

# Mathematical simulation of separating work tool technological process

*Sylwester Tabor*<sup>1\*</sup>, *Aleksandr Lezhenkin*<sup>2</sup>, *Serhii Halko*<sup>2</sup>, *Aleksandr Miroshnyk*<sup>3</sup>, *Stepan Kovalyshyn*<sup>4</sup>, *Aleksandr Vershkov*<sup>2</sup> and *Olena Hryhorenko*<sup>2</sup>

<sup>1</sup> Faculty of Production and Energy Engineering, University of Agriculture in Krakow, Balicka 116B, 30-149 Krakow, Poland

<sup>2</sup> Dmitriy Motorny Tavsia State Agrotechnological University, 72310, Melitopol, Ukraine

<sup>3</sup> Kharkiv Petro Vasylenko National Technical University of Agriculture, 61002, Kharkiv, Ukraine

<sup>4</sup> Faculty of Mechanics and Power Engineering, Lviv National Agrarian University, 80381, Lviv, Ukraine

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**Abstract.** The article describes the technological process of separation of the heaped-off pile of wheat. A mathematical model in the form of regression equations was obtained based on a full-factor experiment. Using the methods of mathematical analysis, the regression equations were investigated for the highest and lowest values. The values of the factors were obtained, for which the response function has the highest values, then the response surfaces were constructed and analyzed.

## 1 Introduction

As it is commonly known from literature [1, 2], the most effective method of harvesting grain crops is their combing at the root with subsequent stationary re-processing. The most problematic link in this technology is the separation of combed heaps. To solve this problem various designs of separating devices have been proposed.

The way for the separation of a culmiferous heap merits consideration. The implementation of this method of separation has allowed us to develop a number of devices [3].

This separating body was mounted in a combine for harvesting grain crops [3]. The field studies [4] proved the effectiveness of the proposed design for the separation of a heaped-off pile of rice.

Works [5, 6] are devoted to the problem of separation, where the results of tests of vibration and gravity separators are discussed.

During the operation of the separating work tools, great attention is paid to the study of the fractional composition of a heap. So studies, which were carried out in harvesting winter wheat with a combine harvester with a double-drum combing device hung on it, showed that a heap of winter wheat contained 35-43% of free grain, 33-47% of dangling

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\* Corresponding author: [sylwester.tabor@urk.edu.pl](mailto:sylwester.tabor@urk.edu.pl)

spikelets and 10-32% of coarse straw impurities. Based on studies [7] there is an average of 53.77% of free grain in battered heaps of winter wheat, 4.13% of dangling spikelets [8], 8.86% of chaff [9] and 33.2% of large straw impurities [10].

Therefore, the circuit solutions [11, 12] using cylindrical sieves with an outer working surface during the separation of the combed heap of cereal grains did not give the desired results in the course of experiments.

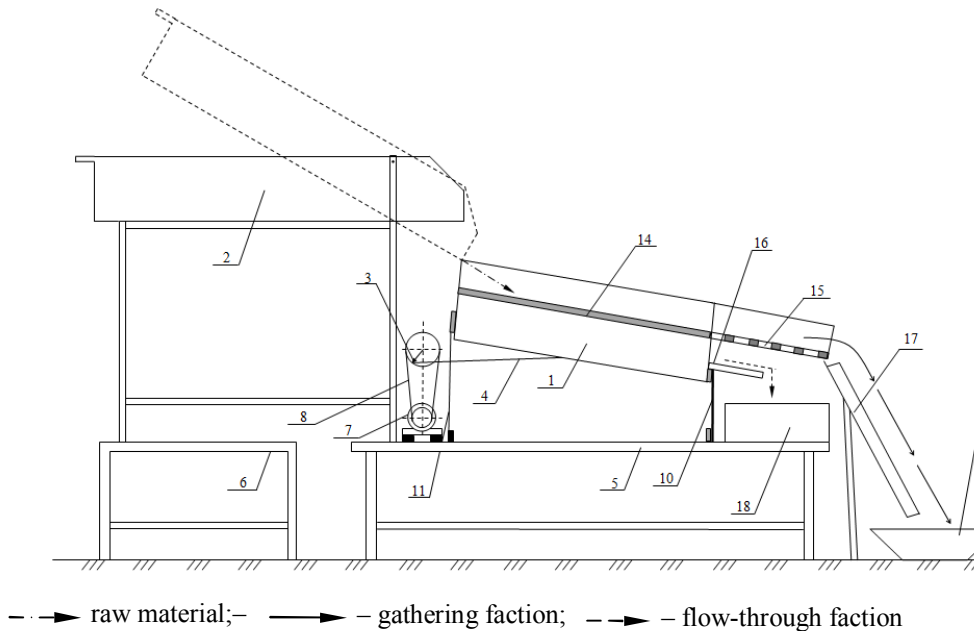
In our opinion, the most appropriate solution for the separation of combed heaps of grain crops is the work tool consisting of a segregator and a flat sieve, which is mounted in the cleaner of the combed heaps [13].

## **2 Material and methods**

The program included: construction of a mathematical model of the technological process of free grain separation experimentally; statistical analysis of the model received as a result; study of the functions of the highest and lowest value response; building response surfaces.

To implement the program, an experimental laboratory unit was developed, the flow chart of which is shown in figure 1.

The technological process of the laboratory unit was as follows. An artificially prepared mixture comprising 60% of free grain, 5% of dangling spikelets, 35% of straw impurities (the method of preparation of the starting material is given in [4]) was loaded onto conveyor 2, from where it came to segregator 14, which, together with a sieve made an oscillatory motion. Due to the oscillations, the straw particles, moving, formed voids through which the grain was sieved down and collected in the lower part of the culmiferous layer. The segregated layer was fed to the sieve 15, where it was divided into two factions. Large straw impurities and dangling spikelets came off the sieve and along the inclined tray 17 fell into the sampler 19. The grain passed through the sieve holes and along the inclined tray 16 fell into the sampler 18 (Fig. 1).



1 - shake mill; 2 - gravity conveyor; 3 - eccentric shaft; 4 - drive arm; 5 – frame; 6. – slide; 7 - electric motor; 8 - multiple vee belt drive; 10,11 – plates; 14 – separator; 15 – sieve; 16 - duct for the flow-through fraction output; 17 - duct for the gathering fraction output; 18,19 - sampler

**Fig. 1.** Technological scheme of the experimental laboratory unit

The mathematical theory of experiment planning was used to study the technological process of the work tool.

Analysis of various experimental plans allowed choosing the most rational plan for solving the tasks. Such a plan was the non-compositional, rotary three-level Box-Benkin plan.

Thus, a full-factor experiment was performed at three levels, for three factors, which was estimated by one response function. In general terms the mathematical model looks as follows:

$$y_1 = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 \quad (1)$$

where

$y_1$  - shall mean the response function characterizing the change in a separation factor;

$x_1$  - shall mean the specific feed of the source material to the work tool;

$x_2$  -shall mean the work tool oscillation frequency;

$x_3$  -shall mean the diameter of sieve holes;

$b_0, b_1, b_2, b_3, \dots$  - shall mean the regression coefficients of the first equation;

$c_0, c_1, c_2, c_3, \dots$  - shall mean the regression coefficients of the second equation.

Using the results of theoretical studies, as well as the analysis of literary sources [14,15], the limits of the factors were identified and listed in Table 1.

The significance of the regression coefficients was determined by Student's criterion, according to the recommendations of. The adequacy of the model was tested by the Fisher criterion.

In accordance with the plan of the experiment, the necessary kinematic and design parameters were set. The oscillation frequency was set using an autotransformer. All experiments were performed in triplicate. Using a table of random numbers, the experiments

were randomized, which ensured the uniform introduction of the element of randomness of the influence of uncontrolled and uncontrollable factors on the response functions.

**Table 1.** Levels of factors variation

The level and range of factors variation	Factors		
	specific feed of material, $\text{kg}\cdot\text{s}\cdot\text{m}^{2-1}$	oscillation frequency, $\text{s}^{-1}$	diameter of holes, mm
	$x_1$	$x_2$	$x_3$
Top level (+)	3,3	55	35
Basic level (0)	2,7	50	25
Lower level (-)	2,1	45	15
Variation interval	0,6	5	10

### 3 Results and discussion

A general view of the mathematical model of this plan is represented by equation (1).

As a result of the calculations, the following values of the confidence intervals for the regression coefficients of the first equation were obtained:

$$\Delta b_0 = \pm 0,000572; \Delta b_{ii} = \pm 0,00043; \Delta b_1 = \pm 0,000215; \Delta b_{ii} = \pm 0,00047$$

When comparing the numerical values of the regression coefficients and their confidence intervals, it turned out that the coefficient  $b_{1,2} = -0,00025$  is less than the confidence interval  $\Delta b_{ii} = \pm 0,00043$ . Consequently, this coefficient turned out to be insignificant and is not included in the model; besides, the coefficient  $b_{13} = 0$  is also not included in the model. All the remaining coefficients were greater than their confidence intervals, from which it follows that these regression coefficients are significant.

To assess the adequacy of the mathematical model obtained, the Fisher criterion was used. To do this, first the calculated value of the Fisher criterion was determined. The calculated value of the Fisher criterion is numerically equal to the ratio of the variance of adequacy to the variance of reproducibility and it is numerically equal to the first equation  $S^2\{y\} = 0,0004$ . The variance of adequacy was also calculated, its numerical value was  $S^2_{\text{ad}} = 0,0000066$  then using the obtained values of the variance of adequacy and variance of reproducibility, we determine the calculated value of the Fisher criterion for the first equation  $F_p = 0,0165$ . Comparison of the table value of the Fisher criterion and the calculated value showed that  $F_m > F_p$  i.e. the model is adequate. Thus, as a result of the implementation of a second-order plan, a mathematical model (2) was obtained in the form of a second-degree polynomial, which adequately describes the process of separation of a combed heap.

$$y_1 = 0,614 - 0,158x_1 - 0,093x_2 + 0,132x_3 - 0,00175x_2x_3 + 0,061x_1^2 - 0,041x_2^2 - 0,0665x_3^2 \quad (2)$$

The equation (2) characterizes the change in the separation coefficient depending on the parameters and modes of operation of the experimental working body. To analyze the equation (2), it is necessary to construct response surfaces.

First of all, we are interested in the highest values of the response function, i.e. the largest values of the separation coefficient. Thus, the task is reduced to determining the values of the factors (specific feed, oscillation frequency and hole diameter) at which the response function takes on the highest values. To solve this problem, it is necessary to investigate the regression equation, which is a function of several variables for the largest and smallest values. In the study of the function of several variables on the largest and smallest value in a closed space, the following algorithm was used:

a) partial derivatives of the first order were found, equated to zero, and the system  $k$  of equations was solved ( $k$  – shall mean the number of variables, and the system does not need

not be linear);

b) after solving the system, critical points  $M_i(x_1, x_2, \dots, x_k)$  were obtained, where  $i = 1, m$ ,  $m$  – shall mean the highest degree of the system equations;

c) the values of the function at these points were determined, though an extremum study was not conducted;

d) the largest and smallest values of the function on each border of the closed space were found;

e) the largest and smallest values were selected from all the calculated ones.

According to the above methodology, we investigate the first regression equation:

$$y_1 = 0,614 - 0,158x_1 - 0,093x_2 + 0,132x_3 - 0,00175x_2x_3 + 0,061x_1^2 - 0,041x_2^2 - 0,0665x_3^2 \quad (3)$$

Take the partial derivatives of this function on the studied factors  $x_1, x_2, x_3$ .

$$\begin{cases} \frac{\partial y_1}{\partial x_1} = -0,158 + 2 \cdot 0,061x_1, \\ \frac{\partial y_1}{\partial x_2} = -0,093 - 0,00175x_3 - 2 \cdot 0,041x_2, \\ \frac{\partial y_1}{\partial x_3} = 0,132 - 0,00175x_2 - 2 \cdot 0,0665x_3. \end{cases} \quad (4)$$

Equate equations (4) to zero and obtain an algebraic system of equations. Solve the system (5) relatively  $x_1, x_2$  and  $x_3$ .

$$\begin{cases} -0,158 + 2 \cdot 0,061x_1 = 0, \\ -0,093 - 0,00175x_3 - 2 \cdot 0,041x_2 = 0, \\ 0,132 - 0,00175x_2 - 2 \cdot 0,0665x_3 = 0. \end{cases} \quad (5)$$

From the first equation of system (5) we find  $x_1$

$$0,122x_1 = 0,158 \Rightarrow x_1 = 1,295.$$

Then we shall determine  $x_2$  and  $x_3$

$$\begin{cases} 0,082x_2 + 0,00175x_3 = -0,093 \\ 0,00175x_2 + 0,133x_3 = 0,132 \end{cases} \begin{cases} 0,00175 \\ -0,082 \end{cases}$$

$$\begin{cases} 0,0001435x_2 + 0,0000030625x_3 = -0,00016275 \\ -0,0001435x_2 - 0,010906x_3 = -0,010824 \end{cases}$$

$$-0,0109029375x_3 = -0,01098675; x_3 = 1,008$$

$$0,082x_2 + 0,001763453 = -0,093; 0,082x_2 = -0,094763453; x_2 = -1,156$$

Thus, as a result of solving the system of algebraic equations (5), we obtained the coordinates of the point  $M(x_1; x_2; x_3) = M(1,295; -1,156; 1,008)$ . However, this point is outside the study area and we discard it. Now we will explore the values of the function at the boundaries of a closed space.

1. Accept that factor  $x_1 = \text{const}$  and denote it by  $c_1$ . Substitute  $c_1$  in the regression equation (3) instead of  $x_1$  and receive the equation of the following form:

$$y_1 = 0,614 - 0,158c_1 - 0,093x_2 + 0,132x_3 - 0,00175x_2x_3 + 0,061c_1^2 - 0,041x_2^2 - 0,0665x_3^2 \quad (6)$$

Next, we shall conduct a study using a similar method. Take the partial derivatives of the response function for variable factors  $x_2$  and  $x_3$ .

$$\begin{cases} \frac{\partial y_1}{\partial x_2} = -0,093 + 0,00175x_3 - 0,082x_2, \\ \frac{\partial y_1}{\partial x_3} = 0,132 - 0,00175x_2 - 2 \cdot 0,0665x_3. \end{cases} \quad (7)$$

Equate each equation of system (7) to zero and as a result receive a system of two linear equations.

$$\begin{cases} -0,082x_2 - 0,00175x_3 - 0,093 = 0, \\ -0,00175x_2 - 0,133x_3 + 0,132 = 0. \end{cases} \quad \text{or} \quad \begin{cases} -0,082x_2 - 0,0175x_3 = 0,093, \\ 0,00175x_2 + 0,133x_3 = 0,132. \end{cases} \quad (8)$$

Find from the system (8) indeterminates  $x_2$  and  $x_3$ :  $x_2 = -1,156$ ;  $x_3 = 1,0008$ .

Accept that  $c_1$  may take the values: -1; -0,5; 0; 0,5; 1. The calculations of the values of the response function  $y_2$  shall be carried out in each of the sections both inside these sections and at the nodal points. In cases the values  $x_2$  or  $x_3$  go beyond the zone of factor space, the value of the response function is not calculated

2. Accept that  $x_2 = c_2 = \text{const}$ . Then the first regression equation of the model (2) shall be

$$y_1 = 0,614 - 0,158x_1 - 0,093c_2 + 0,132x_3 - 0,00175c_2x_3 + 0,061x_1^2 - 0,041c_2^2 - 0,0665x_3^2 \quad (9)$$

Take the partial derivatives of  $y_1$  according to factors  $x_1$  and  $x_3$ .

$$\begin{cases} \frac{\partial y_1}{\partial x_1} = -0,158 + 0,122x_1, \\ \frac{\partial y_1}{\partial x_3} = 0,132 - 0,00175c_2 - 0,133x_3. \end{cases} \quad (10)$$

Equate each equation of system (10) to zero.

$$\begin{cases} -0,158 + 0,122x_1 = 0, \\ 0,132 - 0,00175c_2 - 0,133x_3 = 0. \end{cases} \quad \text{or} \quad \begin{cases} 0,122x_1 = 0,158, \\ 0,00175c_2 + 0,133x_3 = 0,132. \end{cases} \quad (11)$$

From equations (11) we define  $x_1$  and  $x_3$ .

$$\begin{cases} x_1 = \frac{0,158}{0,122} = 1,295, \\ x_3 = \frac{0,132 - 0,0175c_2}{0,133}. \end{cases} \quad (12)$$

The second equation of the system (12) includes a constant  $c_2$ , which can take the values: -1; -0,05; 0; 0,5; 1. These values are taken from the construction of the model and correspond to the points of the closed space. Substituting the values  $c_2$  into equation (9), we

define  $x_3$ . As a result of the calculations the following is received:

$$(x_3)_{|_{c_2=1}} = 1,124; (x_3)_{|_{c_2=-0,5}} = 1,058; (x_3)_{|_{c_2=0}} = 0,992; (x_3)_{|_{c_2=0,5}} = 0,927; (x_3)_{|_{c_2=1}} = 0,861. \quad (13)$$

The obtained results are substituted into the regression equation (3) and the values of the response function  $y_1$  are determined.

3. Accept that  $x_3 = c_3 = \text{const}$  and substitute  $c_3$  into the regression equation (3), we obtain:

$$y_1 = 0,614 - 0,158x_1 - 0,093x_2 + 0,132c_3 - 0,00175x_2c_3 + 0,061x_1^2 - 0,041x_2^2 - 0,0665c_3^2$$

$$\begin{cases} \frac{\partial y_1}{\partial x_1} = -0,158 + 0,122x_1 \\ \frac{\partial y_1}{\partial x_2} = -0,093 - 0,00175c_3 - 0,082x_2. \end{cases} \quad (13)$$

Equate each equation of system (13) to zero.

$$\begin{cases} -0,158 + 0,122x_1 = 0, \\ -0,093 - 0,00175c_3 - 0,082x_2 = 0. \end{cases} \quad (14)$$

From the first equation of the system (14)  $x_1 = 1,295$ . From the second equation of the system (14) we determine  $x_2$

$$x_2 = \frac{-0,093 - 0,0175c_3}{0,082} \quad (15)$$

Accept that  $c_3$  changes within the limits from -1 to 1, i.e.  $c_3 = -1$ ,  $c_3 = -0,5$ ,  $c_3 = 0$ ,  $c_3 = 0,5$  and  $c_3 = 1$ . Substitute in equation (15) the values that can take  $c_3$ , and as a result we receive the following values  $x_2$ , i.e.:

$$(x_2)_{|_{c_3=-1}} = -1,113; (x_2)_{|_{c_3=0,5}} = -1,145; (x_2)_{|_{c_3=0}} = -1,123; (x_2)_{|_{c_3=1}} = -1,155; (x_2)_{|_{c_3=0}} = -1,134$$

As a result, we obtain the values  $x_1$ ,  $x_2$ ,  $x_3$ , which are substituted into the regression equation (3). After calculations, the values of the response function  $y_1$  are obtained at  $x_3 = \text{const}$ .

The results of calculations of the functions of the response  $y_1$ , showed that the greatest value of the response function  $y_1 = 0,95225$  was taken when  $x_1 = -1$ ,  $x_2 = -1$  and  $x_3 = 1$ , therefore when constructing the surface of the response function, these values of the factors were taken.

At the first stage studies have been conducted of the response function at the maximum and minimum value, as a result, the values of factors at which the response function (equation 2) take the highest value were determined. Then one of the factors was recorded at the value at which the response function had the greatest value, and the other two factors were given different values from -1 to +1. The construction of response surfaces was performed using Maple software.

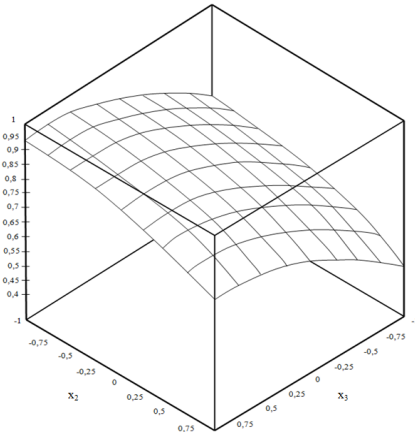
As a result of the construction, three response surfaces were obtained. The response surface  $x_1, x_2$  of the separation coefficient is shown on fig.2. The greatest value of the separation coefficient 0.95, is achieved when  $x_1 = -1$ ,  $x_2 = -1$  and  $x_3 = 1$ , when switching to natural values of factors, this means that the greatest value of the separation coefficient will be

at a specific diameter of oscillation frequency, i.e. with the diameter of oscillation frequency graphically

Fig. 3 shows that the highest separation coefficient was reserved for the diameter of oscillation frequency of 0.15 mm and a sieve hole diameter of 0.15 mm.

Considering the separation coefficient will be achieved provided that  $x_2 = -1$  and  $x_3 = 1$ , while  $x_1 = -1$  being reserved.

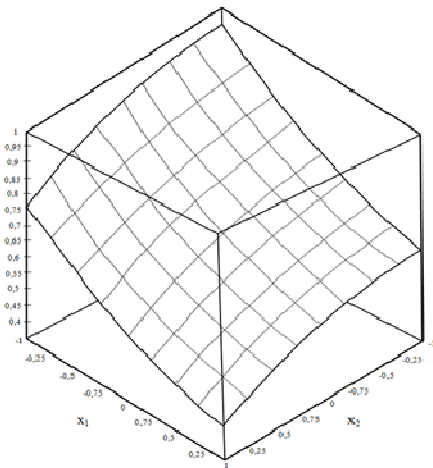
As the oscillation frequency increases and the diameter of the sieve holes decreases, the separation factor decreases, and at an oscillation frequency of  $55 \text{ s}^{-1}$  and a sieve hole diameter of 15 mm it is 0.5. The response surface  $x_2x_3$  clearly shows how the separation factor changes as the factors  $x_2$  and  $x_3$  change, i.e. the sieve oscillation frequency and the sieve hole diameter. Three response surfaces  $x_1x_3$ ,  $x_1x_2$  and  $x_2x_3$  graphically characterize the process of changing the separation coefficient.



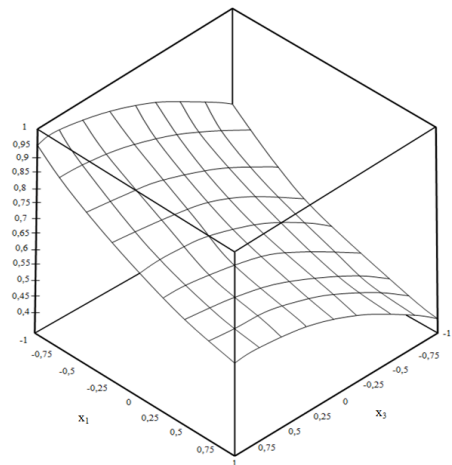
oscillation frequency of  $45 \text{ s}^{-1}$  and a sieve hole diameter of 15 mm in the specific feed and oscillation frequency reaches 0.45, while at an oscillation frequency of  $55 \text{ s}^{-1}$  and a sieve hole diameter of 15 mm it reaches 0.5. The response surface  $x_1x_2$  clearly shows how the separation factor changes as the factors  $x_1$  and  $x_2$  change, i.e. the sieve oscillation frequency and the sieve hole diameter.

The figure clearly shows that the highest separation factor is achieved provided that  $x_1 = -1$  and  $x_3 = 1$ , while  $x_2 = -1$  being reserved in the specific feed and a decrease in the oscillation frequency is registered at  $x_2 = -1$  the value of the separation coefficient is 0.45, while at  $x_1 = 1$  and  $x_3 = -1$  it reaches the smallest value of 0.4, oscillation frequency of  $45 \text{ s}^{-1}$  and a sieve hole diameter of 15 mm.

As Fig. 4 shows, the largest separation coefficient will be achieved provided that  $x_2 = -1$  and  $x_3 = 1$ , while  $x_1 = -1$  being reserved.



**Fig. 2.** Response surface  $x_1x_2$  of the separation coefficient, factor  $x_3$  is reserved for  $x_3 = 1$



factor  $x_2$  is reserved for  $x_2 = -1$



**Fig. 4.** Response surface  $x_2x_3$  of the separation coefficient, factor  $x_1$  is reserved for  $x_1 = -1$

In all three cases, the greatest value of the separation coefficient is observed at a specific feed of combed heap to the sieve of  $2.1 \text{ kg}\cdot\text{s}\cdot\text{m}^{-2}$ , the sieve oscillation frequency of  $45 \text{ s}^{-1}$  and sieve hole diameter of 35 mm, with such parameters and operating modes of experimental work tools the separation factor is 0.95.

## 4 Conclusions

1. As a result of the implementation of the three-level Box-Benkin plan, we received the data which, by calculation, made it possible to determine the mathematical model ratios, which establish the dependence of the qualitative indicators of the separation of the work tool on the parameters and modes of its operation.
2. When conducting a statistical analysis of the significance of the regression ratios, it was established that the ratios  $b_{12} = -0.00025$  and  $b_{13} = 0$  were less than the confidence interval  $\Delta b_{ij} = \pm 0.00043$ , which means that these factors are insignificant, all other ratios were greater than the confidence interval, which implies that these factors are significant. The evaluation of the adequacy of the model by the Fisher criterion showed that the model is adequate and this makes it possible to use it for further analysis.
3. The analysis of the regression model made it possible to substantiate the rational parameters of the work tool: diameter of openings is 35 mm; specific feed of the combed heap  $2.1 \text{ kg}\cdot\text{cm}^{-2}$ ; oscillation frequency  $45 \text{ s}^{-1}$ .

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