was first realized by Carpooling, which was used as a rationing tactic in the United States during the Second World War. Later, the first professional carsharing service is proposed by SEFAGE in Zurich in 1948. This concept have evolved dramatically due to the oil and energy crisis in the 1970s and today, the concept of car-sharing systems is developed and more attractive by using internet, mobile, and information technologies. Car-Sharing services are one of the solutions that currently spreading in big cities over the world such as Autolib', Bluely, I-Move in France, Bundesverband CarSharing, Share a Starcar, Stattauto München in Germany, Car2Go, Okanagan CarShare, Zipcar in Australia, Buffalo CarShare, City CarShare, eGo CarShare, Ithaca Carshare in United States, etc.). The concept is becoming a mainstream transportation solution with more than a million users in over 26 countries [12].

In its basic structure, a car-sharing network consists of a set of parking and charging stations and a fleet of electric vehicles to be used in a self-service mode by customers (users). For management of such systems, there are different possible organizations including commercial and business companies, public agencies, and cooperatives. In the field of transportation, electric car-sharing systems has emerged as an excellent

mobility to cope with environmental issues affecting nowadays several dense urban areas and big cities in the world. Many schemes of the system have been implemented and can be classified into "one-way" and "two-way" systems, according to whether the users (i.e., customers) should return the rented car at a different or at the location they picked it up. "Two-way systems" require users to return the cars to their original pick up stations. Although relatively easy to manage, this operational mode is restrictive for some users. In order to make car-sharing service more flexible, the rented vehicles in "one-way" systems can be returned at any station destination.

## 2. Challenges and issues

Despite their apparent success over the world, carsharing systems present several and crucial strategic and operational problems. So, to help professional organisation and managers of such "complex" urban transportation networks, modelling, performance evaluation methods, optimization models and simulation tools are requested. For a more comprehensive literature review, the reader can refer to [1, 5, 12].

The existing works in this research field addresses several issues including modelling, analysis and optimization of strategic, tactical and operational decisions. Strategic issues aim to optimize the number, size and allocation of stations, and the number of the electric-cars to be deployed [2, 4, 7]. Operational issues are notably related to the rebalancing operations to improve the level of service and to make the system more profitable [3, 9, 13]. About the literature, different

<sup>\*</sup>Corresponding author: k.labadi@ecam-epmi.com

Figure 1. Car-Sharing design network

approaches and methodologies have been proposed, including mixed integer programming approaches, heuristic optimization approaches, data mining and statistical analysis models and simulation-optimization methodologies. However, as for bike sharing systems [6, 8], the recent and extensive literature on car-sharing systems is dominated by operational research (OR) approaches for solving some specific optimization issues such as routing, planning, scheduling, assignment and resource allocation problems.

For global performance evaluation and analysis of such complex systems, they may well be described through a discrete event dynamic model (SED). They can be characterized by a space of discrete states (e.g., number of available cars in a station) and dynamic changes of states according to some discrete events (e.g., return or departure of cars to (from) a station by user over time). Among the formalisms used to model complex dynamical systems, (stochastic) Petri Nets (PN) have arisen as a practical formalism for modelling DES, widely used thanks to their graphical and mathematical foundation, ready to be exploited for modelling, simulation, theoretical analysis, as well as performance evaluation [9, 11, 14, 16].

In this work, in line with our\* previous original works on modelling, simulation and analysis of public bike sharing systems by using stochastic Petri nets (with marking dependent arc weights) [6, 8, 15], electric car sharing systems are viewed as discrete event systems (DES). Despite their similar concept, the exploitation and management of electric car sharing systems is more complex than bicycle sharing systems due to their specific operational constraints and activities including battery and energy availability; locations of charging stations, regulation of the stations by moving car by car, more road car incidents/ accidents and then car maintenance operations.

# 3. Electric Car-Sharing system (ECSS)

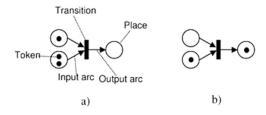
The basic concept of car sharing systems is simple: a fleet of cars can be shared between several users, which can drive a car when they need it. Even the idea is conceptually simple, in the last years several business and operational modes are adopted. Car-sharing systems can be classified in different configurations according to the rental conditions. In this work, we consider the case of "one-way" systems allowing people to park the cars anywhere in the covered area, whereas "round-trip" systems force the user to return the car to the location where it was picked-up. The type of vehicle (combustion, electric or hybrid cars) affects also the system's use. In our case, one-way Electric Car sharing Systems are considered, because of their increased flexibility towards the user compared to round-trip systems and their eco-friendly characteristics. On the other hand, the conception and management of such transportation systems poses several ecological additional challenges with respect to those based on traditional combustion vehicles, mainly related with the limited autonomy allowed by current battery technology.

As shown in Figure 1, we consider a realistic configuration of an Electric Car-Sharing system. The network system is structured around a city and its surrounding commercial and industrial area. It contains N stations (parks), noted by S = {S1, S2, ..., Sn} located around the city and one car-sharing center, located outside the city center, for management and maintenance operations of the system. Each station Si represents a private parking space with its limited (Ci) car parks (places) which are equipped by electric charging points. A car, when available, can be taken out from any station and returned to the same or any other station (one-way rental mode), provided that there is an available place in the station. After parking a vehicle at a given station, it becomes available for other users after a delay useful for

battery charging and eventually for short maintenance operations locally performed by the service provider. Furthermore, car troubles and accidents that need to a "long" interventions and maintenance operations are not negligible unlike bike sharing systems. In this case, the concerned cars are moved to the maintenance park by the maintenance service. The system network contains a parking space that can be used by the maintenance service and also considered as another specific station for users. This parking space is strategically useful for linking the city to the commercial and industrial zone.

# 4. A Petri Net modelling of an ECSS

This work is developed in line with our previous works, based on the modelling and performance analysis power of (stochastic) Petri nets for discrete event systems including modelling, control and balancing of bicycle sharing systems [8, 6]. This section is dedicated to develop a realistic stochastic Petri net model for discrete event simulation and performance analysis of electric car sharing systems, taking into account their organization (Figure 1) and dynamic behaviour (e.g., station capacities, battery recharging, car maintenance, ...).



**Figure 2.** Illustrative example of a simple Petri net - (a) Before the transition firing - (b) After the transition firing

#### 4.1. Basics of Petri Nets

This sub-section is dedicated for basic definitions and description of Petri nets [11]. A Petri net can be defined as is bipartite graph with places and transitions connected by directed arcs. Graphically, places are represented as circles and transitions by rectangles or bars (see Figure 2). The tokens, denoted by dots, contained in places can move from input places to output places by firing the transitions, over the time (firing delays associated to the transitions). At each firing of an enabled transition, the number of tokens removed/added is determined by the weights of the arcs connecting the transition with the corresponding places. A transition can be fired only if it is enabled. The enabling condition means all preconditions for the corresponding activity are fulfilled. In other words, there are enough tokens available in the input places of the transition. The signification of a Petri net model and the interpretation of its component depends on the system to be modelled. In the general case, the transitions are used to model "start" or "end" of events (actions, operations, processes and/or activities) and the occurrence of each event is represented by firing the corresponding transition (activity execution). The directed arcs between the places and the transitions are used to represent material, resource, information, or control flow directions. The ordinary Petri net formalism has been developed in different categories in order to extend its modelling power of diverse complex dynamical systems [14]. In this work, stochastic/ timed Petri nets are chosen to model ECSS.

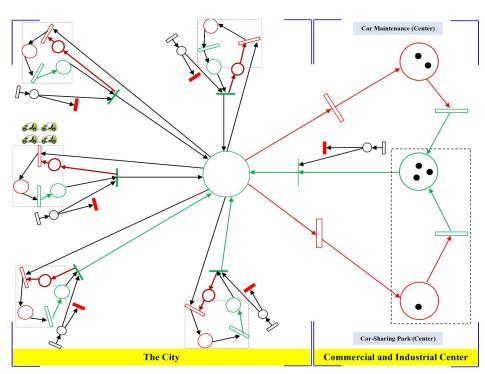


Figure 3. The Petri net model of the ECSS

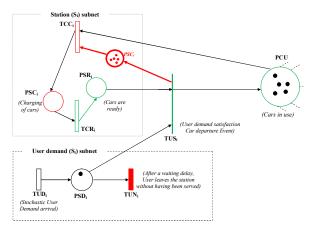


Figure 4. The "car station" and "user demand" subnets

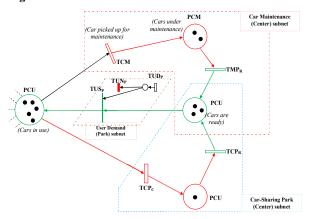


Figure 5. The "car station" and "user demand" subnets

Table 1. Interpretation of places of the model

Places	Meaning	
PCU	Its marking represents cars in use by customers. In other words, it corresponds to the number rented cars, at a given moment.	
PSC <sub>i</sub>	Its marking represents rented cars parked at the station $S_i$ but still under charging (or "short maintenance) operations and then not yet available for users.	
PSR <sub>i</sub>	Its marking represents cars parked at the station $S_i$ , ready and then available for users.	
PSC <sub>i</sub>	Its marking represents the number of free places in the station. It is used to limit the capacity of the station $S_{\rm i}$ .	
$PSD_i$	Its marking represents users (demands) waiting -during a given random delay- for available and ready cars in the station $S_i$ .	
PSM	Represents the maintenance service and its marking corresponds to cars waiting for "long" maintenance by the car maintenance service.	
PCP	Represents the "station Park" used by the maintenance service and users. Its marking represents cars ready and available for users.	

#### 4.2. A Petri Net model of an ECSS

The Figure 3 and the Tables (1) and (2) allow readers, familiar with Petri nets, to quickly gain an understanding

of the developed stochastic Petri Net model of the typical electric car sharing system represented in Figure 1. The model shows distinguishable parts representing different functions and operations according to the system description given in the previous section. The main function of each subnet is represented in a modular way by Figures 4 and 5. All components of the model are described in Tables 1 and 2.

**Table 2.** Interpretation of transitions of the model

Transitions	Meaning and events / Type	
$TUD_i$	User demand arrival at station S <sub>i</sub> (stochastic	
	transition)	
$TUN_i$	User demand "not satisfied" at station S <sub>i</sub> . The	
	user leaves the station without having been	
	served. (Deterministic transition)	
$TUS_i$	User demand "satisfied" and then "user	
	departure" from station S <sub>i</sub> . (immediate	
	transition)	
TCCi	Car return to station S <sub>i</sub> by user. Then, the car	
	is in a charging situation (eventually for	
	"short" maintenance) (stochastic transition)	
$TCR_i$	Car is ready and available for other users	
	after its charging operation (stochastic	
	transition)	
TCM	Car picked up for "long" maintenance (e.g.	
	due to an incident/ accident) by service	
	provider (stochastic transition).	
$TMP_R$	After maintenance, car is parked at the	
	station Park. It is ready and available for	
	users (stochastic transition)	
$TCP_C$	Car return to station Park by user. This car	
	will be in charging operation (stochastic	
	transition).	
$TCP_R$	The car is ready and available for other users,	
	after its charging operation (stochastic	
	transition).	
$TUD_P$	User demand arrival at station park	
-	(stochastic transition)	
$TUN_P$	User demand "not satisfied" at Park. The user	
	leaves the station without having been served	
	(stochastic transition)	
$TUS_P$	User demand "satisfied" and then "user	
	departure" from station park (immediate	
	transition)	

## 4.3. Performance evaluation and analysis

For the performance evaluation of any modelled system by using stochastic/timed Petri nets, analytical or simulation techniques are possible. The analytic approach is applicable if the associated stochastic process has a finite number of states and its stochastic nature allows for analytical resolution under some assumptions. For complex and / or large systems, including car sharing systems, discrete event simulation techniques may be required. The simulation process leads to generate State/Time trajectories representing the dynamic behaviour of the modelled system. Based on its formal enabling and firing rules, the simulation process consists in the execution of the Petri net, dynamically calculating the remaining enabling times, the new markings (states) and the desired performance indices. As illustrated in Figure 3, each cycle corresponds to the firing of one transition (event) and the simulation time is increased accordingly to the nature of transition fired

after its required execution delay and associated policy (restart or continue process). At the end of the simulation, after a fixed duration (Tsim), the state/ time trajectories are used to calculate different performance indices of the modelled system. Several performances can be formally expressed on the basis of the average marking of the places (e.g., average number of available cars in each station), the firing rates of the transitions (e.g., incident and maintenance rates), and the mean sojourn time of tokens in a given place (e.g., turnover / use of cars).

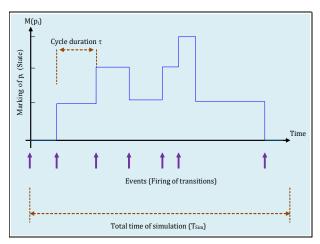


Figure 6. The State/Time Trajectory with Petri Nets.

Formally,

Average marking of a place p<sub>i</sub>:

$$\overline{M(P_i)} = \frac{1}{N_{Rep}} \sum_{k=1}^{N_{Rep}} \left( \frac{\sum (M(p_i)_k * \tau_k)}{T_{Sim}} \right)$$
(1)

- Effective firing rate of a transition t<sub>i</sub>:

$$\overline{F(t_j)} = \frac{1}{N_{Rep}} \sum_{k=1}^{N_{Rep}} \left( \frac{NF(t_j)_k}{T_{Sim}} \right)$$
 (2)

- Mean sojourn time of tokens in a place p<sub>i</sub>:

$$\overline{S(P_i)} = \frac{1}{N_{Rep}} \sum_{k=1}^{N_{Rep}} \left( \frac{\sum (M(p_i)_k * \tau_k)}{NT_k} \right)$$
(3)

Where, as illustrated in Figure 6:

- T<sub>sim</sub> represents the total simulation time
- N<sub>Rep</sub> represents the number of replications
- k is the indice of the k<sup>th</sup> replication
- $\tau_k$  is the duration of the cycle
- M(p<sub>i</sub>)<sub>k</sub> is the number of tokens at the beginning of the cycle
- NF( $t_i$ )<sub>k</sub> is the number of firings of a transition  $t_i$

 NT<sub>k</sub> is the number of tokens that have passed through this place until the current cycle.

# 5. Real-life application of the model

To demonstrate the applicability and relevance of the developed model and approach, a real-life application on the case of YELOMOBILE in "La Rochelle" (France) is developed. By using data from:

https://play.google.com/store/apps/details?id=com.yelom obile.larochelle&hl=en\_US (e.g. station capacities, number of cars ...) and google-map (e.g, distances between stations, localisation, ...), the Petri net model is experimented and validated for several scenarios and parameters (demands, capacities, accident rate, ...).

To demonstrate the capability of the model to reproduce and analyze the system, some results about the dynamic behavior of the model are given graphically in Figure 8, where for each station, the average sojourn time and/ the average number of available cars are estimated by using the equations (3) and (1).



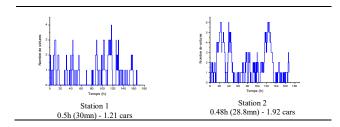
Figure 7. Yelomobile – La Rochelle (France) « Google map » https://play.google.com/store/apps/details?id=com.yelomobile.l arochelle&hl=en US

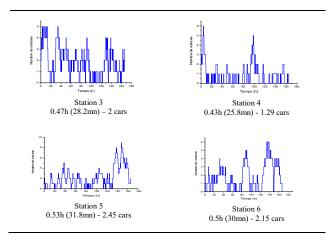
**Table 3.** The stations

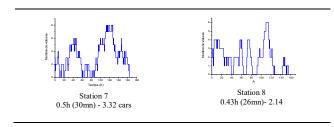
Station	Designation	Capacity
S1	Médiathèque	6
S2	Minimes	6
S3	Piscine	5
S4	Préfecture	6
S5	Gare	9
S6	Pôle Techno forum	6
S7	Place Saint Maurice	6
S8	Marché de Portneuf	6
S9	Arsenal	12
S10	Verdun	9
S11	Aytré	6
S12	Villeneuve-Les-Salines	6
S13	EIGSI	6

# 6. Conclusion

Based on the formal modelling and analysis power of stochastic Petri nets, in this paper an original discrete event model is developed for electric car sharing systems (ECSS) for performance evaluation, analysis and simulation purposes. The model takes into account their complex dynamic behaviour, organization and parameters including capacities of the stations, battery and energy availability, locations of charging stations and also their car maintenance activities, not negligible compared to the case of bike-sharing systems. To demonstrate the applicability and relevance of the developed model and approach, a real-life application on the case of YELOMOBILE in "La Rochelle" (France) is presented. More results according to different scenarios are presented during the oral presentation of this work.







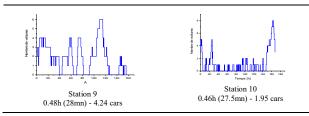


Figure 8. Discrete Event Simulation results

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