Mathematical problems of reliability assurance the building constructions

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Abstract. The paper deals with a mathematical model of console type based on the nonlinear differential equation having a mobile feature of the General solution (or a mobile singular point). The presence of mobile singular points indicates affiliation of this type of equations to the class of intractable in the general case in of quadratures. This fact, taking into account the interpretation of mobile singular point as the coordinate of structural failure, actualizes the development of an analytical approximate method for solving nonlinear differential equations. Taking into account these features for of structural analysis increases the authenticity of results and reliability of construction.

1. Introduction

To solve the problems of reliability assessment and prediction of survivability of the building structure, it is necessary to have a mathematical model, which is represented by analytical expressions. Mathematical models allow us to evaluate the characteristics of errors in programs and predict the reliability of structures in the design and operation. Confirmation of great attention to mathematical modelling are publications [1-18] in the study of technical processes, in solving economic problems [19-20], solving problems in medicine [21, 22].

2. Representation of model for the solution

For the structural analysis of the console type design with possible effects on it, a mathematical model based on a nonlinear differential equation is considered:

$$y'' = \frac{M_x}{EJ_x}\sqrt{(1+(y')^2)^3} + F_x \tag{1}$$

here M_x – bending moment; EJ_x – stiffness uniform beam; F_x – external action.

Equation (1) allows for structure simplification by replacing variables $y' = Y$, $M_x/(EJ_x) = \psi(x)$

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$$Y' = \Psi(x)\sqrt{(1+Y^2)^3} + F(x) \tag{2}$$

The domain of existence of the solution of equation (2) consists of the domain of analyticity and the neighborhood of mobile singular points. If the theorem of existence and uniqueness of the solution is proved in one of the papers for the domain of analyticity, the proof of the theorem of existence and uniqueness of the solution in the vicinity of mobile singular points is given in this paper.

3. Task solution

In the present work one of the main problems of the analytical approximate method is solved – proof of the existence and uniqueness of the solution in the vicinity of mobile singular point. The presented theorem has no analogues in the classical literature and allows to obtain the solution of the subsequent problems of the analytical approximate method. In the given proof the author's technology is used [14].

Consider the Cauchy problem for equation (2) with an initial condition.

$$Y(x_0) = Y_0 \tag{3}$$

3.1. Theorem

Let the following conditions be true:

1) x^* – mobile singular point of the Cauchy problem (2) – (3);

2) functions $\Psi(x)$, $F(x) \in C^\infty$ in the area $\left|x - x^*\right| < \rho_1$, $\rho_1 = \text{const} \neq 0$;

3) there are $M_{1,n}$ and $M_{2,n}$, for which conditions are met

$$\left|\frac{\psi^{(n)}(x^*)}{n!}\right| \le M_{1,n} \qquad \left|\frac{F^{(n)}(x^*)}{n!}\right| \le M_{2,n} \qquad \forall\, n = 0, 1, 2, \ldots$$

where is $M_{1,n}$ and $M_{2,n}$ are some constants.

Then there is a unique solution of the problem (2) – (3) in the form

$$Y(x) = (x - x^*)^{-1/2} \sum_0^\infty C_n (x - x^*)^{n/2} \tag{4}$$

in the area $\left|x - x^*\right| < \rho_2$, wherein

$$\rho_2 = \min\left\{\rho_1, \frac{1}{9(M+1)^2}\right\} \qquad M = \max\{\left|Y_0\right|, M_{1,n}, M_{2,n}\} \qquad n = 0, 1, \ldots$$

3.2 The proof

According to the theorem we have

$$\psi(x) = \sum_0^\infty D_n (x - x^*)^n$$

$$F(x) = \sum_0^\infty A_n (x - x^*)^n \tag{5}$$

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because x^* is a regular point for $\psi(x)$ and $F(x)$. Consider structure for the function $Y(x)$.

$$Y(x) = (x - x^*)^{\rho} \sum_0^{\infty} C_n (x - x^*)^n \tag{6}$$

Here we introduce the notation:

$$Y^2(x) = \sum_0^{\infty} C_n^* (x - x^*) \quad 1 + Y^2 = \sum_0^{\infty} C_{1,n}^{**} (x - x^*)^{n+2\rho}$$

$$C_{1,n}^{**} = C_n^* \qquad \forall n = 0, 1, 2, \ldots \qquad (1 + y^2)^2 = \sum_0^{\infty} C_{2,n}^{**} (x - x^*)^{n+4\rho}$$

$$C_{1,2}^{**} = 1 + (2C_2C_0 + C_1^2) \quad C_n^* = \sum_{i=0}^{n} C_{n-i} C_i \quad C_{2,n}^{**} = \sum_{i=0}^{n} C_{1,i}^{**}$$

$$(1 + y^2)^3 = \sum_0^{\infty} C_{3,n}^{**} (x - x^*)^{n+6\rho} \qquad \sqrt{(1 + Y^2)^3} = \sum_0^{\infty} B_n (x - x^*)^{n+3\rho}$$

$$(1 + Y^2)^3 = \sum_0^{\infty} B_n^* (x - x^*)^{n+6\rho} \qquad B_n^* = \sum_{i=0}^{n} B_{n-i} B_i$$

Substitute (5) and (6) in equation (2). Taking into account the designations we obtain:

$$\sum_0^{\infty} C_n (n + \rho)(x - x^*)^{n+\rho-1} = \sum_0^{\infty} D_n (x - x^*)^n \sum_0^{\infty} B_n (x - x^*)^{n+3\rho} + \sum_0^{\infty} A_n (x - x^*)^n \tag{7}$$

From (7) follows

$$n + \rho - 1 = n + 3\rho \quad \Rightarrow \quad \rho = -\frac{1}{2}$$

Then the structure of the series in (6) takes the form (4) and from (7) we obtain recurrence relations for the coefficients:

$$C_n \left(\frac{n-1}{2} \right) = \sum_{i=0}^{n} B_{n-i} D_i + A_{n-3} \text{ for } n = 3, 5, 7, \ldots \tag{8}$$

Herewith

$$C_n \left(\frac{n-1}{2} \right) = \sum_{i=0}^{n} B_{n-i} D_i \text{ for } n = 0, 1, 2, 4, 6, \ldots$$

From the obtained recurrence relations, we have it $C_0 = \pm\sqrt{1/(2D_0)}$, then $B_0 = -C_0^3$. Consider the option for $C_0 = \sqrt{1/(2D_0)}$. We get

$$C_1 = 0 \;\; B_1 = 0 \, , \;\; C_2 = -\frac{1}{\sqrt{2}} \frac{D_1}{D_0^{3/2}} \;\;\; B_2 = 0 \, , \;\; C_3 = \frac{A_0}{1 + 3C_0^2} \;\;\; B_3 = -\frac{3C_3}{4D_0^2}$$

Expressions of subsequent coefficients allow us to hypothesize estimates for C_n and B_n :

$$\left| C_n \right| \le 3^{n-3} M (M+1)^{n-2} \tag{9}$$

$$\left| B_n \right| \le 3^{n-2} M (M+1)^{n-2}$$

We prove the validity of the estimates in the case of $n = 2k+1$. From the recurrence relation (8) we obtain

$$C_{2k+3} \left(\frac{2k+2}{2} \right) = \sum_{i=0}^{2k+3} B_{2k+3-i} D_i + A_k$$

or

$$C_{2k+3}(k+1) = B_{2k+3} D_0 + \sum_{i=1}^{2k+2} B_{2k+3-i} D_i + A_k$$

Considering that $B_1 = B_2 = 0$, will have been

$$C_{2k+3}(k+1) = B_{2k+3} D_0 + \sum_{i=3}^{2k} B_{2k+3-i} D_i + A_k \tag{10}$$

From the designations entered above

$$B_{2k+3}^{*} = \sum_{i=0}^{2k+3} B_{2k+3-i} B_i$$

In turn to $B_{2k+3}^{*} = C_{3,2k+3}^{**}$, whereupon

$$B_{2k+3} B_0 + \sum_{i=1}^{2k+2} B_{2k+3-i} B_i = C_{3,2k+3}^{**}$$

or, given $B_1 = B_2 = 0$,

$$B_{2k+3} = \frac{1}{B_0} \left(C_{3,2k+3}^{**} - \sum_{i=3}^{2k} B_{2k+3-i} B_i \right) = \frac{1}{B_0} \left(\sum_{i=0}^{2k+3} C_{2,2k+3-i}^{**} C_{1,i}^{**} - \sum_{i=3}^{2k} B_{2k+3-i} B_i \right) =$$

$$= \frac{1}{B_0} \left(C_{2,2k+3}^{**} C_{1,0}^{**} + \sum_{i=1}^{2k+2} C_{2,2k+3-i}^{**} C_{1,i}^{**} - \sum_{i=3}^{2k} B_{2k+3-i} B_i \right) =$$

$$= \frac{1}{B_0} \left(\left(\sum_{i=0}^{2k+3} C_{1,2k+3-i}^{**} C_{1,i}^{**} \right) C_{1,0}^{**} + \sum_{i=1}^{2k+2} C_{2,2k+3-i}^{**} C_{1,i}^{**} - \sum_{i=3}^{2k} B_{2k+3-i} B_i \right) =$$

$$=\frac{1}{B_0}\left(C^{**}_{1,2k+3}(C^{**}_{1,0})^2+\left(\sum_{1}^{2k+2}C^{**}_{1,2k+3-i}C^{**}_{1,i}\right)C^{**}_{1,0}+\sum_{i=1}^{2k+2}C^{**}_{2,2k+3-i}C^{**}_{1,i}-\sum_{i=3}^{2k}B_{2k+3-i}B_i\right)$$

Continuing the process of transition to one index for the coefficients and getting rid of the asterisks in the notation, we obtain

$$B_{2k+3}=\frac{1}{B_0}\left(C_{2k+3}C_0^5+\left(\sum_{l=2}^{2k+1}C_{2k+3-l}C_l\right)C_0^4+\sum_{i=1}^{2k+2}\left(\sum_{l=0}^{2k+3-i}C_{2k+3-i-l}C_l\right)\left(\sum_{l=0}^{i}C_{i-l}C_l\right)C_0^4+\right.$$

$$\left.+\sum_{i=1}^{2k+2}\left(\sum_{j=0}^{2k+3-i}\left(\sum_{l=0}^{2k+3-i-j}C_{2k+3-i-j-l}C_l\right)\left(\sum_{l=0}^{j}C_{j-l}C_l\right)\left(\sum_{l=0}^{i}C_{i-l}C_l\right)\right)-\sum_{i=3}^{2k}B_{2k+3-i}B_i\right) \tag{11}$$

Substitute the resulting expression for B_{2k+3} in (10) and, performing the identity transformations, we obtain

$$C_{2k+3}=\frac{1}{(k+1)+C_0^2}\cdot\frac{1}{B_0}\left(\left(\sum_{l=2}^{2k+1}C_{2k+3-l}C_l\right)C_0^4+\sum_{i=1}^{2k+2}\left(\sum_{l=0}^{2k+3-i}C_{2k+3-i-l}C_l\right)\left(\sum_{l=0}^{i}C_{i-l}C_l\right)C_0^4+\right.$$

$$\left.+\sum_{i=1}^{2k+2}\left(\sum_{j=0}^{2k+3-i}\left(\sum_{l=0}^{2k+3-i-j}C_{2k+3-i-j-l}C_l\right)\left(\sum_{l=0}^{j}C_{j-l}C_l\right)\left(\sum_{l=0}^{i}C_{i-l}C_l\right)\right)-\sum_{i=3}^{2k}B_{2k+3-i}B_i\right)$$

After substitution in the last ratio of the estimates for the coefficients C_n and B_n, the execute of identical transformations, we obtain

$$\left|C_{2k+3}\right|\le\frac{1}{k+1}\left(3^{2k-4}(k+1)(M+1)^{2k+3}+3^{2k-7}M(M+1)^{2k}+\right.$$

$$+3^{2k-10}M(M+1)^{2k-9}\sum_{i=1}^{2k+2}(i+1)(2k+3-i)\left(\frac{2k+3-i}{2}\right)+$$

$$\left.+3^{2k-4}M(M+1)^{2k-4}\right)\le 3^{2k}M(M+1)^{2k+1}$$

We prove the estimate for B_{2k+3}. From (11) we obtain

$$\left|B_{2k+3}\right|\le\left|C_{2k+3}C_0^2+\left(\sum_{l=2}^{2k+1}C_{2k+3-l}C_l\right)C_0+\sum_{i=2}^{2k+1}\left(\sum_{l=0}^{2k+3-i}C_{2k+3-i-l}C_l\right)\left(\sum_{l=0}^{i}C_{i-l}C_l\right)C_0+\right.$$

$$\left.+\frac{1}{C_0^3}\sum_{i=2}^{2k+1}\left(\sum_{j=0}^{2k+3-i}\left(\sum_{l=0}^{2k+3-i-j}C_{2k+3-i-j-l}C_l\right)\left(\sum_{l=0}^{j}C_{j-l}C_l\right)\right)\left(\sum_{l=0}^{i}C_{i-l}C_l\right)-\frac{1}{C_0^3}\sum_{i=3}^{2k}B_{2k+3-i}B_i\right|$$

Taking into account the expressions for C_0, C_1, B_1, B_2 and estimates for coefficients C_n, we obtain from last after the transformations:

$$\left|B_{2k+3}\right|\le 3^{2k}M(M+1)^{2k+1}\left(1+\frac{1}{3^5(M+1)^5}+\frac{1}{3^{13}(M+1)^6}+\right.$$

$$+\frac{1}{3^{12}(M+1)^{3}}+\frac{1}{3^{3}(M+1)^{4}}\Bigg)\le 3^{2k+1}M(M+1)^{2k+1}$$

Analogous make sure of the estimates and in the event. Taking into account the estimates (9) for the coefficients, on the basis of a sufficient sign of convergence of the power series, we obtain the convergence region for the correct part of the series (4):

$$\left|x-x^{*}\right|<\frac{1}{9(M+1)^{2}}$$

which completes the proof.

As a result, when nonlinear differential equations are used in calculations, the theorem guarantees the existence and uniqueness of the solution in the vicinity of mobile singular point, as well as, implicitly way, the existence of the most a mobile singular point.

3. Conclusion

The result obtained in this paper is necessary for the development of the mathematical theory of finding a mobile singular point with a given accuracy and the development of software for computer calculations. The technology of theorem proving allows to obtain an analytical approximate solution in the vicinity of a mobile singular point, i.e. to determine the longitudinal coordinate of the local fracture site of a cantilever-type structure under quasi-static load.

Thus, taking into account the peculiarities of solving nonlinear differential equations for of structural analysis increases the authenticity of results and reliability of design and, as a consequence, allows you to get results that are close to the real behavior of the structure.

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