The hydraulic resistance coefficient in the conditions of simultaneous effect of Re, Fr and $\frac{B}{h}$

George Volgin*

Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

Abstract. One of the most important tasks of engineering hydraulics is to determine the energy loss during the motion of the fluid flow. The study of the question of whether the patterns of hydraulic resistances are similar in a calm and turbulent flow is relevant in the design of hydraulic structures. In most cases, a turbulent regime of fluid motion is observed in various applications, but to date, the theory of turbulence is not considered complete. When designing hydraulic structures, inaccuracies in the existing calculation methods can lead to a decrease in the efficiency and reliability of the entire spillway structure as a whole. The need for an integrated approach to the analysis of the impact on the hydraulic resistance of

various factors is noted (degree of spread $\left(\frac{B}{h}\right)$), the degree of turbulence

(Re) and the degree of flow roughness (Fr)), which is not always provided by known dependencies and methods of calculation. On the basis of our own experimental data, a new formula for calculating the hydraulic resistance of turbulent flows in smooth channels was obtained. The functional dependence of the hydraulic resistance coefficient on the

parameters $\left(\frac{B}{h}\right)$, Re and Fr is obtained.

1. Introduction

The question of hydraulic resistances when moving fluid in channels and pipes has been considered for a long time. There are numerous empirical formulas of General nature, as well as specialized laws recommended for certain types of structures. The issue under consideration has been developed on the basis of the doctrine of similarity and studies of the problem of turbulence. As a result of these works appeared semi-empirical formulas justified by modern ideas about the nature of hydraulic resistances and confirmed by experimental data of high accuracy [1 - 20].

^{*} Corresponding author: volgin-gv@mail.ru

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Studies show the existence of various hypotheses about the relationship of the coefficient of hydraulic resistance in open flows with either the Reynolds number or the Froude number. There is also an opinion about the influence on the coefficient of hydraulic

resistance of the channel geometry, namely the spread parameter $\left(\frac{B}{h}\right)$. However, the

universal dependence has not yet been obtained [10, 15, 17, 20].

2. Method and materials

Fundamental studies of the hydraulic resistance coefficient in pressure pipes made by Nikuradze I. in the 30s of the 20th century showed the presence of several areas with different behavior of the hydraulic resistance coefficient depending on the flow rate (Reynolds number) and the state of the inner surface of the pipe [6 - 8]. On the basis of experimental researches of Nikuradze I. formulas for calculation of coefficient of hydraulic resistance were received.

One area in which the hydraulic resistance coefficient decreases with increasing speed has been called the "hydraulically smooth resistance area". Such an area is typical for the flow in which the magnitude of the roughness projections is less than the thickness of the viscous sublayer. For the coefficient of hydraulic resistance, when the hydraulically smooth regime, the Blasius G. proposed the formula:

$$\lambda = \frac{0.3164}{\text{Re}^{0.25}} \tag{1}$$

On the basis of experimental data for this resistance region Nikuradze I. obtained the following dependence:

$$\lambda = 0.0032 + \frac{0.221}{\mathrm{Re}^{0.237}} \tag{2}$$

The movement of fluid in open channels is characterized by a number of factors that distinguish it from the pressure: a variety of cross – sectional shapes, the existence of a free surface, the influence of the slope on the flow structure, the presence of two possible Statescalm and stormy. The change in flow rate during non-pressure movement leads to a change in the depth of the flow, and, consequently, to a change in the area of the living section. The relationship between depth and flow is not linear, but is a complex relationship. Consequently, when changes in the flow rate is changed simultaneously and the speed, and $\begin{pmatrix} B \end{pmatrix}$

the parameter $\left(\frac{B}{h}\right)$ [10, 11]. The question of studying the influence of $\left(\frac{B}{h}\right)$ on the behavior of λ was considered by many scientists. There have been proposed various dependencies of the behavior of $\lambda = f\left(\frac{B}{h}\right)$, however, the universal dependence is still not received.

The experiments of Zegzda A. showed that in open channels there are the same areas of resistance as in pipes, and also the regularities established by the experiments of Nikuradze I. are preserved [9]. However, later, a number of researchers noted some difference in these dependencies in non-pressure and pressure motion (A. Egorov, G. Clegan, O. Aivazyan, R. Pauli, V. Vedernikov, A. Altshul) [1 - 5, 13, 14, 18]. Recommendations were proposed on

the need to take into account the peculiarities of the flow in open channels due to the presence of a free surface and the influence of the cross-section shape.

As can be seen from the formulas (1) and (2), the only criterion affecting the coefficient of hydraulic resistance is the Reynolds number. On the other hand, the coefficient of hydraulic resistance is related to the coefficient of Chezi through the dependence of the

 $C = \sqrt{\frac{8g}{\lambda}}$ [17]. Using the universal formula A. Chezi $C = \frac{Q}{\omega\sqrt{Ri}}$, the formula for

calculating the coefficient of hydraulic resistance in an open channel is obtained [12, 14, 19]:

$$\lambda = \frac{8gRi}{U^2} \tag{3}$$

To obtain the regularities of the effect of Reynolds numbers, the degree of flow stormy

(Fr) and the flow spread parameter $\left(\frac{B}{h}\right)$ on the hydraulic resistance coefficient, an

experimental study was conducted. The studies were conducted by the author in the laboratory "Hydraulics and Hydromechanics" of Hydraulic Engineering and Power Plant Construction, Moscow State University Of Civil Engineering (National Research University) in the period from 2014 to 2017. The walls of the tray are made of glass, the bottom of the channel is made of polished stainless steel. The channel length of 12.5 meters. For measurements, a section with uniform motion in the flow was selected. An electromagnetic flow meter with a measurement accuracy of 0.5% was used to measure the flow. To measure the depth of the flow used electronic depth meters with a measurement accuracy of 0.5 mm. Experimental data were obtained using a rectangular hydraulic tray with an automatic Jack lifting system. During the experiments, the bottom slope changed 9 times. For each slope conditions were selected under which you can get the maximum range of changes in the numbers of Froude and Reynolds. 154 experiments were conducted on 10 slopes, in which the Reynolds numbers varied from 23 to 308 thousand, the Froude numbers from 0.5 to 6, the channel spread from 2 to 20. In the analysis of experimental data, the calculation of the hydraulic resistance coefficient was based on the formula (3).

3. Results and Discussion

In the analysis of experimental data, the influence of the flow rate on the dependence $\lambda = f(\text{Re})$ was estimated. The experimental data were divided into groups where Fr < 1 (calm flow) and Fr > 1 (stormy flow) (figure 1). The graph shows that at a steady state of the flow the obtained experimental points are close to the values calculated by the formulas of Blasius and Nikuradze in the entire range of measurements. However, in a rapid flow, it is seen that at the Reynolds numbers of more than 100 000 experimental points deviate significantly from the equations of (1) and (2). It is experimentally established that with increasing Froude numbers, the effect of the Reynolds number on the value of the hydraulic resistance coefficient decreases.



Fig. 1. Comparison of experimental data with the equations of Blasius (1), and Nikuradze (2) for calm and stormy flow

To assess the effect of flow stormy, the experimental data were divided into ranges by Froude numbers. Figure 2 shows the results of calculation λ for a calm flow, which are close to the results obtained by I.Nikuradze for smooth pipes. In a turbulent flow (Figure 3) it can be seen that in a stormy flow for identical Reynolds numbers, as the degree of flow stormy (Fr) increases, the values of the hydraulic resistance coefficient (λ) increase. Analysis of the data also shows that for fixed values of the numbers Re and $\left(\frac{B}{h}\right)$, the value of λ increases in proportion to the number (Fr)^{0.25}.



Fig. 2. Analysis is of calm flow experimental data



Fig. 3. Analysis of stormy flow experimental data

To analyze the effect of the parameter $\left(\frac{B}{h}\right)$ on the hydraulic resistance coefficient (3), the experimental data were grouped by ranges $\left(\frac{B}{h}\right)$ and the step between these ranges from 1 to 20 $\left(\frac{B}{h}\right)$. The graph of dependence $\lambda = f\left(\frac{B}{h}\right)$ is shown in figure 4. From the analysis of the experimental data, it can be concluded that with the growth $\left(\frac{B}{h}\right)$ the coefficient of hydraulic resistance also increases.



Fig. 4. The graph of dependence $\lambda = f\left(\frac{B}{h}\right)$

Figure 5 presents the experimental data and assesses the effect of the Reynolds number on the hydraulic resistance coefficient at different degrees of flow turbulence and the influence of the flow spread parameter $\left(\frac{B}{h}\right)$. The graph shows that the nature of the change λ in calm and stormy flows is different. In the region of a calm flow, it is clearly noticeable that with the growth of Froude numbers, the values of the hydraulic resistance coefficient decrease. When analyzing the effect $\left(\frac{B}{h}\right)$ on the value of the hydraulic resistance coefficient, it is clear that the higher value λ corresponds to the experimental data, where the values $\left(\frac{B}{h}\right)$ are greater, and the lower value λ – smaller values $\left(\frac{B}{h}\right)$. It can be concluded that the influence of the parameter $\left(\frac{B}{h}\right)$ on the dependence $\lambda = f(Fr)$ is insignificant for a calm flow. In the field of stormy flow (Fr > Fr_{critical}) the behavior of the hydraulic resistance coefficient changes. Thus, the values λ gradually increase to Fr ≈ 3.5 and then do not change with increasing numbers Fr. However, at values $\left(\frac{B}{h}\right) < 10$ the influence of flow spatiality is observed. Thus, it can be concluded that when calculating the coefficient of hydraulic resistance in open uniform flows, the influence of the spread parameter $\left(\frac{B}{h}\right)$ must be taken into account.



Fig. 5. Experimental data analysis. Determining the effect of spread parameter. The graph of $\lambda = f(Fr)$

To assess the simultaneous effect of hydraulic flow characteristics $\left(\operatorname{Re}, \operatorname{Fr}, \frac{B}{h}\right)$ on the

hydraulic resistance coefficient (λ) , a graph was constructed (figure 6). The ratio of the hydraulic resistance coefficient calculated by the formula (3) to the hydraulic resistance coefficient calculated by the Blasius formula (1) is given along the ordinate axis. On the x-

axis is the value of the dependence of the dimensionless complex $\Pi = \left(\operatorname{Fr} \cdot \frac{h}{B} \right)$



Fig.6. Effect of dimensionless complex Π on λ .

In figure 6, the experimental data were grouped by ranges of Reynolds numbers. The characteristic similarity of the experimental data for $\text{Re} < 10^5$ with the Blasius formula was revealed. You can see that by setting $\Pi < 0.75$ experimental values of the coefficient of hydraulic resistance λ are approaching the values calculated by the Blasius formula $\lambda_{\rm B}$. The deviation from the line corresponding to the Blasius formula (1), which is observed at $\Pi > 0.75$, can be explained by an increase in the Froude numbers and the influence of the spatiality of the flow. As a result of the experimental study, a dimensionless complex reflecting the simultaneous effect of Re, Fr, $\frac{B}{L}$ is obtained:

$$K = \frac{\mathrm{Fr}}{\mathrm{Re}} \cdot \frac{h}{B} \tag{4}$$

For values $\Pi < 0.75$ when calculating the coefficient of hydraulic resistance, it is recommended to use the Blasius formula. For values $\Pi > 0.75$, the dependence for calculating the hydraulic resistance coefficient λ in the form of experimental data is obtained:

$$\lambda = \frac{0.3164}{\text{Re}^{0.25}} \left[4 \cdot \left(\text{Fr} \cdot \frac{h}{B} \right)^{0.25} - 2 \right]$$
(5)

As a result of the experimental study of the simultaneous effect of Re, Fr, $\frac{B}{h}$ on the

hydraulic resistance coefficient, the formula (5) recommended for the calculation under the condition $\Pi > 0.75$ was obtained.

4. Conclusions

1. Experimental research and comparison of experimental data with formulas of Nikuradze and Blasius for calculation of coefficient of hydraulic resistance in open smooth channels is carried out. It is found that the experimental data for $Re < 100\,000$ are quantitatively close to these formulas. When $Re > 10^5$, the Nikuradze and Blasius formulas give an underestimated value of λ . Thus, the experimental data with $Re > 10^5$ do not obey the laws described by Blasius and Nikuradze. It is also experimentally proved that with increasing Froude numbers, the effect of the Reynolds number on the value of the hydraulic resistance coefficient decreases.

2. For the first time the analysis of experimental data on several criteria affecting the coefficient of hydraulic resistance allowed us to obtain a calculation formula of the form

$$\lambda = f\left(\operatorname{Re}, \operatorname{Fr}, \frac{B}{h}\right)$$

3. A new formula for calculating the coefficient of hydraulic resistance in open channels in the area of smooth resistance, increasing the accuracy of the calculations from 3 to 5%.

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