Dynamic calculation of nonlinear oscillations of viscoelastic orthotropic plate with a concentrated mass

Dadakhan Khodzhaev[1,*]

[1]Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39 Kari Niyazov Kochashi, Tashkent, 100000, Uzbekistan

Abstract. Plates, panels and shells made of composite material with fixed objects in the form of an additional mass have found a wide use due to their viscoelastic and strength properties. An analysis of their dynamic behavior indicates a significant effect of inhomogeneity of an associated mass type on their strength. The problem of oscillations of a viscoelastic orthotropic rectangular plate with an associated mass is considered according to the Kirchhoff-Love hypothesis in a geometrically nonlinear statement. This problem is reduced to solving the systems of nonlinear integro-differential equations with singular relaxation kernels, solved by the Bubnov-Galerkin method in combination with a numerical method based on the use of quadrature formulas. The numerical values of the approximate solution have been calculated in the Delphi programming environment. At wide range of changes in physicomechanical and geometrical parameters, the behavior of the plate has been studied. The effect of viscoelastic and inhomogeneous material properties, concentrated mass and their location on the oscillatory process of a rectangular plate is shown.

1 Introduction

Intensive development of modern industry has led to a decrease in the consumption of material for structures and machines. In manufacturing the structures with such properties as the lightness, durability and reliability, the most acceptable is the use of composite materials, which allow not only significantly improve the performance characteristics, but in some cases to create the structures that could not be implemented with the use of traditional materials. At the same time, the procedure for calculating and designing structures made of composite materials is rather complicated; it requires the consideration of their real properties. Therefore, the problems of strain, dynamic stability and oscillations of thin-walled structures made of composite materials are of great interest.

Thin-walled structures such as plates, panels and shells often play the role of a bearing surface, to which certain elements of the structure are fixed. Such elements are pads,

[*] Corresponding author: dhodjaev@mail.ru

fasteners and instrument units; in theoretical consideration of the problems, they are interpreted as an additional mass rigidly fixed to the systems and concentrated in the points.

The problems of oscillations and dynamic stability of elastic plates and cylindrical panels with a concentrated mass in various statements have been considered in [1-5]. There the problems have been solved either in a linear statement or only individual properties of structure material have been taken into account.

The study of nonlinear oscillations and dynamic stability of orthotropic plates and cylindrical panels with no account of a concentrated mass in various statements is given in [6-11].

In [12] the natural oscillations of a rectangular plate, two adjacent edges of which are clamped, and the other two are free (CCFF-plate) are studied. The deflection function is chosen as a sum of two hyperbolic trigonometric series. The first eight natural frequencies are founded. The paper provides accuracy analysis and its comparison with other familiar results.

Nonlinear oscillations of a viscoelastic cylindrical panel with a concentrated mass are considered in [13].

The aim of this work is to study the nonlinear oscillations of viscoelastic orthotropic rectangular plates with a concentrated mass.

2 Statement of the problem

Consider a viscoelastic orthotropic shell of a thickness h carrying a concentrated mass M_i at points with coordinates (x_i, y_i), $i=1,2,\dots,I$.

Let us construct a mathematical model of the problem of nonlinear oscillations of a viscoelastic orthotropic shell with a concentrated mass in a geometrically nonlinear statement.

Physical dependence between stresses $\sigma_x, \sigma_y, \tau_{xy}$ and strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ is taken in the following form [14, 15]:

$$\sigma_x = B_{11}\left(1-R_{11}^*\right)\varepsilon_x + B_{12}\left(1-R_{12}^*\right)\varepsilon_y, \quad (x \leftrightarrow y, 1 \leftrightarrow 2), \quad \tau_{xy} = 2B\left(1-R^*\right)\gamma_{xy}, \quad (1)$$

where R^*, R_{ij}^* – are the integral operators with relaxation kernels $R(t)$ and $R_{ij}(t)$, respectively:

$$R^*\varphi = \int_0^t R(t-\tau)\varphi(\tau)d\tau, \quad R_{ij}^*\varphi = \int_0^t R_{ij}(t-\tau)\varphi(\tau)d\tau, \quad i,j=1,2,$$

$$B_{11} = \frac{E_1}{1-\mu_1\mu_2}, \quad B_{22} = \frac{E_2}{1-\mu_1\mu_2}, \quad B_{12} = B_{21} = \mu_1 B_{22} = \mu_2 B_{11}, \quad B = \frac{G}{2}.$$

Here E_1, E_2 – are the elasticity modulus in x- and y-axes directions; G is the shear modulus; μ_1, μ_2 – are the Poisson's ratios; here and hereinafter, the symbols $(x \leftrightarrow y)$, $(1 \leftrightarrow 2)$ indicate that the remaining relations are obtained by circular substitution of indices.

The relationship between the strains in the middle surface $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ and displacements u, v, w in x, y, z directions, with account of initial imperfections, is taken in the form [15]:

$$\varepsilon_x = \frac{\partial u}{\partial x} - k_x\left(w - w_0\right) + \frac{1}{2}\left[\left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial w_0}{\partial x}\right)^2\right], \tag{2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - k_y\left(w - w_0\right) + \frac{1}{2}\left[\left(\frac{\partial w}{\partial y}\right)^2 - \left(\frac{\partial w_0}{\partial y}\right)^2\right], \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y},$$

where $w_0 = w_0(x, y)$ is the initial deflection of the shell, $k_x, k_y = const$ is the curvature of the middle surface of the shell.

The bending M_x, M_y and torsion H moments of the shell per unit length of the element edges, with account of (1), have the form [15]:

$$M_x = -\frac{h^3}{12}\left[B_{11}\left(1 - R_{11}^*\right)\frac{\partial^2(w - w_0)}{\partial x^2} + B_{12}\left(1 - R_{12}^*\right)\frac{\partial^2(w - w_0)}{\partial y^2}\right], \; (x \leftrightarrow y, 1 \leftrightarrow 2),$$

$$H = -\frac{Bh^3}{3}\left(1 - R^*\right)\frac{\partial^2(w - w_0)}{\partial x \partial y}. \tag{3}$$

When deriving the equation of motion of an element of a viscoelastic orthotropic shell with a concentrated mass, we will proceed from equation [15]:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \frac{m}{h}\frac{\partial^2 u}{\partial t^2} = 0, \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} - \frac{m}{h}\frac{\partial^2 v}{\partial t^2} = 0,$$

$$\frac{q}{h} + \frac{1}{h}\left(\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2}\right) + k_x\sigma_x + k_y\sigma_y + \frac{\partial}{\partial x}\left(\sigma_x\frac{\partial w}{\partial x} + \tau_{xy}\frac{\partial w}{\partial y}\right) +$$

$$+ \frac{\partial}{\partial y}\left(\sigma_y\frac{\partial w}{\partial y} + \tau_{xy}\frac{\partial w}{\partial x}\right) - \frac{m}{h}\frac{\partial^2 w}{\partial t^2} = 0. \tag{4}$$

The effect of the concentrated mass on the viscoelastic shell is inertial in nature and is taken into account in the equation of motion (4) with the Dirac δ-function [1]:

$$m(x, y) = \rho h + \sum_{i=1}^{I} M_i \delta(x - x_i)\delta(y - y_i), \tag{5}$$

where ρ is the density of the shell material.

Taking into account (5), substituting (1) and (3) into (4), the following system of integro-differential equations in partial derivatives is obtained:

$$B_{11}\left(1 - R_{11}^*\right)\frac{\partial \varepsilon_x}{\partial x} + B_{12}\left(1 - R_{12}^*\right)\frac{\partial \varepsilon_y}{\partial x} + 2B\left(1 - R^*\right)\frac{\partial \gamma_{xy}}{\partial y} - \left[\rho + \frac{1}{h}\sum_{i=1}^{I} M_i \delta(x - x_i)\delta(y - y_i)\right]\frac{\partial^2 u}{\partial t^2} = 0,$$

$$B_{22}\left(1 - R_{22}^*\right)\frac{\partial \varepsilon_y}{\partial y} + B_{21}\left(1 - R_{21}^*\right)\frac{\partial \varepsilon_x}{\partial y} + 2B\left(1 - R^*\right)\frac{\partial \gamma_{xy}}{\partial x} - \left[\rho + \frac{1}{h}\sum_{i=1}^{I} M_i \delta(x - x_i)\delta(y - y_i)\right]\frac{\partial^2 v}{\partial t^2} = 0,$$

$$\frac{h^2}{12}\left\{B_{11}\left(1-R_{11}^*\right)\frac{\partial^4(w-w_0)}{\partial x^4}+\left[8B\left(1-R^*\right)+B_{12}\left(1-R_{12}^*\right)+B_{21}\left(1-R_{21}^*\right)\right]\frac{\partial^4(w-w_0)}{\partial x^2\partial y^2}+\right. \quad (6)$$

$$\left.+B_{22}\left(1-R_{22}^*\right)\frac{\partial^4(w-w_0)}{\partial y^4}\right\}-k_x\left[B_{11}\left(1-R_{11}^*\right)\varepsilon_x+B_{12}\left(1-R_{12}^*\right)\varepsilon_y\right]-k_y\left[B_{22}\left(1-R_{22}^*\right)\varepsilon_y+B_{21}\left(1-R_{21}^*\right)\varepsilon_x\right]-$$

$$-\frac{\partial}{\partial x}\left\{\frac{\partial w}{\partial x}\left[B_{11}\left(1-R_{11}^*\right)\varepsilon_x+B_{12}\left(1-R_{12}^*\right)\varepsilon_y\right]+2B\frac{\partial w}{\partial y}\left(1-R^*\right)\gamma_{xy}\right\}-$$

$$-\frac{\partial}{\partial y}\left\{\frac{\partial w}{\partial y}\left[B_{22}\left(1-R_{22}^*\right)\varepsilon_y+B_{21}\left(1-R_{21}^*\right)\varepsilon_x\right]+2B\frac{\partial w}{\partial x}\left(1-R^*\right)\gamma_{xy}\right\}-\frac{q}{h}+$$

$$+\left[\rho+\frac{1}{h}\sum_{i=1}^{I}M_i\delta(x-x_i)\delta(y-y_i)\right]\frac{\partial^2 w}{\partial t^2}=0,$$

where $\varepsilon_x,\varepsilon_y$ and γ_{xy} are determined from relationship (2).

Mathematical models obtained using the system (6) with the corresponding boundary and initial conditions take into account the viscoelastic properties and the inhomogeneity of the shell material. This system of integro-differential equations in partial derivatives with five different relaxation kernels is quite general. In a particular case, it is possible to obtain from this system various equations by various theories.

At $k_x=k_y=1/R$, the equations for the viscoelastic orthotropic spherical shell are obtained; at $k_x=0, k_y=1/R$ for the viscoelastic orthotropic circular cylindrical shell and at $k_x=k_y=0$ for the viscoelastic orthotropic plate.

3 The method of solution

Consider the case of nonlinear oscillations of a viscoelastic orthotropic rectangular plate with a concentrated mass hingedly supported at the edges. In this case, mathematical model of the problem is described by the system of equations (6) at $k_x=k_y=0$, the solution of which satisfying the boundary conditions of the problem by the Bubnov-Galerkin method, is sought in the form [16-20]:

$$w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$

$$w_0(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{0nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}, \quad (7)$$

$$u(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}u_{nm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$

$$v(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}v_{nm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b},$$

where $w_{nm}=w_{nm}(t)$, $u_{nm}=u_{nm}(t)$ and $v_{nm}=v_{nm}(t)$ are the unknown functions of time.

Substituting (7) into the system of equations (6) and performing the Bubnov-Galerkin procedure relative to the unknowns w_{nm}, u_{nm} and v_{nm},, a system of nonlinear integro-differential equations is obtained.

Introducing the following dimensionless value into the resulting system we get

$$\frac{w_{kl}}{h}, \frac{w_{0kl}}{h}, \frac{u_{kl}}{h}, \frac{v_{kl}}{h}, \omega t, \frac{qb^4}{\sqrt{E_1E_2}h^4}, \quad \frac{R(t)}{\omega}, \frac{R_{ij}(t)}{\omega}, i, j=1,2$$

and keeping the same notations, a system of nonlinear integro-differential equations is obtained relative to dimensionless values u_{kl}, v_{kl} and w_{kl} :

$$\sum_{n=1}^{N}\sum_{m=1}^{M}A_{klnm}\ddot{u}_{kl}+\left[\frac{6\Delta\delta^2k^2}{\pi^2\lambda^2\eta}\left(1-R_{11}^*\right)+\frac{6g\delta^2l^2(1-\mu_1\mu_2)}{\pi^2\eta}\left(1-R^*\right)\right]u_{kl}=$$

$$=-\frac{6\delta^2kl}{\pi^2\lambda\eta}\left[\Delta\mu_2\left(1-R_{12}^*\right)+g(1-\mu_1\mu_2)\left(1-R^*\right)\right]v_{kl}+$$

$$+\frac{6\Delta\delta}{\pi^3\lambda^3\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}ni^2\mu_{nik}\beta_{mjl}\left(1-R_{11}^*\right)\left(w_{nm}w_{ij}-w_{0nm}w_{0ij}\right)+$$

$$+\frac{6\Delta\delta\mu_2}{\pi^3\lambda\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}mij\gamma_{nik}\alpha_{mjl}\left(1-R_{12}^*\right)\left(w_{nm}w_{ij}-w_{0nm}w_{0ij}\right)+$$

$$+\frac{6g\delta(1-\mu_1\mu_2)}{\pi^3\lambda\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}\left(mij\gamma_{nik}\alpha_{mjl}+nj^2\mu_{nik}\beta_{mjl}\right)\left(1-R^*\right)\left(w_{nm}w_{ij}-w_{0nm}w_{0ij}\right),$$

$$\sum_{n=1}^{N}\sum_{m=1}^{M}B_{klnm}\ddot{v}_{kl}+\left[\frac{6\delta^2l^2}{\pi^2\Delta\eta}\left(1-R_{22}^*\right)+\frac{6g\delta^2k^2(1-\mu_1\mu_2)}{\pi^2\lambda^2\eta}\left(1-R^*\right)\right]v_{kl}=$$

$$=-\frac{6\delta^2kl}{\pi^2\Delta\lambda\eta}\left[\mu_1\left(1-R_{21}^*\right)+\Delta g(1-\mu_1\mu_2)\left(1-R^*\right)\right]u_{kl}+$$

$$+\frac{6\delta}{\Delta\pi^3\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}mj^2\beta_{nik}\mu_{mjl}\left(1-R_{22}^*\right)\left(w_{nm}w_{ij}-w_{0nm}w_{0ij}\right)+$$

$$+\frac{6\delta\mu_1}{\Delta\pi^3\lambda^2\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}nij\alpha_{nik}\gamma_{mjl}\left(1-R_{21}^*\right)\left(w_{nm}w_{ij}-w_{0nm}w_{0ij}\right)+$$

$$+\frac{6g\delta(1-\mu_1\mu_2)}{\pi^3\lambda^2\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}\left(nij\alpha_{nik}\gamma_{mjl}+n^2j\beta_{nik}\gamma_{mjl}\right)\left(1-R^*\right)\left(w_{nm}w_{ij}-w_{0nm}w_{0ij}\right),$$

$$\sum_{n=1}^{N}\sum_{m=1}^{M}C_{klnm}\ddot{w}_{kl}+\left[\frac{\Delta k^4}{2\lambda^4\eta}\left(1-R_{11}^*\right)+\frac{2gk^2l^2(1-\mu_1\mu_2)}{\lambda^2\eta}\left(1-R^*\right)+\right.$$

$$\left.+\frac{\Delta\mu_2k^2l^2}{2\lambda^2\eta}\left(1-R_{12}^*\right)+\frac{\mu_1k^2l^2}{2\Delta\lambda^2\eta}\left(1-R_{21}^*\right)+\frac{l^4}{2\Delta\eta}\left(1-R_{22}^*\right)\right]\left(w_{kl}-w_{0kl}\right)=$$

$$= \sum_{n,i=1}^{N}\sum_{m,j=1}^{M} w_{nm}\left[\frac{6\Delta\delta}{\pi^3\lambda^3\eta} a_{nmijkl}\left(1-R_{11}^*\right)+\frac{6g\delta(1-\mu_1\mu_2)}{\pi^3\lambda\eta} c_{nmijkl}\left(1-R^*\right)+\right.$$

$$\left.+\frac{6\mu_1\delta}{\pi^3\lambda\Delta\eta} f_{nmijkl}\left(1-R_{21}^*\right)\right]u_{ij} + \sum_{n,i=1}^{N}\sum_{m,j=1}^{M} w_{nm}\left[\frac{6\Delta\delta\mu_2}{\pi^3\lambda^2\eta} b_{nmijkl}\left(1-R_{12}^*\right)+\frac{6g\delta(1-\mu_1\mu_2)}{\pi^3\lambda^2\eta} d_{nmijkl}\left(1-R^*\right)+\right.$$

$$\left.+\frac{6\delta}{\pi^3\Delta\eta} e_{nmijkl}\left(1-R_{22}^*\right)\right]v_{ij} - \sum_{n,i,r=1}^{N}\sum_{m,j,s=1}^{M} w_{nm}\left[\frac{3\Delta}{16\lambda^4\eta} h_{nmijklrs}\left(1-R_{11}^*\right)-\frac{3\Delta\mu_2}{16\lambda^2\eta} p_{nmijklrs}\left(1-R_{12}^*\right)+\right.$$

$$+\frac{3g(1-\mu_1\mu_2)}{8\lambda^2\eta} g_{nmijklrs}\left(1-R^*\right)+\frac{3}{16\Delta\eta} q_{nmijklrs}\left(1-R_{22}^*\right)-$$

$$\left.-\frac{3\mu_1}{16\Delta\lambda^2\eta} r_{nmijklrs}\left(1-R_{21}^*\right)\right]\left(w_{ij}w_{rs}-w_{oij}w_{0rs}\right)+\frac{96\alpha_{kl}(1-\mu_1\mu_2)}{\pi^6\eta kl} q.$$

The resulting system of integro-differential equations is solved by a numerical method proposed in [8] and based on the use of quadrature formulas. The Koltunov-Rzhanitsyn singular kernels with three rheological parameters $(A, \beta \text{ and } \alpha)$ [14] are used as the relaxation kernels:

$$\Gamma(t) = Ae^{-\beta t} t^{\alpha-1}, (0 < \alpha < 1)$$

4 Results and analysis

The convergence of the Bubnov-Galerkin method is investigated in the paper. In calculating the deflection values, the 5 first harmonics are held (N=5, M=1). Calculations have shown that further increase in the number of terms does not have a significant effect on the oscillations amplitude of a viscoelastic orthotropic rectangular plate.

Figure 1 shows the dependence of the deflection in the center of elastic (curve 1) and viscoelastic plates (curves 2, 3) on time.

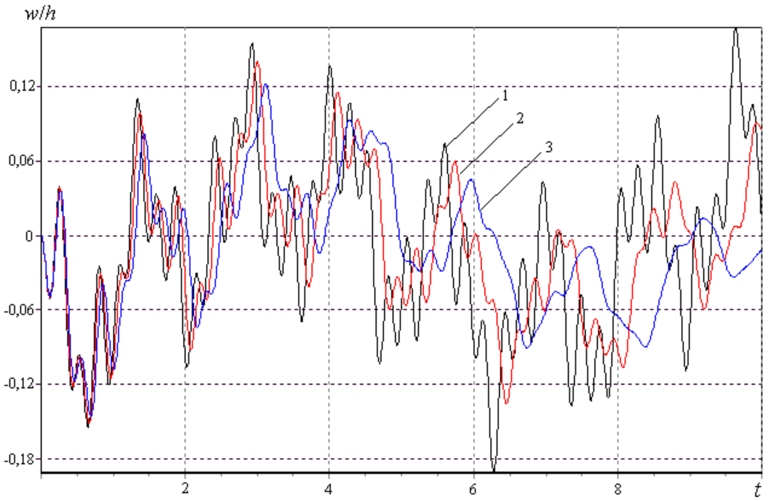

Fig. 1. The dependence of the deflection on time, A=0 (1); 0.05 (2); 0.1 (3).

It is seen that an account of viscoelastic properties of the plate material leads to the attenuation of the oscillatory process. In the initial period, the solutions of elastic and viscoelastic problems differ little, but over time, the viscoelastic properties begin to have a significant effect.

The influence of a concentrated mass in the center of the plate on the oscillatory process is shown in Figure 2. It is seen that an increase in a concentrated mass leads to a decrease in the amplitude of oscillations. It should be noted that in the particular case, when there is no concentrated mass in the center of the plate (M_1=0), the results obtained coincide with the ones given in [11].

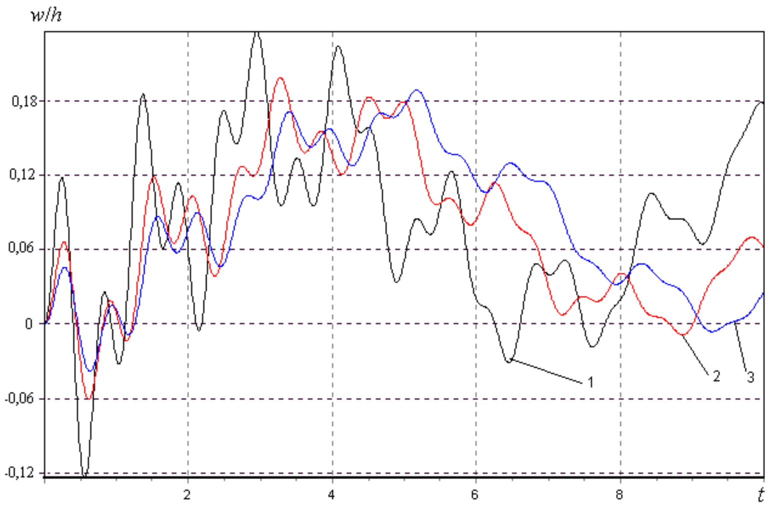

Fig. 2. The dependence of the deflection on time, M_1=0 (1); 0.1 (2); 0.2 (3).

The influence of geometric nonlinearity on the frequency and amplitude of oscillations of a viscoelastic plate has also been studied. Figure 3 shows the graph of function w for nonlinear (curves 1 - λ=1; q=1; w0=10-1; 2 - λ=1; q=3; w0=10-4 and 3 - λ=3; q=1; w0=10-4) and linear cases (curves 4 - λ=1; q=1; w0=10-1; 5 - λ=1; q=3; w0=10-4 and 6 - λ=3; q=1; w0=10-4).

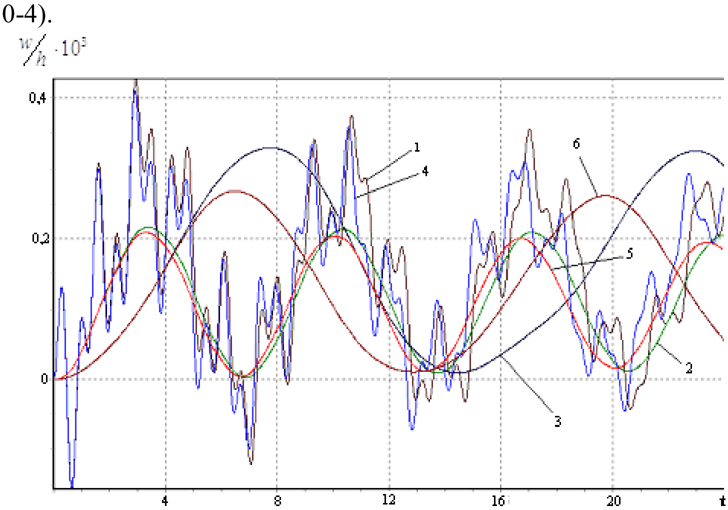

Fig. 3. The dependence of the deflection on time.

Studies have shown that in the absence of initial imperfections and external loads, the results of calculations obtained in cases of linear and nonlinear problems coincide. In this case, the problems can be solved in a linear statement. However, with an increase in the ratio between geometrical parameters of the plate λ with external loads q and initial imperfections w0, the differences in the obtained results are observed. In this case, the problem must be solved in a nonlinear statement.

5 Conclusions

Nonlinear oscillations of viscoelastic orthotropic rectangular plates with a concentrated mass have been studied in the paper.

The calculations have shown that:

- in cases of viscoelastic plates, an increase in a concentrated mass leads to a more intensive decrease in the amplitude of oscillations as compared to elastic case;
- in both elastic and viscoelastic cases, as a concentrated mass moves away from the center of the plate, the oscillation frequency increases;
- depending on the values of geometrical and physical parameters of the plates, it is necessary in calculations to choose the appropriate theory (linear or nonlinear one).

References

1. C.L. Amba-Rao, J.Appl.Mech. **31**, 550-551 (1964)
2. P.D. Cha, Journal of Sound and Vibration **207**, 593-596 (1997)
3. R. Chen, J.Appl.Mech. (Trans. ASME, Ser. E) **3(37)**, 874-875 (1979)
4. P.A. Bondarev, Ukrainian Mathematical Journal **1(26)**, 61-68 (1974)
5. D. Hui, Journal of Applied Mechanics **51**, 383-390 (1984)
6. M.M. Mirsaidov, R.A. Abdikarimov, N.I. Vatin, V.M. Zhgutov, D.A. Khodzhaev, B.A. Normuminov, Magazine of Civil Engineering **6(82)**, 112–126 (2018) doi: 10.18720/MCE.82.11
7. R. Abdikarimov, D. Khodzhaev, N. Vatin, MATEC Web of Conferences **245**, 01003 (2018) doi: 10.1051/matecconf/201824501003
8. A.F. Verlan, R.A. Abdikarimov, Kh. Eshmatov, Electronic modeling **32(2)**, 3-14 (2010)
9. R.A. Abdikarimov, D.A. Khodzhaev, Magazine of Civil Engineering **5**, 83-94 (2014) doi: 10.5862/MCE.49.9
10. R.A. Abdikarimov, V.M. Zhgutov, Engineering and Construction Journal **6(16)**, 38-47 (2010)
11. R.A. Abdikarimov, B.A. Khudayarov, International Applied Mechanics **50(4)**, 389-398 (2014) doi:10.1007/s10778-014-0642-x
12. M.V. Sukhoterin, S.O. Baryshnikov, T.P. Knysh, R.A. Abdikarimov, Magazine of Civil Engineering **6**, 81-94 (2018) doi: 10.18720/MCE.82.8
13. D.A.Khodzhaev, R.A.Abdikarimov, N.I. Vatin, MATEC Web of Conferences **245**, 01001 (2018) doi: 10.1051/matecconf/201824501001
14. M.A. Koltunov, *Creep and relaxation* (Higher School, Moscow, 1976)
15. S.A. Ambartsumyan, *Theory of anisotropic shells* (Science, Moscow, 1961)

16. T. Sultanov, B. Yuldoshev, E. Toshmatov, J. Yarashov, R. Ergashev, M. Mirsaidov MATEC Web of Conferences **265**, 04015 (2019) doi: 10.1051/matecconf/201926504015
17. M.M. Mirsaidov, T.Z. Sultanov, R.A. Abdikarimov, A.N. Ishmatov, B.Sh. Yuldoshev, E.S. Toshmatov, D.P. Jurayev, Magazine of Civil Engineering **1**, 101–111 (2018) doi: 10.18720/MCE.77.9
18. M.M. Mirsaidov, R.A. Abdikarimov, N.I. Vatin, V.M. Zhgutov, D.A. Khodjaev, B.A. Normuminov, Magazine of Civil Engineering **6**, 112-126 (2018) doi: 10.18720/MCE.82.11
19. O.M. Smirnova, Procedia Engineering, **172,** 1039-1043 (2017)
20. O.M. Smirnova, Magazine of Civil Engineering, **6**, 12-22 (2016)