# Analysis of thermal states of a flat-solar collector using state variables

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**Abstract.** This paper analyses the thermal states of the flat solar collector using the state variable method. The knowledge of the transient states describing the state of the solar heating installation under operating conditions enables the analysis of the heat exchange process and the development of control strategies. The analysis of variable states under operating conditions requires the development of a mathematical model describing the dynamic properties of the whole solar heating installation and the definition of the variables describing the state of the system from the energy balance perspective. The paper presents a method enabling the analysis of variables in the condition of a solar heating installation and a single solar collector based on the comparison of two models: an analogue model developed by the Equivalent Thermal Network method and a digital model, developed on the basis of performance data by the Parametric Identification method.

## 1 Introduction

Designing a continuous control system requires knowledge of the dynamics of the controlled object. In case of thermal objects such as flat-plate solar collector, they are determined on the basis of step response determined during normative tests carried out in accordance with ISO 9806:2017 [1]. The standard only specifies the methodology for determining the equivalent time constant, omitting the gain factor and the delay time [2]. The dynamic properties of a collector may be determined using numerous methods based on the energy balance of the collector and enabling the analysis of its thermal conditions [3-6]. However, due to the variable working point and thermal load, the dynamic properties of the collector under operating conditions may differ from those determined during normative tests or those determined in simulation tests [7]. Therefore, it is important to determine the dynamic properties of a solar heating installation under operating conditions. Such possibilities are provided by the diagnostic method of the solar heating installations based on the comparison of two models: a digital model of differential equations developed on the basis of operating data using the method of parametric identification (PI) and an analogue model derived from the energy balance [2]. The method allows to determine the values of collector parameters under operating conditions treated as characteristic values, among

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others: heat loss coefficient $U_L$, heat removal factor $F_R$, heat capacity $(mc)_e$. A more detailed analysis of the heat transfer process in a flat-plate liquid collector is provided by the diagnostic method of solar heating installations based on a comparison of a digital model developed based of operating data using the method of parametric identification and an analogue model in the form of Equivalent Thermal Network (ETN).

## 2 PI and ETN based diagnostic method

In the case of thermal objects, the description of system dynamics obtained as a result of parametric identification may be related to individual components of the object using the thermoelectric analogy. A thermal object can be represented by means of a quadripole consisting of thermal resistances (resistor) and thermal capacitance (capacitor) connected in series to each component of the controlled object (flat-plate liquid collector), which form the electrical circuit of the object based on the so-called Beuken model. The thermal flux (electric current) flowing through individual elements of the electric circuit causes voltage (temperature) drops in individual nodes of the electric circuit. Having the transfer function of the object determining its dynamics under operating conditions and its equivalent description resulting from the laws of physics presented by means of an electric circuit, both models can be compared and behaviors of individual variables may be linked to the individual components of the object by means of the model of state variables.

The general form of the model of state variables is presented by two matrix equations: the equation of state (1) and the equation of output (2). To determine the state transition matrix, the values of all state variables must have a value of '0' except for one state variable, which must have a value of '1'. Then the Aij state matrix element is the response of the i-th state variable to the initial condition of the j-th state variable at zero initial values of all other state variables. Therefore, it is possible to analyze transient states of state variables caused by an abrupt change of the i-th variable.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) \tag{2}$$

where:
A - system matrix
B - input matrix
C - output matrix
D - transfer matrix

## 3 Flat-plate solar collector model using the ETN method

The ETN method has been used by many researchers to analyses the thermal states of a flat-plate solar collector. However, the most complete methodology for analyzing thermal conditions of a flat-plate solar collector using the ETN method has been presented by Chochowski [8]. Chochowski distinguished three homogeneous bodies in the construction of a flat solar collector: the glass cover, the absorber and the operating medium. He presented a solar collector using a thermal diagram in the form of an electric quadripole (Fig. 1). The heat flux (electric current) flowing through the branches of the circuit containing thermal resistances (resistors) causes a spatial distribution of temperature (voltage drops) corresponding to the heat exchange that takes place in the analyzed system.

The first node (Fig. 1) represents the glass cover with an average temperature T1, where, as a result of the absorption of part of the solar irradiance falling on the collector, energy with P1 power is generated halfway through the cover. The second node represents the absorber with an average temperature T2, on the surface of which thermal energy is generated with P2 power as a result of absorption of solar irradiance passing through the glass cover. The third node represents the operating medium which, by flowing through the solar collector, reaches the average temperature T3, determined from the following relationship:

$$T_3 = \frac{1}{2}\left(T_{fp} + T_{fk}\right) \tag{3}$$

Between the network nodes and the loss flow center (environment), thermal streams P flow through the surface thermal resistances R and drifting. The thermal resistances visible in the electrical diagram (Fig. 1) have the following physical meaning:

R1,0 - thermal resistance between the front cover and the environment [K/W]
R1,2 - thermal resistance between the front cover and the absorber [K/W]
R2,0 - thermal resistance between the absorber and the environment, calculated in the direction of the collector bottom [K/W]
R2,3 - thermal resistance between the absorber and the working medium [K/W]
(mc)1 - thermal capacity of the node 1 [J/K]
(mc)2 - thermal capacity of the node 2 [J/K]
(mc)3 - thermal capacity of the node 3 [J/K]

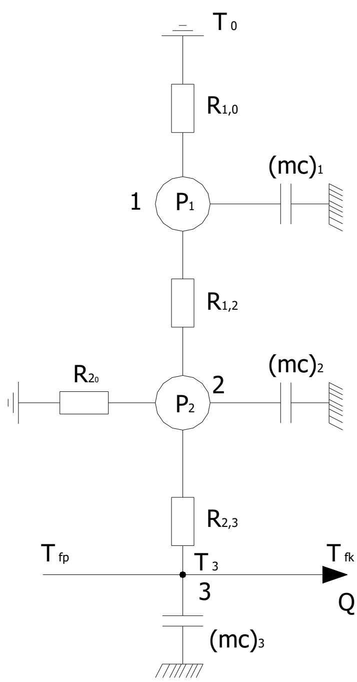


**Fig. 1.** Thermal diagram of the collector for transient state.

where:
$T_{fp}$ - inlet temperature of medium [K]
$T_{fk}$ - outlet temperature of medium [K]

The thermal diagram of a flat-plate solar collector presented in Figure 1 was considered as the basic one, allowing for a general analysis of phenomena occurring during heat exchange between three bodies (including two energetically active bodies) and the environment.

The solution to ETN using the nodal analysis is a system of linear equations, combining equations of all nodes and branches of the medium. The general form of the equation describing the transient state of a solar collector is presented using equations of nodes containing thermal capacities (4).

$$(mc)_i \frac{dT_i}{dt} + T_i \sum_{j=1}^{n+1} \Lambda_{ij} - \sum_{j=1}^{n+1} \Lambda_{ij} T_j = P_i + \Lambda_{i0} T_0 \tag{4}$$

The solution of the above equation means solving the system of linear equations containing equations of all common nodes and equations of all branches of the medium, which takes the matrix form of (5). Matrix c (6) is a diagonal matrix of thermal capacities of particular components and matrix Λ (7) is a matrix of thermal conductivities of particular network nodes.

$$c \frac{d\bar{T}(t)}{dt} + \Lambda \bar{T}(t) = \bar{P}(t) \tag{5}$$

$$c = \begin{bmatrix} (mc)_1 & 0 & 0 \\ 0 & (mc)_2 & 0 \\ 0 & 0 & (mc)_3 \end{bmatrix} \tag{6}$$

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix} \tag{7}$$

Multiplying both sides of the equation 5 by the inverse matrix c-1 results in a matrix of the solar segment state expressed as the relation (8).

$$\frac{d\bar{T}(t)}{dt} = -\left(c^{-1} * \Lambda\right) * \bar{T} + c^{-1} * \bar{P} \tag{8}$$

By substituting matrices 6 and 7 to equation 8, we obtain the matrix notation (9), which can be transformed into the form of 10 by performing arithmetic operations.

$$\frac{d}{dt}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} mc_1 & 0 & 0 \\ 0 & mc_2 & 0 \\ 0 & 0 & mc_3 \end{bmatrix}^{-1} * \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix} * \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} mc_1 & 0 & 0 \\ 0 & mc_2 & 0 \\ 0 & 0 & mc_3 \end{bmatrix}^{-1} * \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \tag{9}$$

$$\frac{d}{dt}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{mc_1} & 0 & 0 \\ 0 & \frac{1}{mc_2} & 0 \\ 0 & 0 & \frac{1}{mc_3} \end{bmatrix} * \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix} * \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{mc_1} & 0 & 0 \\ 0 & \frac{1}{mc_2} & 0 \\ 0 & 0 & \frac{1}{mc_3} \end{bmatrix} * \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \tag{10}$$

The solution to the matrix of state variables is therefore a matrix of the system and a matrix of the input expressed by equation 11.

$$\frac{d}{dt}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\Lambda_{11}}{mc_1} & \frac{\Lambda_{12}}{mc_1} & \frac{\Lambda_{13}}{mc_1} \\ \frac{\Lambda_{21}}{mc_2} & \frac{\Lambda_{22}}{mc_2} & \frac{\Lambda_{23}}{mc_2} \\ \frac{\Lambda_{31}}{mc_3} & \frac{\Lambda_{32}}{mc_3} & \frac{\Lambda_{33}}{mc_3} \end{bmatrix}}_{\text{matrix of the system}} * \begin{bmatrix} T_1(t) \\ T_2(t) \\ T_3(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{P_1}{mc_1} \\ \frac{P_2}{mc_2} \\ \frac{P_3}{mc_3} \end{bmatrix}}_{\text{matrix of the input}} * u(t) \tag{11}$$

The state matrix expresses the ratio of thermal conductivities of individual network nodes to their thermal capacity. Temperatures of individual homogeneous bodies of a solar collector are the state variables, while the ratio of the thermal power emitted in the node to its thermal capacity is the column matrix of the input.

## 4 Parametric model of a flat-plate solar collector

The parametric identification method allows to create a model depending on the number of inputs and outputs of the model in the form of an equation or a system of differential equations. The parametric model was developed on the basis of operating data recorded during the operation of a solar heating installation consisting of 5 flat-plate liquid collectors, thermally loaded by a plate heat exchanger with a 500 $dm^3$ buffer tank. Propylene glycol was used as the medium in the primary circuit of the exchanger, while water was used in the secondary circuit. To measure the solar radiation intensity (solar irradiance) a class 1 pyranometer was used, while temperature was measured using class A PT1000 sensors. The system operation control algorithm was implemented in the PLC, which was connected by means of the modus protocol to the computer with the SCADA program installed. The parametric model of the system was developed based on the methodology developed by Obstawski [11].

**Fig. 2.** Diagnosed solar water heating system.

During the identification process, the solar collector was treated as a two-input and one-output object. The input signals were assumed to be the daily course of solar irradiance and the input temperature of the medium. The output signal was the output temperature of the working medium. Therefore, the dynamics of thermal transitions of the solar segment are reflected by two components in the form of functions G1(s) and G2(s). Function G1(s) enables analysis of the dynamics of thermal transitions caused by changes in irradiance, while function G2(s) presents the influence of changes in the input temperature of the medium caused by hot water decomposition on the output temperature of the medium. The course of the output temperature signal y(t) consists of two components whose amplitude and time course depend on the input function and the collector components' temperatures achieved in the moment t (12).

$$y(t) = y1(t) + y2(t) \quad (12)$$

where:
y1(t) - component of output temperature dependent on changes in solar irradiance and temperature level of components of the solar collector
y1(t) - component of output temperature dependent on variations in the medium input temperature and temperature level of components of the solar collector

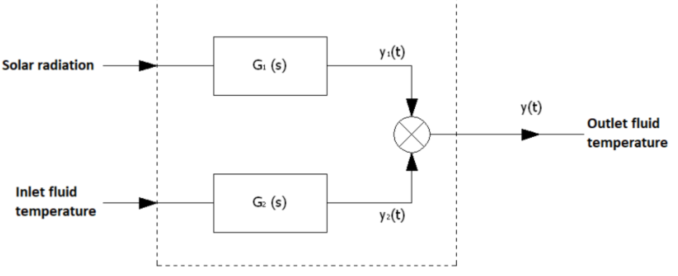


**Fig. 3.** Flat-plate collector battery model.

The model was created based on operating data presented in Figure 4.

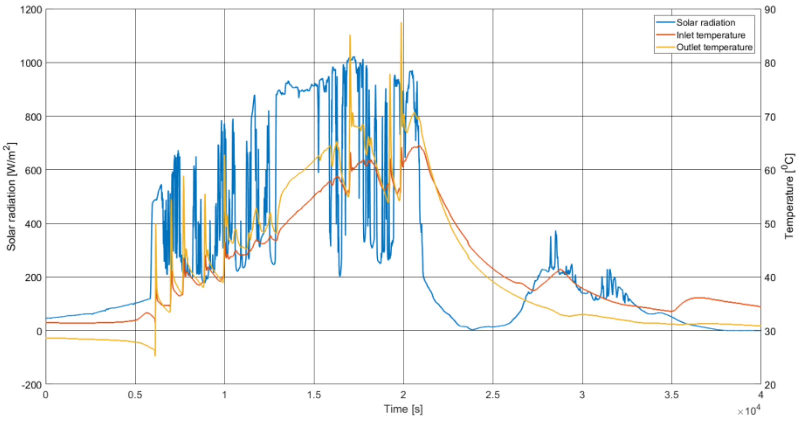


**Fig. 4.** Daily distribution of solar irradiance and temperatures on July 23, 2004.

The parametric model of collector battery having the form of a system of differential equations has been transformed into discrete transmittance form, and then to the form of continuous transmittance using the Tustin method. The main path of the model was described with the use of transfer function G1 (13), while the main path of the model was described with the use of transfer function G2(s) (14). It should be noted that the transfer (transmittance) function describing individual paths of the model has the same specific equation.

$$G_1(s)=\frac{1{,}01*10^{-6}s^2+1{,}55*10^{-6}s+2{,}2*10^{-8}}{s^3+2{,}78*10^{-3}+2{,}55*10^{-4}s*8{,}81*10^{-7}} \tag{13}$$

$$G_2(s)=\frac{1{,}14*10^{-2}s^2+1{,}56*10^{-4}s+6{,}24*10^{-7}}{s^3+2{,}78*10^{-3}+2{,}55*10^{-4}s*8{,}81*10^{-7}} \tag{14}$$

Having the specific equation of transfer functions and applying the direct decomposition, the equation of the state of the flat-plate collector segment can be notated using the dependence 15.

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\-8{,}81*10^{-7} & -2{,}55*10^{-4} & 2{,}78*10^{-3}\end{bmatrix}*\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}*u(t) \tag{15}$$

The outputs of the individual components can be related to state variables and input signals by means of the output equation, which is presented as dependence 16.

$$y(t)=\begin{bmatrix}y_1(t)\\y_2(t)\end{bmatrix}=\begin{bmatrix}2{,}2*10^{-8} & 1{,}55*10^{-6} & 1{,}01*10^{-6}\\6{,}24*10^{-7} & 1{,}56*10^{-4} & 1{,}44*10^{-2}\end{bmatrix}*\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\end{bmatrix}+\begin{bmatrix}0 & 0\end{bmatrix}*u(t) \tag{16}$$

In order to determine the courses of the state variables for the given initial conditions, a state transition matrix must be determined. The function state transition matrix is calculated by converting the state matrix A according to the dependence 17.

$$[sI-A]-1 \tag{17}$$

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where: sI is a diagonal matrix of the same dimensions as the system matrix.

To determine the state transition matrix, all initial conditions of state variables must be set to zero, except for the initial conditions of one variable. Then the response of each state variable to the given initial condition of that variable is determined. In the analyzed case, the state transition matrix is expressed as equation 18.

$$[sI - A]^{-1} = \left[ \frac{\begin{matrix} s^2 + 2,78*10^{-2} + 2.55*10^{-4} & -8,81*10^{-7} & -8,81*10^{-7}s \\ s + 2,78*10^{-2} & s^2 + 2,78*10^{-2}s & -2,55*10^{-4}s - 8,8*10^{-7} \\ 1 & s & s^2 \end{matrix}}{s^3 + 2,78*10^{-2}s^2 + 2,55*10^{-4}s + 8,81*10^{-7}} \right] \quad (18)$$

Comparing the system matrix (11) reflecting the physics of heat exchange of the solar collector and the state transition matrix (18) developed on the basis of operational measurements, it is possible to assign by analogy individual expressions to the state variables (temperatures of: glass cover, absorber, operating medium) and to determine their course for the given initial conditions. To do this, the original of the state transition matrix must be determined using the reverse Laplace transform. Since the individual transforms of the function state transition matrix (18) are in a non-reducible form, they can be represented by simple fractions according to the dependence (19).

$$Y(s) = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2} + ....... + \frac{r_n}{s - p_n} \quad (19)$$

where
$r_n$ - transfer function zero
$p_n$ - transfer function pole

Then the original transform is determined by a general formula, which can be easily arrayed for a specific time interval (19).

$$y(t) = \sum_{i=1}^{n} r(i) * \exp(t * p(i)) \quad (19)$$

where:
t - time vector

In the presented analysis, the courses of phase trajectories of variables determined for excitation of the system by change of the state of the T1 variable from "0" to "1" will be discussed. The time courses of the status variables to the given initial conditions can be presented in the form of a matrix notation (20).

$$\begin{bmatrix} T_1(t) \\ T_2(t) \\ T_3(t) \end{bmatrix} = \Phi(t) * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} L^{-1}\{G_{11}(s)\} \\ L^{-1}\{G_{21}(s)\} \\ L^{-1}\{G_{31}(s)\} \end{bmatrix} \quad (20)$$

Figure 5 shows the course of the phase trajectory of the glass cover temperature to the step change of the T1 variable state. The course of the trajectory shows that following the abrupt input function, the temperature of the glass cover reaches the steady state after 750

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seconds and its value drops to zero. A change in the state of the variable T1 significantly affects the state variable T2, which is the absorber temperature. When unbalanced from the steady state, the temperature of the absorber tends in the opposite direction to that intended, to reach the steady state after 820 seconds (Figure 6). The trajectory of the variable T3, which is the temperature of the working medium, is very interesting. As a result of excitation of the system by changing the state of the variable T1, in the first phase the temperature of the working medium drops in the opposite direction to the intended one and then rises to reach the steady state after the maximum time of 950 seconds (Fig. 7). The T3 state variable trajectory resembles the response of a non minimum phase object to a step input function.

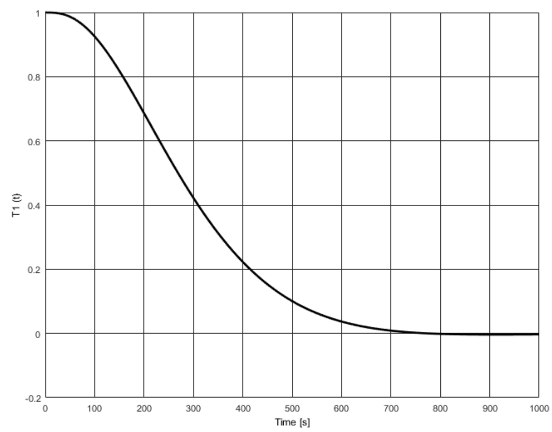


**Fig. 5.** Trajectory of phase variable T1 - glass cover temperature.

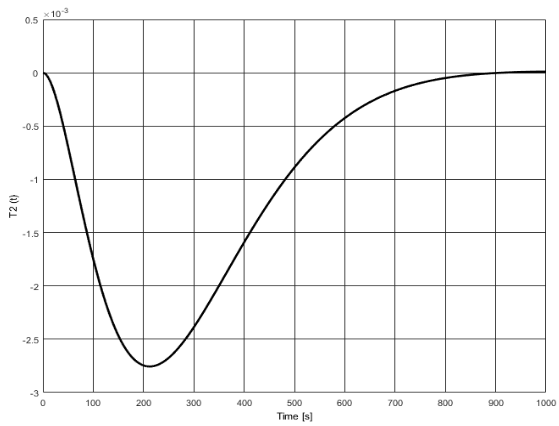


**Fig. 6.** Trajectory of phase variable T2 - absorber temperature.

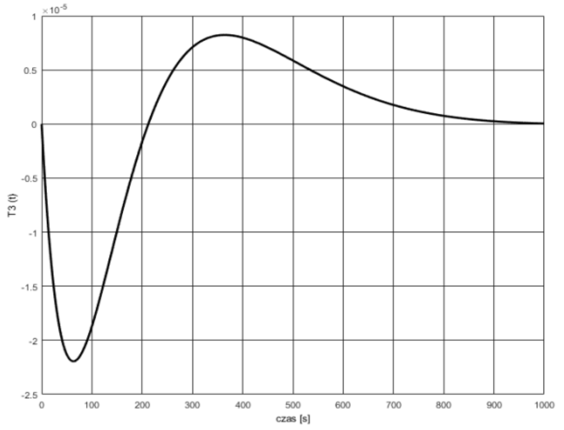


**Fig. 7.** Trajectory of phase variable T3.

With the trajectories of individual state variables it is possible to draw the vector of the system state on a phase plane (Fig. 8). Therefore, it is possible to analyze the influence of a given variable (in the analyzed case, the temperature of the glass cover of the collector) on the other variables.

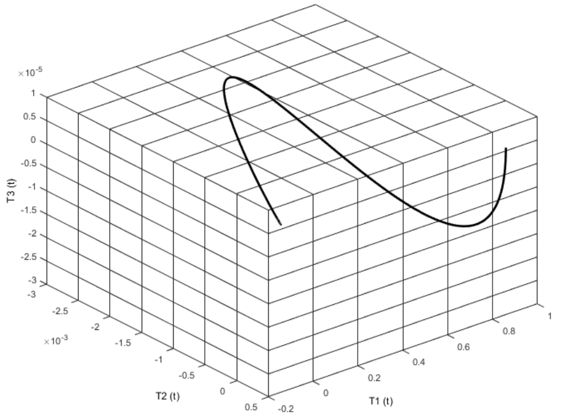


**Fig. 8.** The course of the trajectory of the state vector [T1(t), T2(t), T3(t)] on the phase plane.

## 5 Conclusions

The presented diagnostic method based on the comparison of two types of models: analog model developed on the basis of electrical analogy and digital model developed on the basis of parametric identification method, enables a detailed analysis of the heat exchange process between the main structural elements of a flat-plate solar collector under operating conditions, which is an advantage. The analysis of trajectories of phase state variables enables the assessment of the collector structure in terms of the dynamics of heat exchange between the main components under operating conditions and enables the design of a continuous control system. Having short-term time series of solar irradiance intensity and operating medium temperatures, it is possible to assess the influence of the system operating parameters determining the working point on the heat exchange process in a flat-plate solar collector.

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## References

1. EN ISO 9806:2017 - Solar energy -- Solar thermal collectors -- Test methods
2. P. Obstawski, T. Bakoń, D. Czekalski, Appl. Therm. Eng. **148,** 420-429 (2019)
3. W. Kon, Z. Wang, X. Li, X. Li, N. Xiao, J. Sol. Energy **86**, 398-406 (2012)
4. N. Amrizal, D. Chemisana, J.I. Rosell, J. Barrau, Appl. Therm. Eng. **94**, 244-250 (2012)
5. J. Buzás, R. Kicsiny, Renew. Energy **68**,146-155 (2014)
6. J. Deng, X. Yang, P. Wang, J. Sol. Energy **114**, 418–426 (2015)
7. P. Obstawski, *Modelling the dynamics of work of a flat-plate solar collector* (PhD Thesis, Warsaw University of Life Sciences, Warsaw, 2007)
8. A. Chochowski, *Thermal States Analysis of a Flat-plate Solar Collector* (SGGW Publishers, Warsaw, 1991)