Study on determination method of hydrodynamic coefficient of Marine pile foundation

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Abstract. The irregular waves are simulated by using standard spectrum. Instantaneous value method, Fourier analysis method, least square method and "harbour hydrological code" are used to determine the moment force of coefficient C_M and drag coefficient C_D. Then C_M and C_D that linearized by Borgman L.E. equation are substituted into Morison equation. The time history curve of the wave force on the pile is calculated and compared with the measured wave force data under the action of irregular wave to analyze the advantages and disadvantages of several methods to determine C_M and C_D. The results show that the comparison between C_M and C_D determined by Fourier analysis and least square method is practical.

1 Introduction

Pile foundation structure is a structure form suitable for offshore waters in offshore wind power foundation [1]. In the calculation of stability of pile foundation, the calculation of wave force is crucial. At present, Morison equation is generally used to calculate the wave force of small diameter pile, and the key is to reasonably determine the moment force coefficient C_M and the drag force coefficient C_D [2].

Sarpkaya[3] found through experiments that C_M and C_D depended on Re number and KC number, and gave the rule that C_M and C_D changed with KC number.

Yu yuxiu [4] studied the wave forces acting on the isolated pile by regular waves and irregular waves, and summarized the changing relation curves of C_M, C_D and KC Numbers.

At present, there are three main ways to determine C_M and C_D. One is to use instantaneous value to calculate the number C_M and C_D, the other is to use Fourier analysis method, the third is to use the least square method to calculate. In this paper, the advantages and disadvantages of these three calculation methods were analyzed by comparing the measured wave force data of pile body under the action of irregular waves with the C_M and C_D stipulated in the harbour hydrological code [5].

2 Determination of moment force coefficient and drag force coefficient

2.1 Instantaneous values of C_M and C_D

According to the Airy wave theory, when $\cos\omega t=0$, velocity u=0 and acceleration a has an extreme value. When $\sin\omega t=0$, acceleration $a=0$ and velocity u has extreme value. The instantaneous wave force data can be used to calculate the value of C_M and C_D.

This method is simple to calculate, but it is necessary to ensure that one of the two values of velocity and acceleration of the instantaneous data is the extreme value and the other is 0. Due to the nonlinear effect of waves, this condition may not be true. In addition, some irregularities of the observed data should be taken into account. In order to reduce the error caused by this irregularity, sometimes multiple values are taken for a small segment of the instantaneous vicinity and averaged after analysis.

2.2 Fourier analysis calculates C_M and C_D

In stable flow, experiment is proved that C_M and C_D are functions of Reynolds number (Re), while Reynolds number changes periodically in wave motion. Keulegan and Carpenter of the United States found that the coefficients C_M and C_D were related to the ratio of drag force and moment force, and defined the KC number as

$$KC = \frac{U_{\max} T}{D} \tag{1}$$

Where, $U_{\max}$ is the maximum velocity of wave water point movement, T is the wave period.

At present, it is believed that the coefficients C_M and C_D are related to Re and KC in the fluctuating water flow.

Keulegan and Carpenter proposed that since the velocity and acceleration in the wave current all change periodically with time, C_M and C_D should also be considered uniformly in a period. In other words, the mean value can be taken by Fourier analysis method in a period, which is called the mean value of Fourier analysis. They believe that the wave force should be an odd function, that is, a different sign within a half period:

$$F(\theta) = -F(\theta+\pi) \tag{2}$$

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$$\frac{2F}{\rho D{U_m}^2} = 2(A_1 \sin\theta + A_3 \sin 3\theta + A_5 \sin 5\theta + \cdots$$
$$+B_1 \cos\theta + B_3 \cos 3\theta + B_5 \cos 5\theta + \cdots) \quad (3)$$

The expression corresponding to Morrison equation can be written as follows:

$$\frac{2F}{\rho D{U_m}^2} = \frac{\pi^2}{KC} C_M \sin\theta + 2\left(A_3 \sin 3\theta + A_5 \sin 5\theta + \cdots\right)$$
$$-C_D \left|\cos\theta\right| \cos\theta + 2\left(B_3 \cos 3\theta + B_5 \cos 5\theta + \cdots\right) \quad (4)$$

$$U = -U_m \cos\theta \quad (5)$$

Take the same terms as Morrison equation, get:

$$\frac{2F}{\rho D{U_m}^2} = \frac{\pi^2}{KC} C_M \sin\theta - C_D \left|\cos\theta\right| \cos\theta \quad (6)$$

It should be noted that C_M and C_D are considered to be independent of θ, and they are a constant when the Re Numbers and KC Numbers are known, and when n>3, the terms A_n and B_n are all 0.

Multiply both sides of equation (6) by cosθ and integrate θ in the range of (0, 2π)

$$C_D = -\frac{3}{4}\int_0^{2\pi} \frac{F\cos\theta}{\rho D{U_m}^2} d\theta \quad (7)$$

Multiply both sides of equation (6) by sinθ and integrate θ in the range of (0, 2π)

$$C_M = \frac{2U_m T}{\pi^3 D}\int_0^{2\pi} \frac{F\sin\theta}{\rho D{U_m}^2} d\theta \quad (8)$$

From the above two equations, the mean values of C_M and C_D obtained by Fourier analysis can be obtained.

2.3 The least square method calculates C_M and C_D

Set F_m as instantaneous measured value, F_c as the calculated value of action, E as its error, so the total error is

$$E^2 = \left(F_m - F_c\right)^2 \quad (9)$$

According to the principle of least square method, the selected C_M and C_D values should meet the following conditions:

$$\begin{cases} \dfrac{dE^2}{dC_D} = 0 \\ \dfrac{dE^2}{dC_M} = 0 \end{cases} \quad (10)$$

2.4 Recommended values for C_M and C_D for national specifications

Table 1 shows the recommended values of C_M and C_D for national norms [7].

Table 1. National specification

	API (USA)	DNV (Norway)	DTI (USA)	Harbour hydrological code(Chian)
C_D	0.6~1.0	0.5~1.2	Use the reliable test results	1.2
C_M	1.5~2.1	2.0		2.0

As can be seen from table 1, the definitions of C_M and C_D vary from country to country. The API defines different C_M and C_D values for the roughness or smoothness of rods. There is no domestic regulation on the smoothness of cylinders, and the values are relatively conservative.

3 Calculation of wave elements

3.1 Instantaneous values of C_M and C_D

3.1.1 The wave spectrum

Random wave is closely related to the selection of frequency, and frequency is closely related to wave characteristics such as wave height and wave frequency. Practical marine spectral materials are drawn according to a large number of measured statistics. The corresponding random wave loads are fitted to reflect the wave conditions in a similar real environment. In order to reflect the actual wave force of the offshore fan, the wave spectrum of China is adopted here [8]. Its form is:

$$S(\omega) = \frac{0.74}{\omega^5} \exp\left(-\frac{2.46}{\omega^2 H_S}\right) \quad (11)$$

Where: ω is circular frequency. H_S is the effective wave height of random wave.

3.1.2. Wave equation

Considering the wave as a stationary random process, it is assumed that the wave is composed of several linear waves with different frequencies, amplitudes, wave Numbers and initial random phases. The form of any constituent wave is as follows:

$$\eta_i(t) = A_i \cos\left(k_i x - \omega_i t + \varepsilon_i\right) \quad (12)$$

Where, $\eta(t)$ is the wave equation. A_i is the amplitude of any constituent wave. k_i is the wave number of any constituent wave. ω_i is the circular frequency of any constituent wave. ε_i is the initial random phase of any constituent wave, and its value is averaged at [0, 2π].

In the equation (12), there is a corresponding relation between wave number and circular frequency, that is, the dispersion relation. According to the free surface condition, they are mutually constrained, and their relationship is as follows:

$$\omega^2 = gk \tanh kd \quad (13)$$

Where d is water depth. In the case of infinite water depth, d goes to infinity, and the dispersion relation is:

$$\omega^2 = gk \quad (14)$$

According to Airy wave theory, using the harmonic superposition method, the wave surface time history equation of the random wave is as follows:

$$\eta(t) = \sum_{i=1}^{M} a_i \cos\left(\omega_i t + \varepsilon_i\right) \quad (15)$$

$$\omega_n = \frac{\omega_{i+1} + \omega_i}{2} \quad (16)$$

$$\Delta\omega = \omega_{i+1} - \omega_i \tag{17}$$

$$a_i = \sqrt{2S(\omega_n)\Delta\omega} \tag{18}$$

Velocity time history equation and acceleration time history equation at any depth *d*:

$$u(t) = \sum_{i=1}^{M} a_i \omega_i \frac{\cosh(k_i z)}{\sinh(k_i d)} \cos(\omega_i t + \varepsilon_i) \tag{19}$$

$$a(t) = \sum_{i=1}^{M} a_i \omega_i^2 \frac{\cosh(k_i z)}{\sinh(k_i d)} \cos(\omega_i t + \varepsilon_i) \tag{20}$$

3.2 Calculation method of wave force

In marine engineering, Morison equation is often used to calculate the wave force of small diameter pile. Morison equation was proposed by Morison et al., university of California, Berkeley in 1950. It is a semi-theoretical and semi-empirical formula. The method considers that the wave force on the structure can be divided into two parts: the velocity force caused by the velocity of the wave particle and the moment force caused by the acceleration of the wave particle. The wave force acting on the pile with unit length is expressed as follows:

$$f(t) = \frac{1}{2} C_D \rho D u(t)|u(t)| + \frac{1}{4} C_M \rho \pi D^2 a(t) \tag{21}$$

Where, C_D is the drag force coefficient. C_M is the moment force coefficient. ρ is the density of seawater. D is the diameter of single pile.

It is not difficult to find from the equation (21) that the drag force is non-linear, so it needs to be linearized through Borgman L.E equation, and the results are as follows:

$$u(t)|u(t)| = \sigma_u \sqrt{\frac{8}{\pi}} \cdot u(t) \tag{22}$$

$$\sigma_u^2 = \int_{\omega_1}^{\omega_2} S(\omega) d\omega \tag{23}$$

σ_u is the key to linearization, and the value of the upper and lower limits of the integral in its calculation formula should refer to the actual project.

In practical calculation, several linear waves of different sizes can be formed by dividing the truncated frequency into segments by the equal-frequency method.

3.3 Project profile

3.3.1 Basic situation

Shanghai East China Sea bridge 100MW offshore wind power demonstration project is located in the east sea area of Shanghai East China Sea bridge, with a total installed capacity of 102MW. 34 SL3000 offshore wind turbines with a capacity of 3MW per unit are installed. D=1.7m in diameter and 20.6m in depth.

3.3.2 Hydrologic condition

The characteristic values of tidal level are shown in Table 2. And Table 3 shows design wave elements.

Table 2. Characteristics of tidal level (national 85 elevation)

Mean sea level (m)	0.23
Mean high water (m)	1.86
Mean low water (m)	-1.34
Design high water level (m)	2.55
Design low water level (m)	-2.09
Extreme high water level (m)	3.68
Extreme low water level (m)	-2.93

Table 3. Design wave elements

Recurrence interval (year)	**Average wave height (m)**	**Mean wave period (s)**	**Wave length (m)**
50	2.83	7.76	74.1
$H_{1\%}$**(m)**	$H_{4\%}$**(m)**	$H_{5\%}$**(m)**	$H_{13\%}$**(m)**
5.81	5.06	4.92	4.24

4 Analysis of computing result

4.1 Time history curve of each wave element

Figure 1 is the wave spectrum diagram.

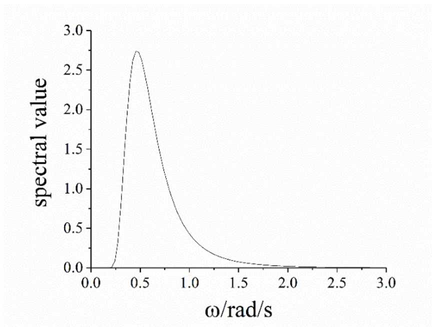

Fig.1. Wave spectrum diagram

As can be seen from figure 1, the change of spectral value after the circular frequency reached 2.5 was almost small, so 0.2< ω <2.5 was taken for calculation.

According to the calculation method in part 3.1, the time history curve of the wave equation is shown in Figure 2, and the time history curve of the velocity and acceleration of the water point is shown in Figure 3 and Figure 4.

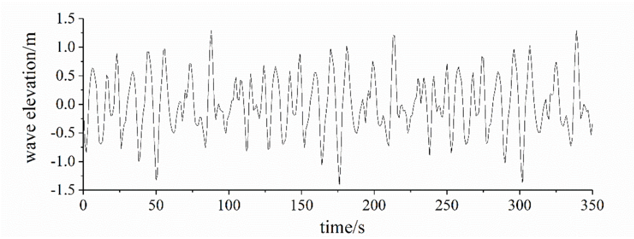

Fig.2. Time history curve of random wave wave surface

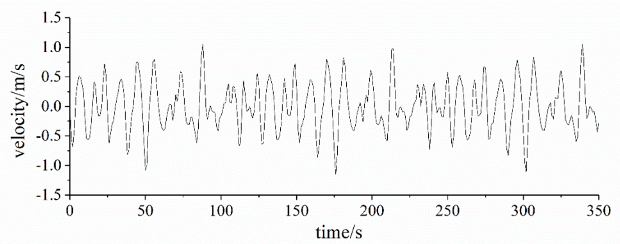

Fig.3. Velocity time-history curve

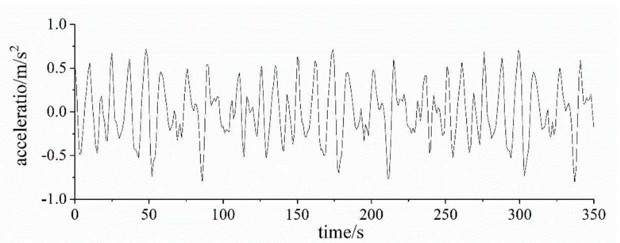

Fig.4. Acceleration time history curve

4.2 Analysis of computing result

C_M and C_D determined by instantaneous value method, Fourier analysis method, least square method and "harbour hydrological code" are shown in Table 4.

Table 4. Values of C_M and C_D

	C_D	C_M
Harbour hydrological code	1.2	2
Least square method	0.58	1.42
Fourier analysis method	0.62	1.56
Instantaneous value method	0.81	1.12

According to C_M and C_D determined by the above methods, the time history curve obtained by substituting into Morison equation is calculated and compared with the measured wave force data on the pile, as shown in Figure 5.

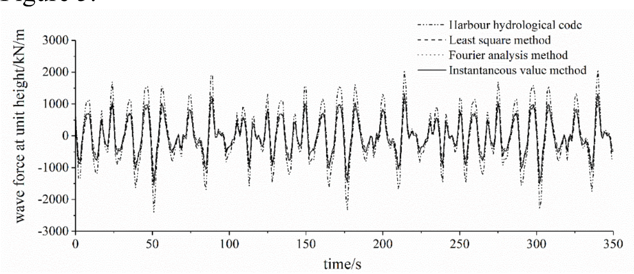

Fig.5. Time history curve of wave force at unit height

It can be seen from the Figure. 5 that the values of C_M and C_D determined by the instantaneous value method.

The harbour hydrological code are quite different from the measured values, while the values of C_M and C_D calculated by the Fourier analysis method and the least square method are quite close to the measured values.

5 Conclusion

The values of C_M and C_D calculated by the instantaneous value method have a large error because the conditions for their values are too harsh and the actual situation is not quite in line with the calculation principle. However, the values of C_M and C_D stipulated in the harbour hydrological code are relatively conservative, and the drag force and moment force calculated according to the domestic code are larger than the measured value. The values of C_M and C_D calculated by Fourier analysis method and least square method are close to the measured values.

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