

# Vibrocreep of concrete

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**Abstract.** We study the creep of statically loaded concrete under stationary vibrations. Our considerations are closely connected with nonlinear rheological equation of mechanical state for concrete. The proposed approach for considered problem used so called energy of entirety. Note that this value can be successively utilized also instead of known Reiner's invariant. In this paper in contrast to the traditional model a material concrete is considered as a union of its links with statistical disturbed strengths. This conception allows to modify Boltzmann's principle superposition of fractions creep deformations. It is notable that the influence of vibrocreep on bearing capacity of the concrete structure is considerable.

## 1. Introduction

The problem a structural safety of buildings are connected with the strength  $R(\tau)$  - basic mechanical characteristic of constructor materials. A material is a thermodynamic system and in consequence of energetic and mass exchange with environment the generating  $R(\tau)$  constraint forces are formed. The accumulated in materials specific energy  $W(\tau)$  is called their energy of entirety [1]. These values define the energetic state of the materials and represent their maximal measure of resistance to a destruction [2]. Since the destructions at moment  $\tau = t$  is generated by instantaneous deformations the corresponding work  $A(t)$  is considered as the energetic parameter of materials, named the energy of its entirety. Note that beside of the force action the magnitude of energy  $W(t)$  essentially depends from the corrosion damages [3]. The destruction of a part of the links under an increasing on cross-sections  $G$  axially loading  $N(\tau)$  implies the redistribution of  $N(\tau)$  on cross-sections  $G(\tau)$  of the capable to resist links.

This generates a non-linear dependence of the deformations on the calculated stresses  $\sigma(\tau) = N(\tau)/F$ , obtained under the assumption of an equal strength for all links. However in fact there is a linear dependence of the deformations of the undamaged links on the so - called structural stresses  $\sigma_s(\tau) = N(\tau)/F(\tau)$ . Here  $F$  and  $F(\tau)$  are the area of  $G$  and  $G(\tau)$  respectively.

Thus we obtain the following correlation

$$\sigma_s(\tau) = S(\tau)\sigma(\tau); S(\tau) = F / F(\tau), \quad (1)$$

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where  $S(\tau)$  is called the function of non-linearity of stress. The constant over of period  $[t_0, t]$  stress  $\sigma_s(\tau)$  generate the deformation

$$\varepsilon(t, t_0) = \varepsilon_{inst}(t, t_0) + \varepsilon_{cr}(t, t_0). \quad (2)$$

Here  $\varepsilon_{inst}(t, t_0) = \sigma_s(t)/E(t)$  is the instantaneous and  $\varepsilon_{cr}(t, t_0) = C(t, t_0)\sigma_s(t)$  is the creep deformations  $E(t)$  is the elasticity module and  $C(t, t_0)$  is the measure of simple creep.

## 2. Position of problem at reduction of equation for vibrocreep factor

An increment  $\Delta\sigma_s(t) = \sigma_s(t) - \sigma_s(t_0)$  impales the creep deformation  $\Delta\varepsilon_{cr}(t, t_0)$ . Since a fraction increment  $\Delta\varepsilon_{cr}(t, \tau_i)$  is in depended from a value and duration of the rest interments  $\Delta\sigma_s(\tau_j)$ ;  $j \neq i$ , we can define  $\Delta\varepsilon_{cr}(t, t_0)$  by Boltzmann's principle superposition [4]. In resent with respect to the structural stresses the next rheological state equation

$$\varepsilon(t, t_0) = \sigma_s(t) \left[ \frac{1}{E(t)} + C(t, t) \right] - \int_{t_0}^t \sigma_s(\tau) \frac{\partial C(t, \tau)}{\partial \tau} d\tau \quad (3)$$

is reduced [5], [6]. Here  $C(t, t)$  is so – called quick creep [7]. The creep of statically loaded concrete was studied in many works (for example in [8 – 21]). The phenomenon it intensification under vibration influence is called vibrocreep of concrete.

The correlation

$$C_v(t, \tau) = K_v C(t, \tau) \quad (4)$$

was proposed in [21], [22] on the basis of experiments. Here  $C_v(t, \tau)$  is the measure of vibrocreep;  $K_v$  is the vibrocreep factor;  $\tau$  is the moment of loading;  $t$  is the moment of observation. The factor  $K_v(\omega, \rho, \gamma)$  is determined by the frequency  $\omega$ , parameter  $\gamma$  of the open surface of concrete and asymmetry of stresses

$$\rho = \sigma_{\min} / \sigma_{\max}. \quad (5)$$

Presented the function  $S(\tau)$  in form

$$S(\tau) = 1 + S_d(\tau) \quad (6)$$

we have

$$\sigma_s(\tau) = \sigma(\tau) + S_d(\tau)\sigma(\tau). \quad (7)$$

The function  $S_d(\tau)$  is determined by the ratio of common energy entirety  $W_e(\tau)$  of all damages links to such energy of all entire links of constructive element. Thus the stress  $\sigma_{sn}(\tau) = S_d(\tau)\sigma(\tau)$  is the addition to in consequence of redistribution of stresses.

The function

$$S(\tau) = 1 + V \cdot \left[ \frac{\sigma(\tau)}{R(\tau)} \right]^\alpha \quad (8)$$

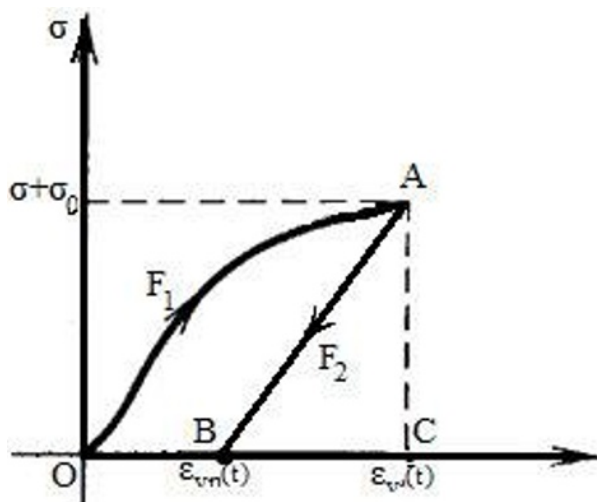
is used in applications, where  $V$  and  $\alpha$  are the empirical parameters. In this case

$$\sigma_{sn}(\tau) = V \cdot \left[ \frac{\sigma(\tau)}{R(\tau)} \right]^\alpha \cdot \sigma(\tau). \quad (9)$$

The following figure present the interpretation of action

$$\sigma(\tau) = \sigma - \sigma_0 \cos(\omega\tau - t_0) \quad (10)$$

at the  $t = t_0 + 2\pi/\omega$  end of one cycle.



**Fig. 1.** A loop of hysteresis

The bounded by loop  $OABO$  of hysteresis area  $F_1$  is the specific dissipate energy  $\Delta W(t)$ ; the area  $F_2$  of triangle  $BAC$  is the specific potential energy  $\Pi(t)$ . These energy is determined only by values  $W_d(\tau)$  and  $W_e(\tau)$  respectively and therefore are invariant on frequency  $\omega$ . Thus we have the relations

$$\frac{d\Delta W(t_0 + 2\pi/\omega)}{d\omega} = 0, \quad (11)$$

$$\frac{d\Delta \Pi(t_0 + \pi/\omega)}{d\omega} = 0. \quad (12)$$

Relation (11) is so – called energetic invariant experimentally discovered by N.I. Davidenkov [23].

According to (6) and the equality (7) we have that the deformation  $\varepsilon_{vlin}(t) = \varepsilon_v(t) + \varepsilon_{vn}(t)$  (see Fig. 1) generate by  $\sigma(\tau)$  stress and at the end of half cycle

$$\varepsilon_{vlin}(t_0 + \pi/\omega) = \frac{\sigma + \sigma_0}{E(t_0 + \pi/\omega)} + K_v(\omega, \rho, \gamma) \int_{t_0}^{t_0 + \pi/\omega} C(t, \tau) d\sigma(\tau). \quad (13)$$

In accord to Fig. 1

$$\Pi(t_0 + \pi/\omega) = \frac{1}{2} \varepsilon_{vlin}(t_0 + \pi/\omega) (\sigma + \sigma_0) \quad (14)$$

and relation (12) implies

$$\frac{d\varepsilon_{vlin}(t_0 + \pi/\omega)}{d\omega} = 0. \quad (15)$$

Because of smallness  $\pi/\omega$  we suppose  $E(t_0 + \pi/\omega) \approx E(t_0)$ .

Now by (13) and (15) we obtain the equation

$$\frac{\partial K_v(\omega, \rho, \gamma)}{\partial \omega} + P(\omega, \rho, \gamma) K_v(\omega, \rho, \gamma) = 0, \quad (16)$$

$$P(\omega, \rho, \gamma) = \frac{\partial Z(\omega)}{\partial \omega} / Z(\omega), \quad (17)$$

$$Z(\omega) = \int_{t_0}^{t_0 + \pi/\omega} C(t, \tau) d\sigma(\tau). \quad (18)$$

**Remark 1.** In work [24], [25] is presented the another deduction of equation (16). In virtue of (16) and (17)

$$\frac{\partial \ln K_v(\omega, \rho, \gamma)}{\partial \omega} + \frac{\partial \ln Z(\omega)}{\partial \omega} = 0, \quad (19)$$

$$\frac{\partial \ln [K_v(\omega, \rho, \gamma) \cdot Z(\omega)]}{\partial \omega} = 0. \quad (20)$$

According to (20) the value

$$I(\omega, \rho, \gamma) = K_v(\omega, \rho, \gamma) \cdot Z(\omega) \quad (21)$$

is invariant on  $\omega$  and since  $\lim_{\omega \rightarrow \infty} K_v(\omega, \rho, \gamma) = 1$  we have

$$K_v(\omega, \rho, \gamma) \cdot Z(\omega) = \lim_{\omega \rightarrow \infty} Z(\omega), \quad (22)$$

$$K_v(\omega, \rho, \gamma) = \lim_{\omega \rightarrow \infty} Z(\omega) / Z(\omega). \quad (23)$$

In a typical case  $C(t, \tau) = C(\infty, t_0) \left[ 1 - \beta e^{-\gamma(t-\tau)} \right]$  and the relation (18) implies

$$Z(\omega) = \frac{C(\infty, t_0) \left[ (\sigma - \sigma_0) \gamma^2 e^{\frac{-\gamma\pi}{\omega}} - (\sigma - \sigma_0) \gamma^2 + \sigma \omega^2 \left( e^{\frac{-\gamma\pi}{\omega}} - 1 \right) \right]}{\gamma^2 + \omega^2}. \quad (24)$$

Since  $\lim_{\omega \rightarrow 0} Z(\omega) = [2\sigma_0 + (\sigma - \sigma_0)\beta]C(\infty, t_0)$  taking into account  $\gamma \approx 10^{-3} \div 10^{-2}$ , from (23) and (24) we have

$$K_v(\omega, \rho, \gamma) = \frac{2\sigma_0 + (\sigma - \sigma_0)\beta}{2\sigma_0(1 - \beta) + \sigma\beta \left( 1 - e^{\frac{-\gamma\pi}{\omega}} \right)}. \quad (25)$$

Evidently from (25) follows

$$\lim_{\omega \rightarrow 0} K_v(\omega, \rho, \gamma) = 1. \quad (26)$$

Finally, since  $\sigma_0/\sigma = \frac{1-\rho}{1+\rho}$  according to (25)

$$K_v(\omega, \rho, \gamma) = \frac{1 + \frac{\rho\beta}{1-\rho}}{1 - \beta + \frac{(1+\rho)}{1-\rho} \beta \left( 1 - e^{\frac{-\gamma\pi}{\omega}} \right)}. \quad (27)$$

### 3. Conclusions

1. In the considered problem the application of notion energy entirety allows to reduce equation relatively vibrocreep factor more plus simply that by previous approaches.

2. The proposed relations can be used also for study the vibrocreep of no uniform stressed concrete.
3. The vibrocreep factor permits to define more exactly the rigidity of normal cross-section.

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