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Conserving heat consumption by modeling and optimizing efficiency of complex heat exchanger systems

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Abstract. Many needs have to be met in human life. One of the key needs is to provide living comfort, which is clearly associated with heat. Analyzing the amount of produced heat, it should be emphasized that the higher the development of a country, the greater the demand for heat. In the era of the debate on the impact of human activities on the climate, it is impossible not to emphasize the importance of conserving energy and heat, and thus the rational management of these goods. The paper proposes a mathematical description of the complex systems of heat exchangers in the form of linear differential equations. Their analytical solution is presented in the form of temperature change of the heat carrier along the heating surface of a four-thread heat exchanger. Analysis of the possible heat exchange cases for eight possible flow systems is presented. In addition, the most effective minimization of heat losses was found.

1 Introduction

The implementation of the linear model of the economy in the world for decades has caused a significant reduction in natural resources, as well as a huge demand for energy. The processes of manufacturing goods were based on raw materials newly obtained from the natural environment, and to a negligible extent on raw materials from recycling. The same was true for the energy demand. New energy generation units were being created, while the amount of energy recovered in the whole structure of energy demand accounted for a small percentage. This linear production concept is associated with enormous environmental costs. The transition to the implementation of a closed-circuit economy eliminates these burdens to a significant extent.

In general, there are heat exchangers in the processes of energy consumption between the energy generation system and its direct use. They are devices whose efficiency is high, nevertheless, not 100%. A slight improvement in the efficiency value of these devices,

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taking into account the amount of consumed energy, would result in huge amounts of conserved energy. What is more, it is well known that nothing is as profitable, both economically and ecologically as conserved energy. An increase in the efficiency of heat exchangers is also the easiest way to “obtain” the cheapest fuel.

2 Heat exchangers in the power industry. Finding conserving in energy consumption

Heat exchangers are widely used in human life – among others in construction, refrigeration, heating, air conditioning, cars, computers or central heating systems for residential buildings. They are generally used where there is heat exchange between the generation system and the reception system. They have the form of large-sized devices, but also small computer exchangers. Another exchanger for many applications in industry and heating systems is the shell and tube exchanger. It is built of a shell in which a tube bundle is placed, where the heat transfer medium flows in the coil, whereas the medium that receives the heat is in the shell.

Energy processes occurring during energy generation or processing are characterized by relatively low efficiency. A significant part of the chemical energy of the fuel is expelled in the form of waste energy, and one of the methods of minimizing energy consumption is to optimize heat recovery. Recycling at least a small part of this lost energy and reusing it for the needs of any other process or technology is another simple way of “obtaining” the cheapest fuel [1]. The recovered heat can be used, for example, for heating a building, providing technological heat for ventilation units, heating hot water and all other technological processes.

3 Mathematical model of a multi-element heat exchanger with any flow structure of medium

Each surface heat exchanger is a four-pole object with two incoming and two outgoing elements (Figure 1).

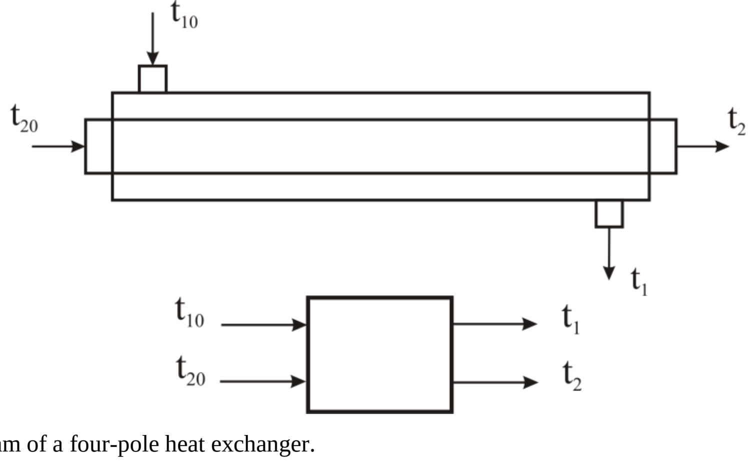

Fig. 1. Diagram of a four-pole heat exchanger.

The heat flux $d\dot{q}$ transmitted by the warmer medium to the colder medium can be expressed by means of the Peclet equation [2, 3, 4, 5]

$$d\dot{q} = k\, dF(t_1 - t_2) \quad (1)$$

where: $d\dot{q}$ – elementary heat flux, W/m^2; k – heat transfer coefficient, W/m^{2}K; dF – elementary surface of heat exchange, m^2; t_1 – warmer medium temperature, K; t_2 – cooler medium temperature, K.

Whereas the energy balance of mediums, corresponding to the elementary surface of the exchanger, takes the form [2, 3, 4, 5]

$$c_1\dot{m}_1 t_1 = d\dot{q} + c_1\dot{m}_1\left(t_1 + dt_1\right) \tag{2}$$

$$c_2\dot{m}_2 t_2 = -d\dot{q} + c_2\dot{m}_2\left(t_2 + dt_2\right) \tag{3}$$

where: c – specific heat, J/(kgK); $\dot{m}$ – mass flux, kg/s.

After the equations (2) and (3) are transformed, equations of temperature changes of warmer and colder factors along the surface of the exchanger are obtained

$$\frac{dt_1}{dF} = -a_1\left(t_1 - t_2\right) \tag{4}$$

$$\frac{dt_2}{dF} = a_2\left(t_1 - t_2\right) \tag{5}$$

where coefficients a_1 and a_2 are expressed as follows

$$a_1 = \frac{k}{c_1\dot{m}_1} \tag{6}$$

$$a_2 = \frac{k}{c_2\dot{m}_2} \tag{7}$$

Equations (4) and (5) form a system of differential equations. Their solution is the following relationships [6]

$$t_1 = C_1 + C_2 \mathrm{e}^{-\left(a_1 + a_2\right)F} \tag{8}$$

$$t_2 = C_1 + C_2 \frac{a_2}{a_1} \mathrm{e}^{-\left(a_1 + a_2\right)F} \tag{9}$$

After considering the boundary conditions

$$F = 0; \quad t_1 = t_{10}; \quad t_2 = t_{20} \tag{10}$$

the values of C_1 and C_2 constants are obtained

$$C_1 = \frac{t_{20} + t_{10}\dfrac{a_2}{a_1}}{1 + \dfrac{a_2}{a_1}} \tag{11}$$

$$C_2 = \frac{t_{10} - t_{20}}{1 + \dfrac{a_2}{a_1}} \tag{12}$$

By substituting constant values C_1 and C_2 for dependences on the medium temperatures t_1 and t_2, the following system of linear equations is obtained

$$\left[\frac{a_2}{a_1} + \mathrm{e}^{-\left(a_1 + a_2\right)x}\right]t_{10} + \left[1 - \mathrm{e}^{-\left(a_1 + a_2\right)x}\right]t_{20} - \left(1 + \frac{a_2}{a_1}\right)t_1 = 0 \tag{13}$$

$$\frac{a_2}{a_1}\left[1 - \mathrm{e}^{-\left(a_1 + a_2\right)x}\right]t_{10} + \left[1 + \frac{a_2}{a_1}\mathrm{e}^{-\left(a_1 + a_2\right)x}\right]t_{20} - \left(1 + \frac{a_2}{a_1}\right)t_2 = 0 \tag{14}$$

The above system of equations can be written in matrix form [7, 8, 9,10]

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} \begin{bmatrix} t_{10} \\ t_{20} \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15}$$

or

$$BT = 0 \tag{16}$$

where: **B** – coefficient matrix, **T** – temperature matrix, **0** – zero column matrix.

In order to generalize the described heat exchange model for the case of the exchanger, which is component of the exchanger system (or constitutes one of the stages of the multistage exchanger), the system of equations (15) should be extended with additional equations

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & 0 & b_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t_{10} \\ t_{20} \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{t}_{10} \\ \tilde{t}_{20} \end{bmatrix} \tag{17}$$

where: $\tilde{t}_{10}$, $\tilde{t}_{20}$ – values of temperatures of warmer and cooler medium at the entry to the system of exchangers (or multistage heat exchanger), K.

In order to further generalize the described model, taking into account any configuration of the flux sequence of mediums in the multistage exchanger system, the following assumptions were made:

- two external mediums with known temperatures are given to the system,
- the system also leaves two external mediums.

For each exchanger constituting an element of the system, a coefficient matrix can write [7, 8, 9, 10]

$$\mathbf{B_i} = \begin{bmatrix} b_{11}^i & b_{12}^i & b_{13}^i & 0 \\ b_{21}^i & b_{22}^i & 0 & b_{24}^i \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{18}$$

where index *i* denotes the number of the exchanger in the system.

The medium exiting the given exchanger can flow into any other exchanger of the system. In order to determine the flow direction of the factors: warmer G, cooler C and a mixture of both mediums GC, the following matrixes are introduced:

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \tag{20}$$

$$GC = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \tag{21}$$

In matrixes (19)-(21) the minus denotes that the medium exits the considered exchanger. The four temperature values of the *i*-th exchanger form a single-column matrix of 4x1 in dimensions

$$\mathbf{T_i} = \begin{bmatrix} t_{10}^i \\ t_{20}^i \\ t_1^i \\ t_2^i \end{bmatrix} \tag{22}$$

Finally, a system of linear equations is obtained, which after appropriate transformations can be presented in the matrix form

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & \dots & A_{1n} \\ A_{21} & A_{22} & A_{23} & \dots & A_{2n} \\ A_{31} & A_{32} & A_{33} & \dots & A_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ A_{n1} & A_{n2} & A_{n3} & \dots & A_{nn} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \dots \\ T_n \end{bmatrix} = \begin{bmatrix} T_{10} \\ T_{20} \\ T_{30} \\ \dots \\ T_{n0} \end{bmatrix} \tag{23}$$

or

$$AT = T_0 \tag{24}$$

where A - block matrix defining the structure of the exchanger system.

4 Calculation results

The matrix of temperatures of working media T and the matrix of coefficients A for the variant of the system from Fig. 2a are presented in the form:

$$T = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix}, A = \begin{bmatrix} -a_1 & a_1 & 0 & 0 \\ a_2 & -(a_2 + a_3) & a_3 & 0 \\ 0 & a_4 & -(a_4 + a_5) & a_5 \\ 0 & 0 & a_6 & -a_6 \end{bmatrix} \tag{25}$$

where: $a_1 = \frac{k_{12}}{c_1 \dot{m}_1}$, $a_2 = \frac{k_{12}}{c_2 \dot{m}_2}$, $a_3 = \frac{k_{23}}{c_2 \dot{m}_2}$, $a_4 = \frac{k_{23}}{c_3 \dot{m}_3}$, $a_5 = \frac{k_{34}}{c_3 \dot{m}_3}$, $a_6 = \frac{k_{34}}{c_4 \dot{m}_4}$, F – heat exchange surface; c – specific heat; k – heat transfer coefficient; $\dot{m}$ – medium flow rate; the single index indicates the number of the medium, the double index of the heat transfer coefficient corresponds to the numbers of the two media between which heat exchange occurs.

The general solution of the system of homogeneous first order differential equations (25) is in the form [3]:

$$T = \sum_{j=1}^{4} c_j \cdot \alpha_j \cdot e^{\lambda_j \cdot F} \tag{26}$$

where: λ – eigen values, α – matrix A eigenvectors, c – constant of integration.
For the given initial conditions $F_0 = 0$, $t_{10} = 100$, $t_{20} = t_{30} = t_{40} = 0$ i $a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 1$ a specific solution of system (1) was obtained for the diagram shown in Fig. 2, the

results of which are presented in the form of temperature change dependencies along the heating surface for four heat carriers.

For each of the eight possible cases of media flow, shown in Fig. 1, the systems of linear differential equations were formulated by analogy. The results of the solutions for each of the eight cases are also shown in Fig. 2 in the form of graphs of media temperature changes along the heating surface.

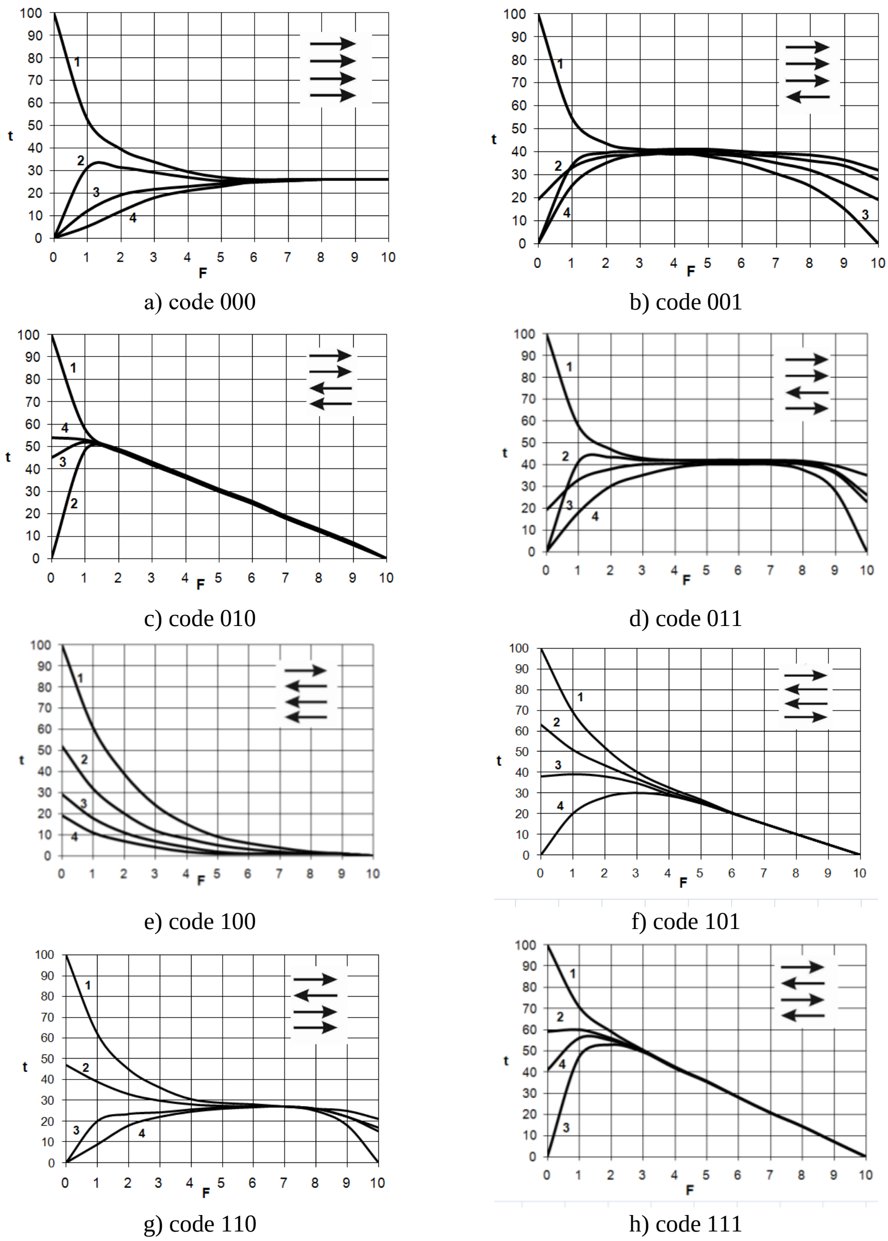

Fig. 2. Graphs of temperature change of four mediums along the heating surface for eight flow variants described by arrows in the graph field. The number in the graph field indicates the number of the medium; the digit of the binary code of the flow structure corresponds to: 0 - co-current flow of two neighboring mediums; 1 - countercurrent flow of two neighboring mediums

5 Conclusions

The heat exchange occurring between two media can be assessed by different efficiency criteria. In the case of a warmer medium, the efficiency of heat exchange can be identified by the value of its final temperature. The lower its value, the more heat has passed to the colder medium, and thus the greater efficiency of the exchanger. At the same time, it means conserving the heat consumption and "acquiring" the cheapest fuel.
The conducted computational analysis showed that the most effective media flow system corresponds to code 100, which makes it possible to ensure a minimum temperature value of the hot medium at the outlet of the device with a minimum value of heat exchange surface.

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