Cascade method of Stretched Elements strengthening by FRP

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Abstract. Subject - analysis of applicability and effectiveness of cascade type models in design of strengthening of stretch elements by gluing on their surface fiber reinforced polymers (FRP). Research objectives – cascade method of stretched elements reinforcement with adhesive joints presentation. Materials and methods – few variants of FRP-reinforcement with application of FEM simulation and analytic approach. A number of diagrams and tables represent the results. Results - a highly efficient and cost-effective method of strengthening the stretched elements to increase their bearing capacity reserves, the features of the bonded joint behavior, the equations and formulae for analysis and design. Conclusions - the name "cascade" reflects the features of the proposed strengthening design. The base element relaxation is gradual with each successive element attached. Analytical expressions in a rather general form are obtained and presented to design the cascade strengthening scheme. Design examples concentrate on the analysis of the adhesive joints application to attach FRP elements. The results suggest the effective use of adhesive joints to strengthen rather strong, including steel, stretched elements. The cascade method eliminates the indispensable use of highly expensive high-strength materials, thereby reducing the cost of reinforcement structures.

1 General

FRP-reinforcement of steel elements (by fiber reinforced polymeric materials) using the adhesive and combined joints can radically simplify and accelerate the technology of reinforcement operation for steel structures in many cases without interrupting the process of their normal service.

The implementation of FRP-reinforcement with adhesive joints involves the reinforced element surface preparation for gluing. Allowing for the real dimensions of building structures, an important parameter is the area of gluing contact surfaces value that may the dimensions of glue layer dictate. In case of rod elements reinforcement, the rational length of the bonding area may be dominant parameter, since the shear stress distribution in the adhesive layer may be extremely uneven along its length. This is especially significant for adhe-

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sive joints in stretched elements strengthening. Surface preparation, as the world experience of strengthening steel elements of bridge structures shows, is the most labor-intensive taking a lot of time operation of the reinforcement process.

Carbon high-strength fibers, having a tensile strength many times exceeding the ultimate tensile strength of base elements, have a modulus of elasticity close to the elastic modulus of the metal of the reinforced elements. However, the total cross-section of high-strength fibers in the reinforcement elements is very insignificant in comparison with the cross-sectional dimensions of the reinforced elements of building structures, and therefore such reinforcement elements can be significantly included in the work of the reinforced element only under pre-tension, which in case of glue joints is highly problematic. Yet, with significant plastic deformations, the polymer elements can be actively involved in the work of the main element, redistributing a significant part of the acting force, while remaining in the elastic state, thus preventing the transition of local damage of elements in the global destruction of the structure.

Allowing for the noted problems, one of its solution factors is the development of a technique for analyzing the polymer reinforcement structure response at the stages of preliminary assignment of parameter values and evaluation of the so obtained layout solution (search through the permissible parameter values for the joint components) and at the stage of taking the final constructive solution. At the first stage, it is reasonable to use the least labor-intensive and time saving FEM and analytical elastic models. The second stage realization may need more accurate calculation methods based on nonlinear models.

This paper presents a special scheme of stretched element strengthening – cascade. It allows fastening not a single but a number of strengtheners sharing the whole discharging force nearly proportionally to their *EF*s.

2 Publications review

Along with traditional methods of reinforcing elements of steel structures using welded and bolted joints, the use of high-strength fiber-reinforced polymer materials (for example, CFRP) can be quite effective, providing, with an insignificant increase in the weight of the structure, that there is no weakening of cross sections, additional stress concentrators or additional potential areas of corrosion, as well as a less labor intensive reinforcement process than traditional ones.

In [1] the authors consider the application of carbon polymers in building structures. The [2] and [3] present the recently obtained results of experimental studies of the behavior of adhesive joints in the strengthening of damaged steel elements working in tension. The work [4] considered principal aspects of ensuring the structure reliability level control by incorporating reserve (intercepting) elements that do not take on the loads in normal service conditions but become active in emergencies and start working as bearing in the overall structure at load level when some elements lose their bearing capacity. Polymer materials, due to their unique properties, are ideal for such applications in stretch zones.

Steel reinforcement elements increase the weight of structures, are prone to corrosion, and lead to the forming of additional zones of stress concentration in the base metal resulting from welding or drilling [5]. Welding may involve additional problems of quality control of welded joints, welding in hard-to-reach places, welding residual stresses, cracking in welded zones subject to heating, and a significant decrease in fatigue strength [6].

Welding in an explosive atmosphere is possible only with long breaks in normal operation of the facility and strict safety measures, which significantly increases the cost of repairs [7].

Polymers have high strength and wear resistance, as well as resistance to aggressive environment. The weight of polymer reinforcement system is several times lower than that of steel of similar reinforcement [8].

At the same time, the resistance of a high-strength fiber to stretching can be many times higher than that of steel, and the modulus of elasticity may be close to or even much higher than that of steel [9].

In [10-18], various aspects of use of polymers in the repair and strengthening of building structures for improving fatigue strength, stability with the use of adhesive and mechanical connections are considered.

The cascade strengthening method under consideration was not managed to be found in any publication in any form though some information about step by step reinforcement by steel elements of some continuous beam stretched flange over support of a bridge in Siberia in the 60-th of last century. On the other hand, there are lots of cable bridges where with one end all cables are fastened to the pylon with other ends distributed along the span homogeneously.

3 Materials and methods

The use of adhesive joints in bearing structures, for all its attractiveness due to a number of features, causes plausible doubts in designers, as a little-studied field. For the confident use of glued joints, the condition is the reflection in the normative documents of the requirements for the physical and mechanical properties of the materials used and the performance of the bearing glue joints that application of existing and/or development of new adhesive compositions must provide and rules and recommendations for design of glue joints.

The analysis of various parameters and properties of the adhesive layer effect on the adhesive joint response and the structure reinforcement effect on structure resistance forecast is possible on the basis of a comparison of the results of numerical and physical experiments with approximate calculations on the basis of analytic relations allowing at relatively little time and effort to consider any required number of different combinations of joint parameters, especially when planning physical experiments [19, 20].

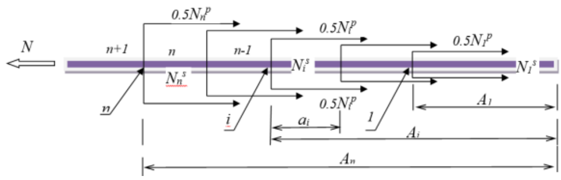

Fig. 1. Cascade scheme

N_i^s – tape stretching force in the interval i; N_i^p - total force in two attached «ribbons» in the interval i; node i - the left node of interval i.

In fig. 1 the reinforcing elements fastened in points 1 to n support the left half length of the stretched base element divided into n+1 intervals under load N. The base element has the mechanical characteristic $(EA)_i^s$ on each interval i, the interval length is a_i. The reinforcing elements have the mechanical characteristics $(EA)_i^p$ on the length A_i. On the right ends all the reinforcing elements and the base element are fixed. The scheme has double symmetry.

The task is to calculate the tension force distributions in all the reinforcing elements and the base element with given geometry and applied force.

The problem can be reduced to solving a $(n+1)$x$(n+1)$ system of linear equations

$$\sum_{k=1}^{i-1} b_{ik}^{ps} \cdot N_k^s + h_{ii}^{ps} \cdot N_i^s - N_{i+1}^s = 0, \qquad i = 1, \dots n;$$

$$N_{n+1}^s = N, \text{ where}$$

$$b_i^p \equiv \frac{(EA)_i^p}{A_i}; \quad b_i^s \equiv \frac{(EA)_i^s}{a_i}; \quad b_{ik}^{ps} \equiv \frac{b_i^p}{b_k^s}; \quad b_{ii}^{ps} \equiv \frac{b_i^p}{b_i^s}; \quad h_{ii}^{ps} = 1 + b_{ii}^{ps}$$

Then the tension force distributions in all the reinforcing elements can be calculated from

$$N_i^p = \sum_{k=1}^{i} b_{ik}^{ps} \cdot N_k^s \qquad i = 1, \dots n$$

For *i* = 1, the first term (sum) in the first equation should be ignored.

The special case is if the base specimen has the damage in the middle [19]. It means that $N_1^s = 0$. Then in the matrix as shown in example the first line and the first column should be excluded thus reducing the order of the problem by 1.

For instance, in case of *n* = 5 the system of equations in the matrix form looks as

$$\begin{vmatrix} h_{11} & -1 & 0 & 0 & 0 \\ b_{21} & h_{22} & -1 & 0 & 0 \\ b_{31} & b_{32} & h_{33} & -1 & 0 \\ b_{41} & b_{42} & b_{43} & h_{44} & -1 \\ b_{51} & b_{52} & b_{53} & b_{54} & h_{55} \end{vmatrix} \blacksquare \begin{vmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ N \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & h_{22} & -1 & 0 & 0 \\ 0 & b_{32} & h_{33} & -1 & 0 \\ 0 & b_{42} & b_{43} & h_{44} & -1 \\ 0 & b_{52} & b_{53} & b_{54} & h_{55} \end{vmatrix} \blacksquare \begin{vmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ N \end{vmatrix}$$

(a) (b)

Fig. 2. Cascade scheme matrix realization for continuous (a) and discontinuous (b) base element with five strengtheners

$$T_i = N_i^p = \sum_{k=1}^{5} b_{ik} \cdot N_k \qquad i = 1, \dots 5$$

The system does not seem difficult to solve even in such a relatively general case. At the same time allowing for the specific structure of the system it can be easily enhanced to higher orders by introducing quite formally some additional lines and columns. Anyone interested can spare some time to experiment with this algorithm.

It should be understood that this model does not take into account the deformability of attachment joints of reinforcement elements. So the analytic results are supposed not to coincide with those of FEM or physical experiments and be useful just for preliminary evaluation of strengthening scheme applicability and effectiveness.

In [19], [20] and a number of publications of many authors is shown that the glue joint has usually a limited working length of several centimeters with a rapidly decaying shear stress distribution in the adhesive layer. So the total strength of the glue joint must be limited by some value. The needed equations can be found in [19] for case (b) of fig. 2 and in [20] the expressions are given to evaluate the effective length and limit force in a single strengthener.

To illustrate the approach to effective strengthening problem solution, consider a simple FEM application.

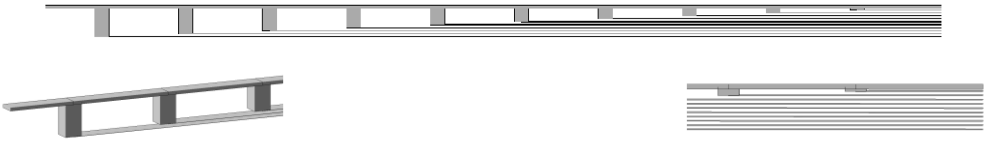

Fig. 2. Scheme. General view, left and right ends

The steel element of total length $2 \cdot (200 + 10 \cdot 300)$ mm under tension force 250 kN applied at the ends having 50x20 mm cross section (250000 N / (50x20) = 250 MPa) is modelled in LIRA-SAPR as shown in fig. 2. Here the strengthening HSFRP tapes each of 50x1 mm cross section are fastened through “absolutely” rigid fastening elements symmetrically on both top and bottom sides of the steel element. The fasteners’ interval is 300 mm.

In fig.2 the bottom left quarter of the whole structure is loaded at the left free end (200 mm) with tension force of 125 kN. cross section is -50x10. All its nodes are secured from vertical displacement and rotation.

4 Analysis results

The results in fig. 3 and table 1 show the tension force values distribution in steel element and strengtheners respectively. The *a*, *b* and *c* variants relate to continuous steel element. The rest three variants *d*, *e* and *f* are for the steel element with a gap in the middle (right end on the model).

The internal tensile force in the steel element decreases with each next interval from left to right and in cases of fig. 4 (the gap) diminishes to zero.

The internal tensile force values in the HSFRP elements decrease with each next interval from left to right in cases of fig. 3 and increase in cases of fig. 4.

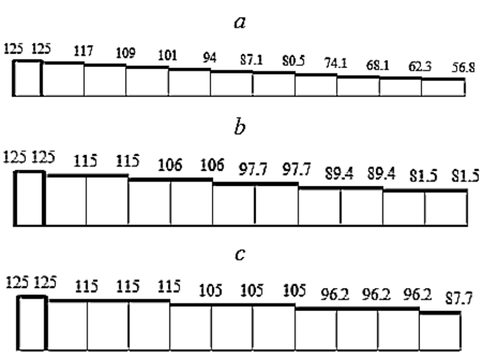

Fig. 3. Scheme

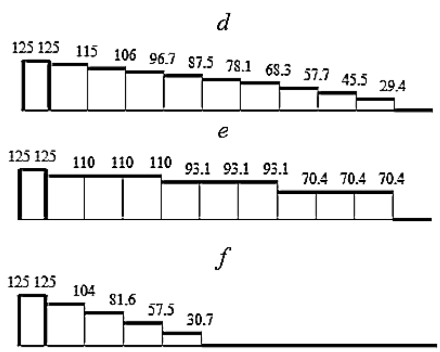

Fig. 4. Scheme

Table 1. Tension force values (kN) in the strengtheners

<table>
<tr><th>Intervals</th><th>a
300 mm</th><th>b
600 mm</th><th>c
900 mm</th><th>d
300 mm</th><th>e
600 mm</th><th>f
300 mm</th></tr>
<tr><td>1</td><td>8,242</td><td rowspan="2">9,514</td><td rowspan="3">10,059</td><td>9,576</td><td rowspan="2">15,647</td><td>21,084</td></tr>
<tr><td>2</td><td>7,900</td><td>9,397</td><td>22,310</td></tr>
<tr><td>3</td><td>7,569</td><td rowspan="2">9,094</td><td>9,288</td><td rowspan="2">16,912</td><td>24,113</td></tr>
<tr><td>4</td><td>7,247</td><td rowspan="3">9,596</td><td>9,275</td><td>26,763</td></tr>
<tr><td>5</td><td>6,935</td><td rowspan="2">8,688</td><td>9,407</td><td rowspan="2">19,564</td><td rowspan="2">30,730</td></tr>
<tr><td>6</td><td>6,633</td><td>9,775</td></tr>
</table>

7	6,339	8,293	9,130	10,563	25,814	
8	6,055			12,217		
9	5,779	7,910		16,118	47,064	
10	5,511		8,512	29,385		
	68,210	43,498	37,296	125,000	125,000	125,000

The value under each column of the table is the sum of tensile forces in strengtheners that equals to the amount of force reduction in the steel strip. Thus the effect of strengthening in case *a*, *b*, *c* is $68{,}210/125 \cdot 100\% = 55\%$, 35% and 30%. The maximum tensile stress level in strengtheners is $8242/(50 \cdot 1) = 165$ MPa, 190 MPa and 201 MPa respectively. In cases *d*, *e* and *f* it reaches the values of 588, 941 and 615 MPa respectively.

In essence, any group of strengtheners could be replaced, although not without possible local damage for the reinforced base element, with one strong enough reinforcement element, especially with pre-tension. But not in case of glue connections.

Here again should be noted that one must take into account the deformability and load bearing capacity of fasteners. The latter is especially significant in the case of glue joints. The strictly limited bearing capacity of glue joints can catastrophically affect the effectiveness of reinforcement.

Consider the same sample but with HSFRP elements bonded to the reinforced element in the same sequence as before. In [19] some useful expressions were presented for a damaged specimen with discontinuity that should be quoted here.

The ultimate value of tension force may be determined from strength conditions of adhesive

$$P \le [P] = \frac{b_p}{\beta} \cdot R_{as}, \qquad \beta = \sqrt{\frac{G_a}{t_a} b_p \cdot \left(\frac{1}{E_s A_s} + \frac{1}{E_p A_p} \right)} \tag{1}$$

R_{as} – glue material shear design resistance, R_{ay} - glue material design resistance, E_s- elasticity modulus of steel, A_s- reinforced beam cross-section area, E_p- elasticity modulus of composite tape material, A_p- tapes cross-section area, G_a - shear modulus of glue material, t_a- glue material layer depth, b_p- tapes width.

Minimal necessary glue layer length

$$d \ge d_{\min} = \frac{1}{2\beta} \ln \frac{1+Y}{1-Y}, \qquad Y \equiv \frac{b_p}{\beta} \cdot \frac{R_{as}}{P} \tag{2}$$

The effective (rational) for accepted value of $P \le [P]$ glue layer length (further lengthening does not result in decrease of shear stress in glue) may be calculated by a number of iterations using the expression

$$\tau_{\max} = P \cdot \frac{\beta}{b_p} \cdot cth(\beta d) \le R_{as} \tag{3}$$

For the above sample the joint parameters are given in table 2.

Table 2. Dimensions and mechanical properties of the reinforcement joint

Steel					*FRP*				
b	*h*	*A*	*E*	*(AE)*$_s$	*b*	*h*	*A*	*E*	*(AE)*$_p$
50	10	500	206000	1,03E+08	50	0,5	25	300000	7500000
mm	*mm*	*mm*^{2}	*MPa*	*N*	*mm*	*mm*	*mm*^{2}	*MPa*	*N*

Glue:	G_a, *MPa*	t_a, *mm*	G_a / t_a	R_{as}, *MPa*
	300	0,5	600	14

So β =0,065508, [P] = 10686 N, d_{min} = 26 mm calculated for applied P = 10 kN. The glue length d = 26 mm results in τ_{max} = 14,00 MPa. A further increase in the length of the gluing does not reduce the shear stresses in the adhesive below 13 MPa.

Comparing this result with data in table 1 (case *d*) allows to suppose that applying more strengtheners bonded with shorter intervals will do.

As for continuous steel element the strengtheners' tensile force values in all three cases *a*, *b* and *c* do not exceed the ultimate value [P] = 10,686 kN.

The deformability of glue fastening could provide some effect of relaxation in strengtheners thus reducing both the shear stresses in the adhesives and effect of strengthening.

Consider some results of numerical experiment with bonded strengtheners. All dimensions, load and mechanical properties are the same as above and the steel element is continuous. Instead of each rigid fastener the glue layer of depth 0,5 mm and 50 mm long is introduced with the same step of 300 mm.

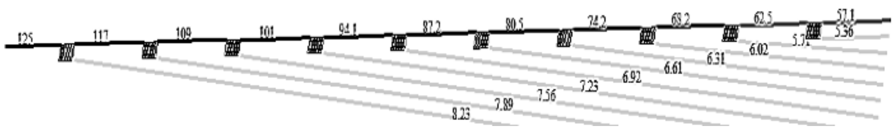

Fig. 5. Tensile forces distribution in the steel element and FRP strengtheners

Fig. 5 is a screenshot of the LIRA-SAPR PC. The scheme is rotated for easy viewing of results. In fact, the looking so massive in the fig. 5 fasteners are nothing else but the glue interlayers (50 mm wide) 50 mm long and 0.5 mm thick, and the strengtheners are FRP strips 50 mm wide and 0.5 mm thick, which are layered on top of each other.

The fig. 5 shows as the tensile force values in the steel element (the bold line at top) gradually decreases from applied 125 kN at the ends to 57,1 kN in the middle. The gray oblique lines show the strengtheners with the tensile force values changing gradually from maximal 8,23 kN at the ends to 5,36 kN in the middle (compare the results with those in fig. 1 and table 1, case *a*).

It is characteristic that the maximum shear stresses in the adhesive are 7.21 MPa (left end), which is significantly less than the limit value of 14 MPa. The maximal normal stress of breakaway is 0,422 MPa.

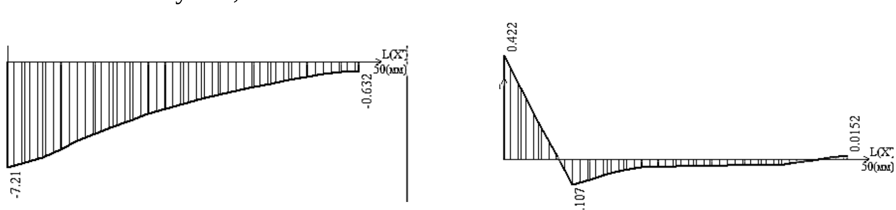

Fig. 6. Shear and breakaway stress (MPa) in the first (left) glue joint

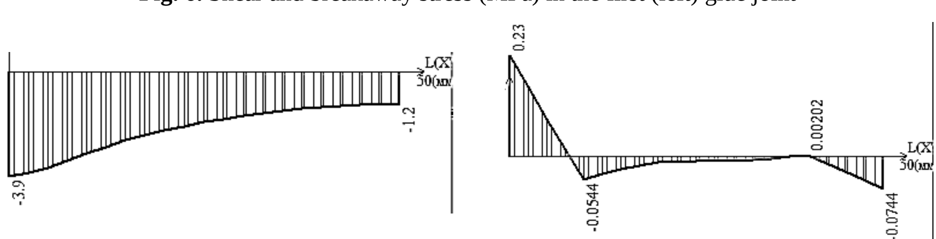

Fig. 7. Shear and breakaway stress (MPa) in the last (right or middle) glue joint

Thus, there is almost a double margin of safety in the most loaded joint.

With preserved all other parameters leaving just the first five left joints leads to max shear stress 7.35 MPa in the first (left) joint and 5.74 MPa in the fifth (last right) joint. The reduced tensile force in the middle part of steel detail is 83.19 kN. The same force values are obtained with on the contrary the five left joints removed. The obtained reduced tensile force in the middle part of steel detail is 81.5 kN without every second joint (five joints with interval 600 mm) while the max shear stress in the first (left) joint is 7.51 MPa.

In case of discontinuous steel element (table 1, case *d*) the results look as follows.

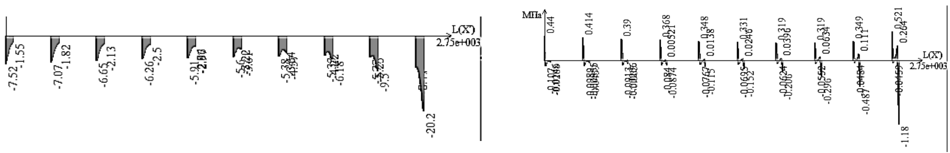

Fig. 8. Shear (left picture) and breakaway stress (MPa) distribution through the glue joints for discontinuous steel element (with a gap in the middle (right end))

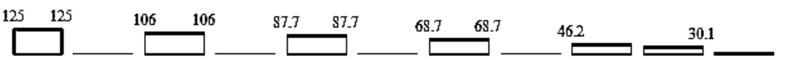

Fig. 9. Tensile forces distribution in the steel element with a gap in the middle (right end)

If all the joints move leftward by 100 mm the max shear stress in the last (right) joint diminishes to 16.4 MPa

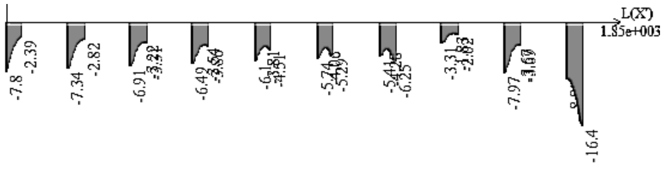

Fig. 10. Shear stress (MPa) distribution through the glue joints for discontinuous steel element (with a gap in the middle (right end)), joint interval 200 mm

If instead of adding or moving the joints just to try to diminish the max shear stress in the last (right) joint by stretching this joint longitudinally it won't help. See the fig. 11.

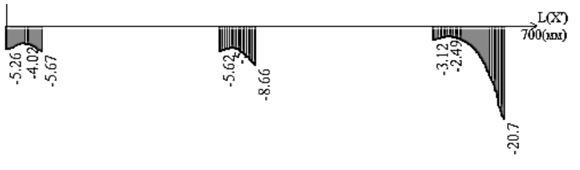

Fig. 11. Shear stress diagrams (MPa) for the last three glue joints, the last is twice longer (100 mm)

Calculations showed that a complete sticking of the middle (rightmost in the diagram) stringer along its entire length to the base element surface only increases the surge of shear stresses over the crack (up to 27 MPa in this sample), and therefore, for the glue joint to work correctly, some partial compensation is required to remove the full discontinuity in base element.

Evidently if there is not the through crack or something alike in the steel element but it is just partially damaged (notched) the situation becomes more favorable.

To conclude, suppose the rightmost section of the steel element (the middle of a full rod) has a weakened section (the cross-sectional area has decreased by half), but there is no complete break of continuity.

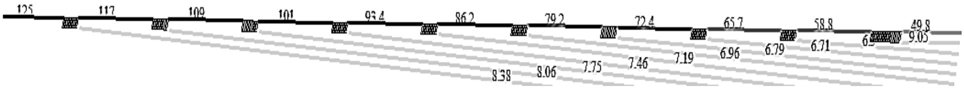

Fig. 12. Tensile forces distribution in the steel element and FRP strengtheners, weakened section

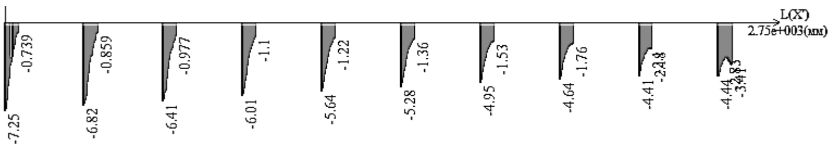

Fig. 13. Shear stress distribution in the glue joints fastening FRP strengtheners, weakened steel cross-section

The steel element tensile stress on the interval with weakened section amounts to 49800 / (50x10/2) = 199,2 MPa. The max tensile stress in FRP amounts to 9050 / (50x1) = 181 MPa.

5 Conclusions

1. A highly efficient and cost-effective method of strengthening the stretched structural elements for a real increase in the reserves of their bearing capacity is suggested.
2. The name "cascade" reflects the features of the proposed strengthening design. The relaxation of the main element increases gradually in stages with each successive strengthener element attached.
3. Analytical expressions in a rather general form are obtained and presented for calculating the cascade strengthening scheme but not allowing for the deformability of joints.
4. The relatively simple FE system let consider a number of design samples to analyze the features of cascade reinforcement system functioning and evaluate the effectiveness of its application for various materials and system parameters. Several examples are included.
5. Examples of the numerical calculation of a stretched steel element are focused on the analysis of the features of the use of adhesive joints for attaching reinforcement elements. The results obtained suggest the possibility of the effective use of adhesive joints to strengthen rather strong, including steel, stretched elements.
6. In a cascade system, with its efficient layout, the total additional resource of the carrying capacity can be quite rationally distributed over a number of individual strengtheners. So, the tension force in each of the strengtheners takes very modest values, and that eliminates the necessity for the indispensable use of highly expensive high-strength materials.
7. Of course, the presented approach so far provides only a basis for further experimental as well as theoretical studies in direction of optimizing and increasing the reliability of design solutions for real practical use.

References

1. I.I. Ovchinnikov, I.G. Ovchinnikov., G.V. Chesnokov, D.A. Tatiev, D.V., Pokulaev, Internet-journal *Naukovedeniye,* Vypusk **3**, may – june (2014)
2. A.R. Tusnin, E.O. Schurov, PGS, № **7**, 69-73 (2017)
3. A.R. Tusnin, E.O. Schurov, PGS, № **9**, 25-29 (2017)

4. A.I. Danilov, PGS, № **8**, 74-77 (2014)
5. Tavakkolizadeh, H. Saadatmanesh, *Journal of Structural Engineering, ASCE,* **129,** 186-196 (2003)
6. S. El-Tawil, E.Ekiz, S. Goel, S.-H. Chao, *Journal of Constructional Steel Research,* **67,** 261-269 (2011)
7. N. G. Tsouvalis, L. S. Mirisiotis, D. N. Dimou, *International Journal of Fatigue*, **31,** 1613-1627 (2009)
8. A. Shaat, D. Schnerch, A. Fam, S. Rizkalla, *Centre for Integration of Composites into Infrastructure* (2003)
9. T.-C Nguyen., Y. Bai, X.-L. Zhao, R. Al-Mahaidi, *Composite Structures*, **93**, 1604-1612 (2011)
10. M. Bocciarelli, P. Colombi, G. Fava, and C. Poggi, *Composite Structures*, **87**, 334-343 (2009)
11. H. Liu, R.Al-Mahaidi, and X. Zhao, Compos. Struct. Vol. **90**, 12-20 (2009)
12. K. A. Harries, A. J Peck., and E. J. Abraham, *Thin-Walled Structure*. Vol. **47**, 1092-1101 (2009)
13. A.K. Patnaik, and C. L. Bauer, *Proceeding of the 4th Advanced Composites for Bridges and* structures *conference*, Calgary, Canada (2004)
14. P. Colombi, A. Bassetti, A. Nussbaumer, *Fatigue Fract Eng Mater Struct*, Vol. **26**, No. 1, 59-67 (2003)
15. B. Täljsten, C.S. Hansen, J.W. Schmidt, *Construction and Building Materials*, **23(4)**, 1665-1677 (2009)
16. E. Ghafoori, M. Motavalli, J. Botsis, A. Herwig, M. Galli, *International Journal of Fatigue*, **44**, pp. 303-315 (2012)
17. E. Ghafoori, A. Schumacher, M. Motavalli, *Engineering Structures*, **45**, 270-283 (2012)
18. E. Ghafoori, M. Motavalli, *Composite Structures*, **101**, 22-34 (2013)
19. A.I. Danilov, O.A. Tusnina, Procedia Engineering, Volume **153**, 124–130 (2016)
20. A.I. Danilov, I.A. Kalugin, Structural mechanics of engineering constructions and buildings, **14** (5), 13 pp. (2018)