Load Bearing Capacity Calculation of the System "Reinforced Concrete Beam – Deformable Base" under Torsion with Bending

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Abstract. It is presented the formulation and solution of the load bearing capacity of statically indeterminable systems "reinforced concrete beam – deformable base" by spatial cross-sections under force and deformation effects. The solution of problem is currently practically absent in general form. It has been established the relationship between stresses and strains of compressed concrete and tensile reinforcement in the form of diagrams. The properties of the base model connections are described based on a variable rigidity coefficient. It is constructed a system of n equations in the form of the initial parameters method with using the modules of the force (strain) action vector. The equations of state are the dependences that establish the relationship between displacements which are acting on the beam with load. Constants of integration are determined by recurrent formulas. It makes possible to obtain the method of initial parameters in the expanded form and, consequently, the method of displacements for calculating statically indefinable systems. The values of the effort obtained could be used to determine the curvature and rigidity of the sections in this way. It is necessary not to set the vector modulus *P*, the deformation is set in any section (the module is considered as an unknown) during the problem is solving. This allows us to obtain an unambiguous solution even in the case when the dependence M–χ has a downward section, i.e one value of moment can correspond to two values of curvature.

1. Introduction

Improving the methods of calculating buildings and structures erected in difficult geotechnical conditions (on subsiding, weak and inhomogeneous soils, undermining and karst areas, etc.) is possible [1–3] based on a more complete use of achievements in the border area structural mechanics of reinforced concrete [1–22], associated with the calculation of complex systems "base – foundation – upper structure", Fig. 1.

At present, the nonlinear work of one or both elements is taken into account [6–8, 19, 20] in a number of cases.

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In this case, the forces are determined in the beam and the base of the given load action. The function of the sections rigidity of the beam is taken according to the current norms. However, since it is about the establishment of efforts in the elements of the system, the total stock of its bearing capacity remains unclear. This is especially important for buildings and structures operating in complex engineering and geological conditions [9, 10, 20], since it is not possible either to identify uneven elements and, therefore, to rationalize the placement of concrete and reinforcement, nor to reasonably decide on the need for expensive protective measures.

2. Models and Methods of Research

The art of liberating a real object (fig. 1) from insignificant features and identifying the design schemes of several levels is the most complicated stage of the modeling process [11, 12]. It is performed a construction of a method for calculating statically indeterminate systems: "reinforced concrete beam – deformable base". It is known that the calculation of such structures is a rather complicated task of the structural mechanics of reinforced concrete [1, 3, 4].
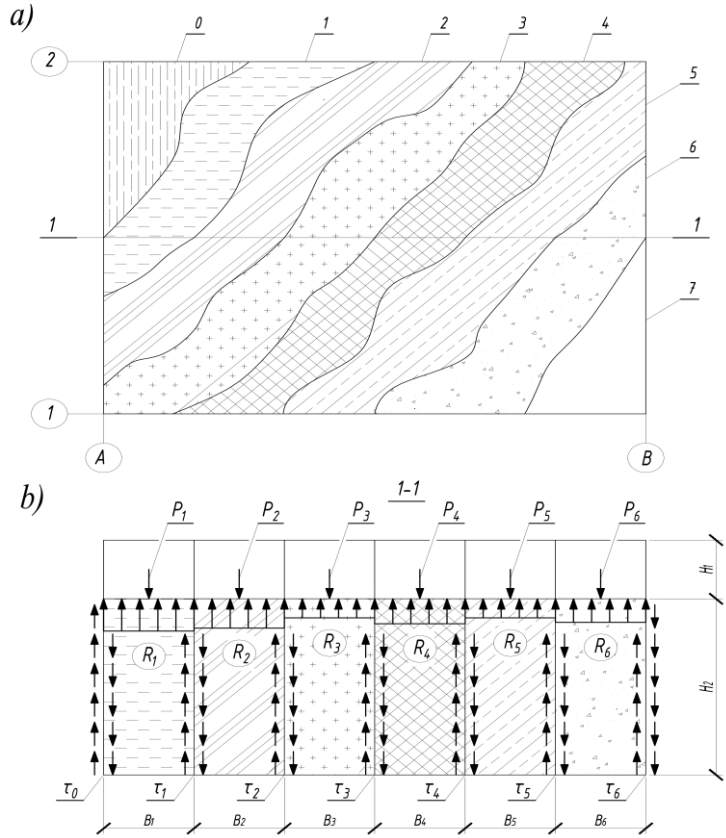

Fig. 1. To take into account the soil conditions in the area of the construction site (*a*) when modeling a statically indeterminate system, "concrete beam – deformable base" (*b*):
0 – dusty sand; 1 – fine (small) sand; 2 – sandy loam; 3 – silty sandy loam; 4 – light sandy loam; 5 – heavy sandy loam; 6 – light sandy clay; 7 – heavy clay;
$P_1...P_6$ – loads from the aboveground part of the building; $\tau_0...\tau_6$ – tangential stresses between the layers of the soil; $B_1...B_6$ – dimensions of soil layers; $R_1...R_6$ – the resistance of soil layers; H_1 – the height above ground part of the building; H_2 – the height of the soil layers

The most reasonable solution of the problem is given in [1, 4, 5] in cases where the bearing design capacity is limited by the resistance of normal and inclined sections.

However, the prerequisites, on which these works are based, are intended mainly for the zone of net and transverse bending and do not take into account the effect of spatial cracks on the strength and rigidity of the beam sections in the zones of joint action of bending, torque and transverse forces [12–18]. They also don't contain the criteria for exhausting the strength of the beam at fracture along the spatial cross-section, which substantially affects the nature of the destruction and the bearing capacity of the system as a whole.

As noted earlier [9, 10], the solution of the strength problem of the system "reinforced concrete beam – deformable base" in terms of spatial cross-sections in the case of force and deformation influences is a rather complicated problem. It solution is practically absent in the general form at present.

In this paper, the problem is formulated as follows: to determine the values of the vectors force and deformation modules, corresponding to the exhaustion of the load bearing capacity of the system "reinforced concrete beam – deformable base" in terms of spatial cross-sections.

The solution begins with establishing the connection between stresses and strains of compressed concrete and stretched reinforcement. Such dependencies can be taken in accordance with fig. 2.

Other prerequisites are used to estimate the stress-strain state at the level of the "section" algorithm under the combined action of bending and torsional moments and shear force, correspond to those adopted in [9, 22, 23].

It is advisable to adopt the base model in accordance with the works [9, 19, 20], where the properties of the links are described on the basis of a variable rigidity coefficient (fig. 2).

$$R_i = k_i y_i a . \tag{1}$$

Here R_i, k_i and y_i – respectively, effort, linear rigidity and displacement in the *i*-th bond of the base.

The bending moment can be expressed in the *i-th* section of the beam for various support schemes by the form:

$$M_i = M_1 + Q_1 L + \left(\frac{L}{n}\right)^2 \cdot \sum_{j=1}^{i-1}(i-j)y_j k_j - q_l \overline{M}_{0i} , \tag{2}$$

where M_1 = moment in termination at $i=1$; $\overline{M}_{0i}$ = moment in the *i-th* section of a given unit vector of external forces.

Torque in the *i*-th section of the beam for various support schemes, can be expressed by the form:

$$M_{t,i} = M_{t,1} + \left(\frac{L}{n}\right)^2 \cdot \sum_{j=1}^{i-1}(i-j)\left[y_j + y_{j,*}\left(\frac{1}{2}-\beta\right)\right]k_j - q_{l,t}\overline{M}_{t,0i} . \tag{3}$$

Performing the appropriate algebraic transformations of formula (2) using [9], we obtain a system of n equations $(2 \le k \le n+1)$ in the form of the initial parameters method:

$$F_k\left[y_1, \varphi_1, M_1, M_{t,1}, Q_1, M_{n+1}, M_{t,n+1} y_i, q_l(q_d), q_l(q_d), q_t(q_{t,d})\right] = 0 . \tag{4}$$

In formula (4) $q_l(q_d)$, $q_t(q_{t,d})$ = the modulus of the force (deformation) action vector.

Turn to the equations of state – the dependencies that establish the relationship between displacements and the load acting on the beam. Ways are possible different to build them.

For example, it is known that an analytical apparatus for calculating statically indeterminable systems can be obtained on the basis of the initial parameters method. If the integration constants are determined not from a system of equations, but by recurrent

formulas, then it becomes possible to obtain, in the expanded form, the method of initial parameters, and hence the method of displacements.

The equilibrium states of normal sections are successively detected in the process of solving the problem. The stress-strain state closes in the zone of a dangerous spatial crack with increasing load, up to the exhaustion of the beam load bearing capacity [11, 18, 22, 23]. The traditional problem of structural mechanics is solved at each stage of the loading of the iterative process: at a given load, internal forces are determined in the system elements.
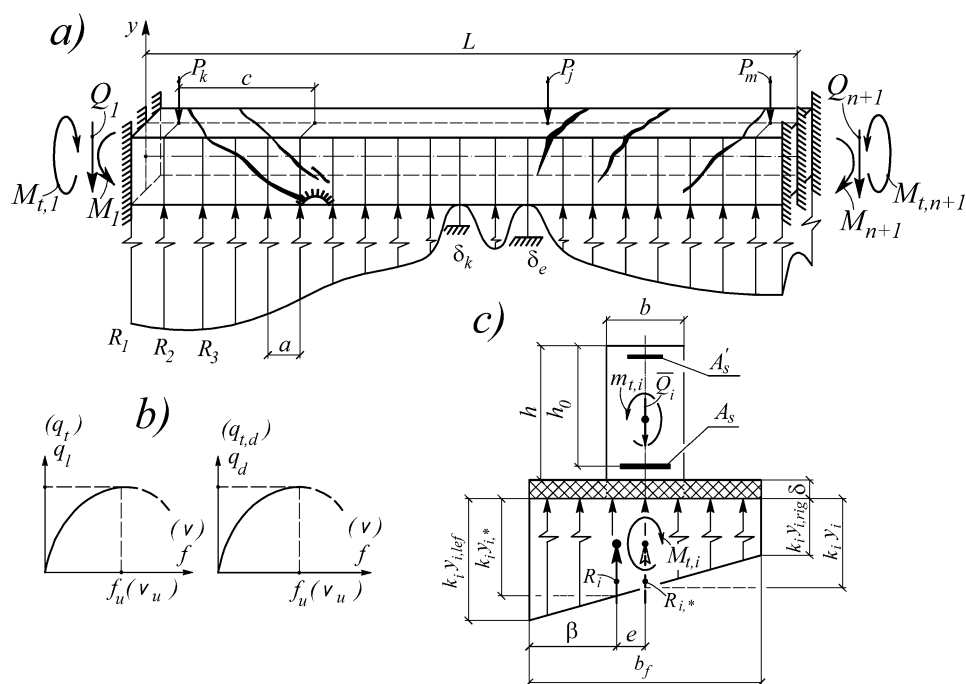

Fig. 2. To the calculation of the "reinforced concrete beam – deformable base" system:
a – calculation scheme; b – diagrams of the system state; c – cross-section of the system;
$R_1...R_{n+1}$ = the reactions in the first and $(n+1)$ bond of the ground base; a = the length of the section on which the "reinforced concrete beam – deformable base" system is broken; δ_k, δ_e = the ground subsidence values; L = the length of the "reinforced concrete beam – deformable base" system; M_1, M_{n+1} = bending moments; M_t, $M_{t,n+1}$ = torques; Q_1, Q_{n+1} = shear forces; c = the horizontal projection of a dangerous spatial crack; P_k, P_j, P_m = the load of above-ground parts of the building; h = the height of the beam; h_0 = the working height of the beam; δ = the base plate thickness; b = the width of the beam; b_f = the width of the soil mass; β = the distance from the left (most) ordinate of the trapezoid of the soil massif to its center of gravity; $e=(0.5-\beta)b_f$, where e is the eccentricity between the center of the beam gravity and the center of the soil mass gravity; $m_{t,i}$ = the values of the internal, redistributed in the "reinforced concrete beam – deformable base" system, the torque of the beam; Q_i = varying shear force; $q_l(q_t)$ and $q_d(q_{t,d})$ = modules of the corresponding power (deformation) effects vector; v = relative twist angle; f = deflection; f_u = the values of deflection at the maximum values of the corresponding modules; A'_s = the area of the upper working reinforcement; A_s = bottom working reinforcement area.

The values of the force thus obtained could be used to determine the curvature, the relative angle of twist, and the rigidity of the sections.

However, since, by analogy with the works [9, 23], the dependencies $M - \chi$, $M_t - v$ have descending sections (i.e., two curvature values can correspond to one moment), the probability of obtaining an ambiguous solution is quite obvious in this case. In this

connection, it is necessary not to set the vector $\overline{P}$ modulus, but the deformation in any section (the module is considered as an unknown).

System (4) generally has $(n+9)$ unknowns:

$$y_i(i=1,2,...n+1), \varphi_1, M_1, M_{t,1}, Q_1, M_{n+1}, M_{t,n+1}, q_l(q_d), q_t(q_{t,d}).$$

Additional conditions for system (4) are the boundary conditions and equilibrium equations compiled for the entire system.

With reference to the case considered on fig. 2, for determining y_i, φ_1, v_1, y_{n+1}, Q_1, M_{n+1}, $M_{t,n+1}$ boundary conditions are used $y_i=0$, $\varphi_1=0$, $v_1=0$, $y_{n+1}=0$, $\varphi_{n+1}=0$, $v_{n+1}=0$ respectively. To determine the unknown bending moment M_1, the equilibrium moment equation is used with respect to a point $(n+1)$:

$$M_1 + Q_1 L + \left(\frac{L}{n}\right)^2 \cdot \sum_{i=1}^{n}(n-i+1)y_i k_i - M_{n+1} - q_l \overline{M}_{0,n+1} = 0. \tag{5}$$

To determine the unknown torque $M_{t,1}$, the equation of equilibrium moments relative to a point $(n+1)$ is used:

$$M_{t,1} + \left(\frac{L}{n}\right)^2 \cdot \sum_{i=1}^{n}(n-i+1)\left[y_i + y_{i,*}\left(\frac{1}{2}-\beta\right)\right]k_i - M_{t,n+1} \pm q_{l,t}\overline{M}_{t,0,n+1} = 0, \tag{6}$$

where β = the distance from the left (most) ordinate of the trapezoid of the soil massif to its center of gravity (Fig. 2, *c*).

Equating the value of the acting bending moment in the section S to the given one, we define the unknown q_l:

$$M_1 + Q_1 L\frac{S-1}{n} + \left(\frac{L}{n}\right)^2 \cdot \sum_{i=1}^{n}(S-i)y_i k_i - q_l \overline{M}_{0,s} = M_{set}. \tag{7}$$

Equating the value of the current torque in the S section to the given one, we determine the unknown $q_{l,t}$:

$$M_{t,1} + \left(\frac{L}{n}\right)^2 \cdot \sum_{i=1}^{n}(S-i)\left[y_i + y_{i,*}\left(\frac{1}{2}-\beta\right)\right]k_i - q_{l,*}\overline{M}_{t,0,s} = M_{t,set}. \tag{8}$$

The lower values q_l and $q_{l,t}$ are chosen to assess the existence of the bearing capacity of the system "reinforced concrete beam – deformable base". So, we have $(n+9)$ equations for solving the system (4).

The used equilibrium equation $\sum Y = 0$ serves to determine the unknown transverse force in the embedment (fig. 2) with $i=n+1$:

$$Q_1 + \frac{L}{n}\sum_{i=1}^{n+1}y_i k_i - q_l \sum_{j=1}^{m}P_j - Q_{n+1} = 0. \tag{9}$$

Having solved system (4), it is possible to determine the magnitudes of the acting moments in the sections of the reinforced concrete bar, provided that the values of B_i, B_t, k_i are known, which in turn depend on the acting forces.

This predetermines the iterative solution to the problem, which is as follows.

1. Choose a disadvantageous section and, setting a fixed curvature and relative twisting angle for it, determine M_s, B_s, M_{st}, B_{st}.

2. Take the initial values of the rigidity $B_{i,0}$, $k_{i,0}$, $B_{i,t,0}$, $k_{i,t,0}$.

3. Formulate and solve a system of linear equations (4).

4. According to the formula (3) and (4) to determine the magnitude of the bending and torsion moments in sections of the reinforced concrete rod.

5. Calculate the curvatures χ and relative twist angles v at the next iteration:

$$\chi_{i,j+1} = \frac{M_{i,j}}{B_{i,j}}, (i = 1,2,\ldots,s-1,s+1,\ldots n+1). \tag{10}$$

$$v_{t,i,j+1} = \frac{M_{t,i,j}}{B_{t,i,j}}, (i = 1,2,\ldots,s-1,s+1,\ldots n+1). \tag{11}$$

For given values of curvature χ and relative twist angles $\chi_{i,j+1}$, $v_{t,i,j+1}$ and the resulting dependence $M - \chi$, $M_t - v$ from [9, 23] it is possible to determine the values of the moments $m_{i,j+1}$ and $m_{t,i,j+1}$.

It should be noted that the torque M_t and bending moment M on the steps of loading, except for the breaking, are expressed by the following relationship [12, 17]:

$$\frac{M_t}{M} = \eta. \tag{12}$$

7. To define new values of section rigidity:

$$B_{i,j+1} = \frac{m_{i,j+1}}{\chi_{i,j+1}}, (i = 1,2,\ldots,s-1,s+1,\ldots n+1). \tag{13}$$

$$B_{t,i,j+1} = \frac{m_{t,i,j+1}}{v_{t,i,j+1}}, (i = 1,2,\ldots,s-1,s+1,\ldots n+1). \tag{14}$$

8. To specify the values of the rigidity coefficient of the base:

$$k_i = F(P_i),$$

where $F(P_i)$ is the base rigidity change function.

9. The calculation of items 3 ... 8 are repeated until the condition:

$$\left| \frac{q_{j+1} - q_j}{q_{j+1}} \right| \leq \xi, \tag{15}$$

where q_{j+1}, q_j are the modules of vectors $q_l(q_d)$, $q_t(q_{t,d})$, respectively, on $j+1$ and i -th iterations; ξ is given accuracy of calculation $(\xi = 0,01 \div 0,001)$.

The resulting value $q_l(q_d)$ is used further in the determination of the stress-strain state of the system from the given effects.

In this case, a successive change in curvature $\chi_{s,set}$ (the relative angle of twist determines the value $q_l(q_d)$ or $q_t(q_{t,d})$, which with a given accuracy is equal to q_{col} (q_{col} is the specified value of the power or deformation impact modulus).

The value of $q_l(q_d)$ is determined, and corresponded to the maximum load on the state curve (Fig. 2, *c*), that is, the criterion

$$\frac{\partial q}{\partial f} = 0 \tag{16}$$

or the destruction of at least one section of a reinforced concrete rod $\chi_i \geq \chi_{i,u}$ or $v_{t,i} \geq v_{t,i,j+1}$.

The found values $q_l(q_d)$, $q_t(q_{t,d})$ are the desired value of the external force or deformation effects. Of these, the worst value is chosen relative to the load bearing capacity of the "reinforced concrete beam – deformable base" system.

In this case, the destruction may occur from crushing, popping, cutting the concrete over a dangerous spatial crack, crushing the beam wall, or as a result of loss of adhesion in the area of anchoring, or from the development of a spatial crack due to the concentration of deformations at its top.

3. Conclusions

1. It is considered the formulation and solution of the bearing capacity problem of statically indeterminable systems "reinforced concrete beam – deformable base" by spatial cross-sections under force and deformation effects. Their the solution is practically absent at present in general form. The relationship between stresses and strains of compressed concrete and tensile reinforcement has been established in the form of diagrams. The properties of the base model connections are described and based on a variable rigidity coefficient.

2. It is constructed a system of *n* equations in the form of the initial parameters method. It is using the modules of the force (strain) action vector. The equations of state are dependences that establish the relationship between displacements and the load acting on the beam. Integral constants are determined not by a system of equations, but by recurrent formulas. Then it becomes possible to obtain the method of initial parameters in the expanded form and, consequently, the method of displacements for calculating statically indeterminable systems.

3. Equilibrium states of normal sections are successively detected in the process of solving the problem. Here the stress-strain state closes in the zone of a dangerous spatial crack with increasing load, up to the exhaustion of the beam load bearing capacity. Moreover, the traditional problem of structural mechanics is solved at each stage of the loading of the iterative process. Internal forces are determined in the system elements at a given load.

The values of the effort obtained in this way could be used to determine the curvature and rigidity of the sections. But it is necessary not to set the vector modulus $\overline{P}$ when solving the problem, it is necessary to set the deformation in any section (the module is considered as an unknown). This allows us to obtain an unambiguous solution even in the case when the dependence $M - \chi$ has a downward section, i.e. one value of moment can correspond to two values of curvature.

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