Analysis of a Fishery Model with two competing prey species in the presence of a predator species for Optimal Harvesting

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Abstract. A harvesting fishery model is proposed to analyze the effects of the presence of red devil fish population, as a predator in an ecosystem. In this paper, we consider an ecological model of three species by taking into account two competing species and presence of a predator (red devil), the third species, which incorporates the harvesting efforts of each fish species. The stability of the dynamical system is discussed and the existence of biological and bionomic equilibrium is examined. The optimal harvest policy is studied and the solution is derived in the equilibrium case applying Pontryagin's maximal principle. The simulation results is presented to simulate the dynamical behavior of the model and show that the optimal equilibrium solution is globally asymptotically stable. The results show that the optimal harvesting effort is obtained regarding to bionomic and biological equilibrium.

1 Introduction

Introducing red devil fish species in freshwater ecosystems has threatened the presence of commercial fish (such as, tilapia and goldfish) in the habitats. In the freshwater ecosystems, red devil (*Amphilophus Labiatus*) growth highly and abundantly. Red devil is as one of the major threats to global biodiversity [1, 2]. The red devil species introduced in the ecosystems through spreading the seeds of the commercial fish. The red devil species is from Nicaraguan lakes. The presence of a red devil fish in the ecosystems has become the leading threat of the survival of other fish in the habitats. Red devil has preyed on other commercial fish, such as carp and tilapia. Red devil fish is a type of predator that interferes the survival of other fish in which as the predator, red devil fish has low commercial value.

A number of the investigations regarding to fisheries resources have been conducted. The fishery model of two competing species was addressed by Clark *et al.* to discuss the aspect bionomics and the policy of optimal harvesting policy [2]. The combined harvesting of two competing species from the aspect of bionomic was also studied and discussed for the optimal equilibrium policy [3-7].

Das *et al.* [8] and Chaudhuri *et al*. [9] proposed the model one prey – one predator with combined harvesting. They analysed the dynamical behaviour of the model and the optimal harvesting policy. They assumed that the growth of both prey and predator are formulated in the logistic terms. Das *et al*. [8] used the functional response of predator that it is limited by density of prey. Rao *et al*. [10] investigated analytically the dynamical behaviour of the model for three species with one prey and two competing predators. They also discussed the policy of optimal harvesting. .

Kar *et al*. [11] analyzed a harvesting model for two competing species taking into account the presence of a predator species which feeds on the two competing species, as the third species which was not harvested. They also analyzed bionomic equilibrium solution and the policy of optimal harvesting.

In this paper, we propose and analyze an ecological model of multispecies harvesting of two competing prey species and a predator species. We consider a combined harvest effort that imposes the exploitation of each species. The growth model of each species is modeled by logistics terms and we consider that the functional response of both competitor and predator of other species is assumed in bilinear term. The existence of the possible equilibrium states of the model and the stability of nontrivial equilibrium state by using a Lyapunov function is discussed. Next, we discuss the possibilities of the existence of a bionomic equilibrium. The optimal harvesting policy is studied and the optimal solution is derived in the nontrivial equilibrium case by using Pontryagin's maximum principle. Finally, some numerical examples are discussed.

2 Model formulation

We consider an aquatic ecosystem, there are two prey species of fish that competes each other for using a common resource. In the presence of a red devil fish, an invasive species (as predator) threat the survival of these prey species. Besides competing for the use of same resources, red devil fish also eats the small other fish. All

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these species are imposed continuously harvesting. We propose the logistic growth function for both two prey species and the predator (that is, the population of each species compete for the same resource), in addition to the third species increases due to in the presence of prey populations. The model equations for three species in which two competing prey species and a predator species with harvesting on all species is given by the following,

$$\begin{aligned}\frac{dx_1}{dt} &= r_1x_1\left(1-\frac{x_1}{K_1}\right)-\alpha_{12}x_1x_2-\alpha_{13}x_1x_3-q_1Ex_1\\ \frac{dx_2}{dt} &= r_2x_2\left(1-\frac{x_2}{K_2}\right)-\alpha_{21}x_1x_2-\alpha_{23}x_2x_3-q_2Ex_2 \quad (1)\\ \frac{dx_3}{dt} &= r_3x_3\left(1-\frac{x_3}{K_3}\right)+\alpha_{31}x_1x_2+\alpha_{32}x_2x_3-q_3Ex_3\end{aligned}$$

where $x_1(t), x_2(t)$ and $x_3(t)$ are the biomass densities of the first prey, the second prey and predator populations with the natural growth rates r_1, r_2 and r_3 respectively. The parameters K_1, K_2 and K_3 are the carrying capacities of the first prey, the second prey and predator populations, respectively. Parameter α_{12} is rate of decrease of the first prey due to the competition with the second prey, α_{13} is rate of decrease of the first prey due to inhibition by the predator, α_{21} is rate of decrease of the second prey due to the competition with the first prey, α_{23} is rate of decrease of the second prey due to inhibition by the predator, α_{31} is rate of increase of the predator due to successful attacks on the first prey, α_{32} is rate of increase of the predator due to successful attacks on the second prey, q_i for $i=1,2,3$ are catch ability coefficient of the first prey, the second prey and predator species respectively. E is the harvesting effort and $q_iEx_i, i=1,2,3$ are the catch-rate functions based on the catch-per-unit-effort hypothesis. In the analysis of the system, we assume that $r_i - q_iE > 0$ for $i=1,2,3$.

3 Model analysis

In the section, we analyze the existence and the stability of equilibria.

3.1 Existence of equilibrium

We determine the conditions for the existence of the equilibrium points of the system (1). By equating the right hand side of the system (1) to zero, we obtain the equilibrium states. The possible equilibrium states are given as follows,

P_0: The state of all washed out $x_1^* = 0; x_2^* = 0; x_3^* = 0$.

P_1: The state in which only the predator survives, two prey species are washed out. The equilibrium state is

$$x_1^* = 0; x_2^* = 0; x_3^* = \frac{K_3(r_3-q_3E)}{r_3}.$$

P_2: The state in which the first prey and predator species survive and the second prey species extinct out, is given by

$$x_1^* = \frac{\alpha_{13}(-r_3+q_3E)+\frac{r_3}{K_3}(r_1-q_1E)}{\alpha_{31}\alpha_{13}+\frac{r_3r_1}{K_1K_3}}; \; x_2^*=0;$$

$$x_3^* = \frac{\alpha_{31}(r_1-q_1E)+\frac{r_1}{K_1}(r_3-q_3E)}{\alpha_{31}\alpha_{13}+\frac{r_3r_1}{K_1K_3}}.$$

It exists when $\alpha_{13}(r_3-q_3E) < \frac{r_3}{K_3}(r_1-q_1E)$.

P_3: The state in which the second prey and predator exist, while the first prey extinct out. It is given by

$$x_1^*=0; \; x_2^* = \frac{\alpha_{23}(-r_3+q_3E)+\frac{r_3}{K_3}(r_2-q_2E)}{\alpha_{32}\alpha_{23}+\frac{r_2r_3}{K_2K_3}};$$

$$x_3^* = \frac{\alpha_{32}(r_2-q_2E)+\frac{r_2}{K_2}(r_3-q_3E)}{\alpha_{32}\alpha_{23}+\frac{r_2r_3}{K_2K_3}}.$$

It exists, when $\alpha_{23}(r_3-q_3E) < \frac{r_3}{K_3}(r_2-q_2E)$.

P_4: The state in which both the first and the second prey exist, while the predator extinct, It is given by

$$x_2^* = \frac{\alpha_{21}(r_1-q_1E)-\frac{r_1}{K_1}(r_2-q_2E)}{\alpha_{21}\alpha_{12}-\frac{r_1r_2}{K_1K_2}}; \; x_3^* = 0.$$

This state exists, when

$$\alpha_{12}(r_2-q_2E) > \frac{r_2}{K_2}(r_1-q_1E), \alpha_{21}(r_1-q_1E) > \frac{r_1}{K_1}(r_2-q_2E),$$

$$\alpha_{21}\alpha_{12} > \frac{r_1r_2}{K_1K_2}.$$

P_5: The co-existence state, the state in which two prey and predator species exists. The equilibrium state is given by,

$$x_1^* = \frac{1}{\overline{X}}\left[\left(-r_1+q_1E\right)\left(\alpha_{32}\alpha_{23}+\frac{r_2r_3}{K_2K_3}\right)\right]$$

$$+\frac{1}{\overline{X}}\left[\left(r_2-q_2E\right)\left(\alpha_{32}\alpha_{13}+\frac{r_3}{K_3}\alpha_{12}\right)\right]$$

$$+\frac{1}{\overline{X}}\left[\left(r_3-q_3E\right)\left(\alpha_{12}\alpha_{23}-\frac{r_2}{K_2}\alpha_{13}\right)\right]$$

$$x_2^* = \frac{1}{\overline{X}}\left[\left(r_1-q_1E\right)\left(\alpha_{31}\alpha_{23}+\frac{r_3}{K_3}\alpha_{21}\right)\right]$$

$$+\frac{1}{\overline{X}}\left[\left(r_3-q_3E\right)\left(\frac{r_1}{K_1}\alpha_{23}-\alpha_{21}\alpha_{13}+\right)\right]$$

$$+\frac{1}{\overline{X}}\left[-\left(r_2-q_2E\right)\left(\alpha_{31}\alpha_{13}+\frac{r_1r_3}{K_1K_3}\right)\right]$$

$$x_3^* = \frac{1}{\overline{X}}\left[\left(r_1-q_1E\right)\left(\alpha_{21}\alpha_{32}-\frac{r_2}{K_2}\alpha_{31}\right)\right]$$

$$+\frac{1}{\overline{X}}\left[\left(r_2-q_2E\right)\left(\alpha_{12}\alpha_{31}-\frac{r_1}{K_1}\alpha_{32}\right)\right]$$

$$+\frac{1}{\overline{X}}\left[\left(r_3-q_3E\right)\left(\alpha_{21}\alpha_{12}-\frac{r_1r_2}{K_1K_2}\right)\right],$$

where,

$$\overline{X} = \left(\alpha_{12}\alpha_{23}-\frac{r_2}{K_2}\alpha_{13}\right)\alpha_{31}+\left(\alpha_{21}\alpha_{13}-\frac{r_1}{K_1}\alpha_{23}\right)\alpha_{32}$$

$$+\frac{r_3}{K_3}\left(\alpha_{21}\alpha_{12}-\frac{r_2r_1}{K_1K_2}\right)$$

3.2 The stability of the equilibrium state

We analyze the global stability of the system (1) by constructing a suitable Lyapunov function. The global stability of the system (1) for the co-existence equilibrium state is given in the following theorem.

Theorem 3.1: The equilibrium state $P_5(x_1^*, x_2^*, x_3^*)$ is globally asymptotically stable if,

(i) $\alpha_{13}=\alpha_{31}, \alpha_{23}=\alpha_{32}$,

(ii) $4\frac{r_1r_2}{K_1K_2} > \left(\alpha_{12}+\alpha_{21}\right)^2$.

Proof: To examine the globally stability of the system (1), we define a Lyapunov function

$$V\left(x_1,x_2,x_3\right) = x_1-x_1^*-x_1^*\ln\left(\frac{x_1}{x_1^*}\right)+x_2-x_2^*-x_2^*\ln\left(\frac{x_2}{x_2^*}\right)$$

$$+x_3-x_3^*-x_3^*\ln\left(\frac{x_3}{x_3^*}\right)$$

We see that V is definite positive. The time derivative of V along the solutions of the system (1), is given by,

$$\frac{dV}{dt} = \frac{x_1-x_1^*}{x_1}\frac{dx_1}{dt}+\frac{x_2-x_2^*}{x_2}\frac{dx_2}{dt}+\frac{x_3-x_3^*}{x_3}\frac{dx_3}{dt}.$$

After simplifying, $\frac{dV}{dt}$ can be rewritten as,

$$\frac{dV}{dt} = -\mathbf{X}^{\mathbf{T}}\mathbf{A}\mathbf{X}\ , \qquad (2)$$

where,

$\mathbf{X}^T = \left[x_1-x_1^*, x_2-x_2^*, x_3-x_3^*\right]$, and matrix $\mathbf{A}$ is

$$\begin{bmatrix} \frac{r_1}{K_1} & -\frac{1}{2}\left(\alpha_{12}+\alpha_{21}\right) & \frac{1}{2}\left(\alpha_{31}-\alpha_{13}\right) \\ -\frac{1}{2}\left(\alpha_{12}+\alpha_{21}\right) & \frac{r_2}{K_2} & \frac{1}{2}\left(\alpha_{32}-\alpha_{23}\right) \\ \frac{1}{2}\left(\alpha_{31}-\alpha_{13}\right) & \frac{1}{2}\left(\alpha_{32}-\alpha_{23}\right) & \frac{r_3}{K_3} \end{bmatrix}$$

We observe that $\frac{dV}{dt}<0$ if the matrix $\mathbf{A}$ is definite positive. The matrix A is definite positive if, $\frac{1}{2}\left(\alpha_{31}-\alpha_{13}\right)=0, \frac{1}{2}\left(\alpha_{32}-\alpha_{23}\right)=0$, and $4\frac{r_1r_2}{K_1K_2}-\left(\alpha_{12}+\alpha_{21}\right)^2>0$. So, we conclude that the co-existence equilibrium is globally asymptotically stable, if $\alpha_{13}=\alpha_{31}$, $\alpha_{23}=\alpha_{32}$ and $4\frac{r_1r_2}{K_1K_2}>\left(\alpha_{12}+\alpha_{21}\right)^2$.

This completes the proof.

3.3 Bionomic equilibrium

The concept of bionomic equilibrium is the concepts that are related to the biological and economic equilibrium. The biological equilibrium is derived from the right hand sides of the system (1) equal zero. The economic equilibrium is obtained by equaling the total revenue that is obtained by selling the harvested biomass with the total needed efforts for harvesting.

Let c is the fishing cost per unit effort, p_i for $i=1,2,3$ are the price per unit biomass of the first, the second and the third species respectively. The net economic revenue at any time t is given by

$$R = \left(p_1q_1x_1+p_2q_2x_2+p_3q_3x_3-c\right)E. \qquad (3)$$

We can obtain the bionomic equilibrium $P\left(x_{1b}, x_{2b}, x_{3b}, E\right)$ that are given in the following equations,

$$x_1 = 0,\ E = \frac{r_1}{q_1}\left(1 - \frac{x_1}{K_1}\right) - \frac{\alpha_{12}}{q_1}x_2 - \frac{\alpha_{13}}{q_1}x_3$$

$$x_2 = 0,\ E = \frac{r_2}{q_2}\left(1 - \frac{x_2}{K_2}\right) - \frac{\alpha_{21}}{q_2}x_1 - \frac{\alpha_{23}}{q_2}x_3 \qquad (4)$$

$$x_3 = 0,\ E = \frac{r_3}{q_3}\left(1 - \frac{x_3}{K_3}\right) + \frac{\alpha_{31}}{q_3}x_1 + \frac{\alpha_{32}}{q_3}x_2$$

Eliminating E from (4), we obtain the nontrivial biological equilibrium. The possibility of the bionomic equilibrium is determined in the following cases.

Case I. When $p_1q_1x_1 + p_2q_2x_2 + p_3q_3x_3 < c$, the fishing cost per unit effort exceeds the total revenue, so the biomass is not harvested.

Case II. When $p_1q_1x_1 + p_2q_2x_2 + p_3q_3x_3 > c$, the fishing cost per unit effort is less than the total revenue, so the biomass can be harvested. The economic equilibrium is given by

$$R = p_1q_1x_1 + p_2q_2x_2 + p_3q_3x_3 - c = 0 \qquad (5)$$

We refer to (4) as the economic equilibrium point. The bionomic solution (x_{1b}, x_{2b}, x_{3b}) is obtained by solving both the equations (4) and (5) simultaneously.

3.4 Optimal harvesting policy

Our objective is to maximize the present revenue value J of a continuous time that is given by

$$J = \int_0^\infty \mathrm{e}^{-\delta t}\left(p_1q_1x_1 + p_2q_2x_2 + p_3q_3x_3 - c\right)E(t)\,dt \quad (6)$$

where δ represents the instantaneous annual rate of discount. In this problem, we maximize J subject to the state equations (1) by applying Pontryagin's maximal principle [6, 11]. The value of the control variable $E(t)$ is between 0 and E_{max}, so that the control function $V(t) \in [0, E_{max}]$. The Hamiltonian of the problem is

$$\begin{aligned} H &= \mathrm{e}^{-\delta t}\left(p_1q_1x_1 + p_2q_2x_2 + p_3q_3x_3 - c\right)E \\ &+ \lambda_1\left(r_1x_1\left(1 - \frac{x_1}{K_1}\right) - \alpha_{12}x_1x_2 - \alpha_{13}x_1x_3 - q_1Ex_1\right) \\ &+ \lambda_2\left(r_2x_2\left(1 - \frac{x_2}{K_2}\right) - \alpha_{21}x_1x_2 - \alpha_{23}x_2x_3 - q_2Ex_2\right) \qquad (6) \\ &+ \lambda_3\left(r_3x_3\left(1 - \frac{x_3}{K_3}\right) + \alpha_{31}x_1x_3 + \alpha_{32}x_2x_3 - q_3Ex_3\right) \end{aligned}$$

where $\lambda_i, i = 1,2,3$ are the adjoint variables.

By using Pontryagin's maximal principle, we write the adjoint equations regarding to optimal equilibrium solution as $\frac{d\lambda_i}{dt} = -\frac{\partial H}{\partial x_i}$ for $i = 1,2,3$. We obtain adjoint equations as follows,

$$\frac{d\lambda_1}{dt} = -\mathrm{e}^{-\delta t}p_1q_1E + \frac{\lambda_1r_1x_1}{K_1} + \lambda_2\alpha_{21}x_2 - \lambda_3\alpha_{31}x_3 \qquad (8)$$

$$\frac{d\lambda_2}{dt} = -\mathrm{e}^{-\delta t}p_2q_2E + \lambda_1\alpha_{12}x_1 + \frac{\lambda_2r_2x_2}{K_2} - \lambda_3\alpha_{32}x_3 \qquad (9)$$

$$\frac{d\lambda_3}{dt} = -\mathrm{e}^{-\delta t}p_3q_3E + \lambda_1\alpha_{13}x_1 + \lambda_2\alpha_{23}x_2 + \frac{\lambda_3r_3x_3}{K_3} \qquad (10)$$

Next, we consider the state variables, x_1, x_2, x_3 as constants. Eliminating λ_1 and λ_3 from (8) and (10), we find the differential equation in λ_2

$$\left(D^3 + a_2D^2 + a_1D + a_0\right)\lambda_2 = A_2\mathrm{e}^{-\delta t} \qquad (11)$$

where,

$$a_2 = -\left(\frac{r_1}{K_1}x_1 + \frac{r_2}{K_2}x_2 + \frac{r_3}{K_3}x_3\right)$$

$$\begin{aligned} a_1 &= \left(\frac{r_1r_2}{K_1K_2} - \alpha_{21}\alpha_{12}\right)x_1x_2 + \\ &\left(\frac{r_1r_3}{K_1K_3} + \alpha_{31}\alpha_{13}\right)x_1x_3 + \left(\frac{r_2r_3}{K_2K_3} + \alpha_{32}\alpha_{23}\right)x_2x_3 \end{aligned}$$

$$\begin{aligned} a_0 &= \left(\alpha_{32}\alpha_{13}\alpha_{21} - \frac{r_1\alpha_{32}}{K_1}\alpha_{23} + \alpha_{31}\alpha_{13}\alpha_{23}\right)x_1x_2x_3 + \\ &\left(\frac{r_3}{K_3}\alpha_{12}\alpha_{21} - \frac{r_2}{K_2}\alpha_{31}\alpha_{13} - \frac{r_1r_2r_3}{K_1K_2K_3}\right)x_1x_2x_3 \end{aligned}$$

$$\begin{aligned} A_2 &= \left[\left(\alpha_{32}\alpha_{13} + \frac{r_3}{K_3}\alpha_{12}\right)x_1x_3 + \alpha_{12}\delta x_1\right]p_1q_1E + \\ &\left[\left(\alpha_{31}\alpha_{12} - \frac{r_1}{K_1}\alpha_{32}\right)x_1x_3 - \alpha_{32}\delta x_3\right]p_3q_3E \\ &-\left(\frac{r_1r_3}{K_1K_3} + \alpha_{31}\alpha_{13}\right)x_1x_3p_2q_2E \\ &-\left(\frac{r_1}{K_1}\delta x_1 + \frac{r_3}{K_3}\delta x_3 + \delta^2\right)p_2q_2E \end{aligned}$$

The solution of the equation (10) is

$$\lambda_2(t) = C_1\mathrm{e}^{m_1t} + C_2\mathrm{e}^{m_2t} + C_3\mathrm{e}^{m_3t} + \frac{A_2}{N}\mathrm{e}^{-\delta t}, \qquad (12)$$

where $C_i, i = 1,2,3$ are arbitrary constants, and $m_i, i = 1,2,3$ are the roots of the equations $a_3m^3 + a_2m^2 + a_1m + a_0 = 0$, and $N = a_0 - a_1\delta + a_2\delta^2 - a_3\delta^3 \neq 0$. It is clear from the equation (11) that $\lambda_2(t)$ is bounded if and only if $m_i < 0, i = 1,2,3$ or $C_i, i = 1,2,3$ identically equal zero. It is difficult to identify whether $m_i < 0, i = 1,2,3$. For this, we assume that $C_i = 0, i = 1,2,3$, then $\lambda_2(t) = \frac{A_2}{N}e^{-\delta t}$. By a similar process, we get $\lambda_1(t) = \frac{A_1}{N} e^{-\delta t}$ and $\lambda_3(t) = \frac{A_3}{N} e^{-\delta t}$ where

$$A_1 = \left[\left(\alpha_{23}\alpha_{31} + \frac{r_3}{K_3}\alpha_{21} \right) x_2 x_3 + \left(\alpha_{21}\delta - \frac{r_2}{K_2}\delta\alpha_{31} \right) x_2 \right] q_2 p_2 E$$

$$- \left[\left(\alpha_{32}\alpha_{23} + \frac{r_2 r_3}{K_2 K_3} \right) x_2 x_3 - \frac{r_2}{K_2}\delta x_2 - \frac{r_3}{K_3}\delta x_3 \right] q_1 p_1 E$$

$$+ \left[\left(\alpha_{32}\alpha_{21} - \frac{r_2}{K_2}\alpha_{31} \right) x_2 x_3 + \alpha_{31}\delta x_3 \right] q_3 p_3 E$$

$+2\delta^2 \alpha_{31} q_2 p_2 E - \delta^2 q_1 p_1 E$, and

$$A_3 = \left[\left(\frac{r_2}{K_2}\alpha_{13} - \alpha_{12}\alpha_{23} \right) x_1 x_2 - \delta\alpha_{13} x_1 \right] q_1 p_1 E +$$

,

$$\left[\left(\frac{r_1}{K_1}\alpha_{23} - \alpha_{21}\alpha_{13} \right) x_1 x_2 - \delta\alpha_{23} x_2 \right] q_2 p_2 E +$$

$$\left[\left(\alpha_{12}\alpha_{21} - \frac{r_1 r_2}{K_1 K_2} \right) x_1 x_2 + \frac{r_2}{K_2}\delta x_2 + \frac{r_1}{K_1}\delta x_1 - \delta^2 \right] q_3 p_3 E$$

We observe that the shadow prices $\lambda_i(t)e^{\delta t}, i = 1,2,3$ of each fish species is bounded when $t \to \infty$ and we get $\lambda_i(t)e^{\delta t} = \frac{A_i}{N} =$ constant, for $i = 1,2,3$.

We maximize the Hamiltonian in the equation (7) for $0 \le E \le E_{max}$. We consider that the optimal equilibrium does not occur either at $E = 0$ or $E = E_{max}$. There is a singular control that satisfies the equation,

$$\frac{\partial H}{\partial E} = e^{-\delta t}\left(p_1 q_1 x_1 + p_2 q_2 x_2 + p_3 q_3 x_3 - c \right) -$$

$$\lambda_1 q_1 x_1 - \lambda_2 q_2 x_2 - \lambda_3 q_3 x_3 = 0, \qquad (13)$$

We conclude that the total harvesting cost per unit effort equals to the discounted value of the future profit at the steady state [5, 11]. By eliminating parameters λ_1, λ_2 and λ_3 into equation (11), we obtain

$$q_1 x_1 \left(p_1 - \frac{A_1}{N} \right) + q_2 x_2 \left(p_2 - \frac{A_2}{N} \right) + q_3 x_3 \left(p_3 - \frac{A_3}{N} \right) = c \qquad (14)$$

The optimal equilibrium solution $x_{1\delta}$, $x_{2\delta}$, , and $x_{3\delta}$ can be solved simultaneously from (4) and (14) for a given value δ. We observe that if $\delta \to \infty$, we have $\frac{A_i}{N} \to 0$. It implies that $p_1 q_1 x_{1\infty} + p_2 q_2 x_{2\infty} + p_3 q_3 x_{3\infty} = c$. Thus, we have $R(x_{1\infty}, x_{2\infty}, x_{3\infty}), E) = 0$. It shows that the net economic revenue is fully lost when the discount rate is infinite. This conclusion is also supported by Kar *et al.* [11] for the combined harvesting of two competing species, while there is no harvesting for predator.

4 Numerical simulations

In this section, we present the simulation results to illustrate the dynamical of the model. We pick up for some parameters value from with $r_1 = 2.09$, $r_2 = 2.07$, $r_3 = 2.1$, $\alpha_{12} = 0.001$, $\alpha_{21} = 0.001$, $\alpha_{13} = 0.01$, $\alpha_{23} = 0.01$, $\alpha_{31} = 0.01$, $\alpha_{32} = 0.01$, $q_1 = 0.03$, $q_2 = 0.03$, $q_3 = 0.03$, $p_1 = 5$, $p_2 = 5$, $p_3 = 2$, $C = 50$ and $\delta = 0{,}05$. In our optimal problem, we have an equilibrium solution that satisfies the necessary conditions of the maximum principle. For the parameter values as is given, we find that both the biological equilibrium $P_5(33,32,116)$ and the bionomic equilibrium $R(39,38,137)$ exist and achieve to the optimal equilibrium solution. We find that the optimal harvesting effort is $E = 5.6$ units, regarding to the optimal equilibrium $(36,35,126)$. The simulations are presented in the following figures.

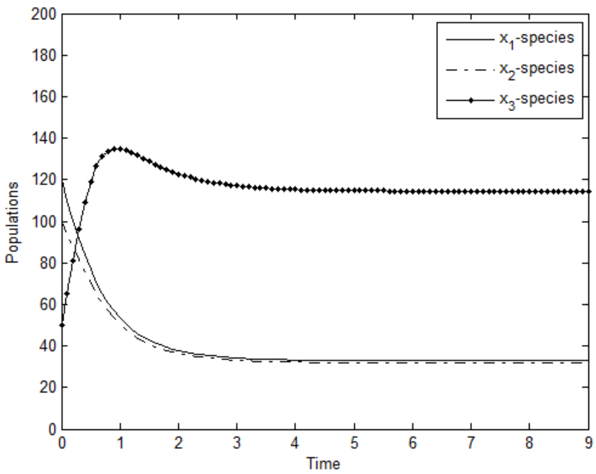

Fig. 1. Graph of the populations against time with initial conditions $x_1 = 120$, $x_2 = 100$, $x_3 = 50$

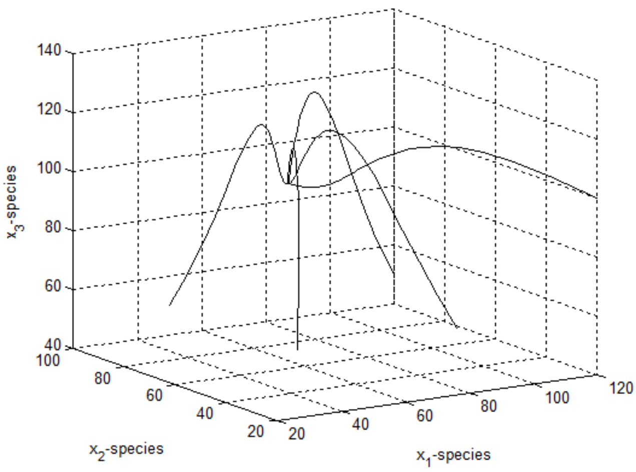

Fig. 2 Graph of the populations corresponding to the optimal harvesting effort $E = 5.6$ units, with different initial conditions.

In figure 1 and figure 2 shows that the trajectories of the solutions close to the optimal equilibrium $P(36,35,128)$. It indicates that the optimal equilibrium solution is globally asymptotically stable.

5 Conclusion

An ecological model was proposed and analysed to study the dynamics for two competing prey species and one invasive species (as predator) in an aquatic habitat. In this paper, we have attempted to study the effects of harvesting of two competing prey species and one predator species, in which predator species eats other two prey species and also the predator competes for using the same resources among this species. The existence of the possible steady states and bionomic equilibrium of the system have been studied. The globally stability of the system in co-existent state is examined by using Lyapunov function.

The optimal harvesting problem was addressed to obtain the optimal equilibrium solution. It is shown that the shadow prices of each biomass remain constant over time, so the boundedness and positivity condition are satisfied. An ecological model is developed from one prey-one predator model, in which the growth of each population is formulated in logistics term for among the same species. For interaction in other species is proposed in bilinear term. In the next research, will be developed for interaction of predator and prey species based on the limited density of prey population, in which it is more reasonable in an ecological system.

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