Construction of equivalents of large equations systems of electrical network based on matrix annulators

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Abstract. An original method of reducing the equations of node voltages is proposed with the aim of equivalence of the electric network. The method is based on the matrix transformations of the indicated equations with the help of matrix annulators. It is shown that this method, in comparison with the traditional one, makes it possible to improve the conditionality of the solved equations by an order of magnitude or more. This has a significant effect on the numerical stability of the resulting electrical network equivalents. The results of reduction of a small, large, and very large system of nodal stress equations are presented.

1 Introduction

It is well known that the feature of calculation modes in electric power systems (EPS) is their multiparametricity. The solution of complete systems of equations covering all the nodes and connections of large power systems, even with simplified models, because of the lack of reliable information on all elements of the network, poses a serious problem, which is complicated by the need to accumulate and store large amounts of information [1] and the corresponding increase in the requirements for the speed of computers [2].

The need to reduce the counting time appears most when carrying out multivariate and multimode calculations in operational control, as well as in planning problems for EPS regimes [3, 4]. Equivalence allows not only to reduce the time of solving the nodal electrical voltage equations (NEVE), but also to reconcile the amount of information and its error.

In a broad sense, the equivalence of EPS is in the transformation (reduction) of a complex mathematical model into a simpler one while preserving the most important (required) properties within a given accuracy. With this approach, the toolkit of Krylov subspaces is widely used [5]. In the narrower sense, with respect to the calculation of EPS modes, equivalence is reduced to the transformation of the replacement circuit and its parameters to a species having a smaller number of nodes and branches and suitable for modeling of the initial EPS modes.

The report proposes an original method of UUN reducing with the purpose to equate an electrical network. The method is based on the NEVE matrix transformations with the help of algebraic objects, called annihilators of matrices.

This method, in comparison with the traditional algorithm based on the Schur complement, makes it possible to improve significantly the conditionality of the solved equations, especially for large (up to 10 000 equations) and very large (up to 100 000 equations) systems of NEVE. It has a significant effect on the accuracy of the received electrical network equivalents and, consequently, on the correctness of numerical models.

2 The traditional approach for NEVE equivalenting

For equivalenting procedure excluded nodes (set *M*) and nodes stored in equivalent units (set *N*) are given [2].

The nodes were renumbered so that from the first node to the *n*-th node there were nodes *N*, then all *m* excluded nodes of the set *M*. In this case, the NEVE were structurally divided into blocks (block matrices and sub-vectors)

$$\begin{bmatrix} \boldsymbol{Y}_{NN} & \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_N \\ \boldsymbol{U}_M \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix}. \quad (1)$$

Here, $\boldsymbol{U}_N$, $\boldsymbol{U}_M$ – the stress subvectors; $\boldsymbol{I}_N$, $\boldsymbol{I}_M$ – current sub-vectors in the vector

$$\begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix}. \quad (2)$$

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Sub-matrix of conductivities $\boldsymbol{Y}_{NN}$ in the conductivity matrix

$$\left[\begin{array}{c|c} \boldsymbol{Y}_{NN} & \boldsymbol{Y}_{MN} \\ \hline \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM} \end{array}\right] \tag{3}$$

is square and covers only the constraints on the set of nodes *N*, and the square submatrix of conductivities is only the connections on the set of nodes *M*. Note that $\boldsymbol{Y}_{NM}$ и $\boldsymbol{Y}_{MN}$ are rectangular matrices.

Expanding the equation (1) up to the selected blocks

$$\begin{aligned} \boldsymbol{Y}_{NN}\boldsymbol{U}_N + \boldsymbol{Y}_{NM}\boldsymbol{U}_M &= \boldsymbol{I}_N, \\ \boldsymbol{Y}_{MN}\boldsymbol{U}_N + \boldsymbol{Y}_{MM}\boldsymbol{U}_M &= \boldsymbol{I}_M, \end{aligned} \tag{4}$$

and expressing from the second equation (4) the stress vector of the excluded nodes $\boldsymbol{U}_M$, it is not difficult to obtain the equation

$$\begin{aligned} &\left(\boldsymbol{Y}_{NN} - \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{Y}_{MN}\right)\boldsymbol{U}_N = \boldsymbol{I}_N - \\ &- \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{I}_M \end{aligned} \tag{5}$$

which with the introduction of new designations

$$\begin{aligned} \boldsymbol{Y}_{NN}^{\ni} &= \boldsymbol{Y}_{NN} - \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{Y}_{MN}, \\ \boldsymbol{I}_N^{\ni} &= \boldsymbol{I}_N - \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}\boldsymbol{I}_M, \end{aligned}$$

is reduced to the equivalent form

$$\boldsymbol{Y}_{NN}^{\ni}\boldsymbol{U}_N = \boldsymbol{I}_N^{\ni}. \tag{6}$$

Thus, the equivalence procedure considered above, for given excluded nodes (set *M*) preserves in the resulting equivalent solution (equivalent) only the nodes of the set *N*.

From a formal point of view, the described equivalence procedure is based on the well-known *Schur complement algorithm* [6, 7]. One of the drawbacks of this approach is the complexity and, often the impossibility of a preconditioning ("regulation" of the conditionality) of the solved equations.

In the next section of the paper, the original method is described, which is alternative to the *Shur complement algorithm*, and then the equivalence method is constructed on its basis.

This method, as it was said earlier, makes it possible to effectively impact on the condition of the NEVE and, as a consequence, reduce the computational errors and increase the correctness of the equivalent solutions obtained.

3 Mathematical justification of the alternative method

The following notation and definitions will be used: $\mathbf{0}_{n\times m}$ – zero size matrix $n\times m$; $\boldsymbol{E}_n$ – unit size matrix $n\times n$; $\cdot^{\mathrm{T}}$ – the transposed matrix; $\cdot^{+}$ – a pseudo-inverse matrix according to Moore-Penrose; $(\cdot)^{\perp}$ – annihilator of the matrix of maximal rank of a given matrix; $\mathrm{rank}\,\cdot$ – the rank of the matrix; $\mathrm{size}\,\cdot$ – dimensions of the matrix (vector dimension); $\mathrm{null}\,\cdot$ – basis of the null space of the matrix; $\mathrm{cond}\,\cdot$ – number of conditionality of the matrix; $\|\cdot\|$ – a given vector norm [6, 7].

In this paper, the so-called left annihilator of matrices is used, which is called the annihilator. Recall [8, 9], that the left annihilator of the maximal rank of matrix $\boldsymbol{M}$ with the size $m\times n$ and rank r is called a matrix $\boldsymbol{M}^{\perp}$, if the both conditions are correct in the same time $\boldsymbol{M}^{\perp}\boldsymbol{M} = \mathbf{0}_{(n-r)\times m}$, $\mathrm{rank}\ \boldsymbol{M}^{\perp} = n - r$.

For simplicity, we shall assume that the annihilators of zero satisfy the orthogonality condition $\boldsymbol{M}^{\wedge}\,\boldsymbol{M}^{\wedge\,\mathrm{T}} = \boldsymbol{E}_{n-r}$.

As a procedure for calculating of matrices annihilators, well-developed methods for computing the null space of a matrix can be used $\mathrm{null}(\boldsymbol{M})$ [6, 10]. In this case $\boldsymbol{M}^{\perp} = \mathrm{null}\ \boldsymbol{M}^{\mathrm{T}\ \mathrm{T}}$.

We will consider the NEVE equation in the following block decomposition:

$$\boldsymbol{A}_1 \,\vdots\, \boldsymbol{A}_2 \left[\begin{array}{c} \mathbf{x}_1 \\ \hdashline \mathbf{x}_2 \end{array}\right] = \mathbf{b} \tag{7}$$

here $\boldsymbol{A}_1$, $\boldsymbol{A}_2$ – are rectangular submatrices of sizes

$$\mathrm{size}\ \boldsymbol{A}_1 = n\times n_1, \quad \mathrm{size}\ \boldsymbol{A}_2 = n\times n_2, \tag{8}$$

in this case $n_1 + n_2 = n$. The decomposition (7), (8) is clearly shown in fig. 1.

The statement: the solution of the linear equation (7) for an invertible block matrix $\boldsymbol{A}_1 \,\vdots\, \boldsymbol{A}_2$ is determined by the equivalent formulas [9]

$$\begin{cases} \mathbf{x}_1 = \boldsymbol{A}_1^{+}\ \mathbf{b} - \boldsymbol{A}_2\mathbf{x}_2\ , \\ \mathbf{x}_2 = \ \boldsymbol{A}_1^{\perp}\boldsymbol{A}_2\ ^{-1}\boldsymbol{A}_1^{\perp}\mathbf{b}. \end{cases} \tag{9}$$

$$\begin{cases} \mathbf{x}_1 = \ \boldsymbol{A}_2^{\perp}\boldsymbol{A}_1\ ^{-1}\boldsymbol{A}_2^{\perp}\mathbf{b}, \\ \mathbf{x}_2 = \boldsymbol{A}_2^{+}\ \mathbf{b} - \boldsymbol{A}_1\mathbf{x}_1\ , \end{cases} \tag{10}$$

here $A_1^{\perp}$, $A_2^{\perp}$ are the left annihilators of zero of the submatrices maximal rank A_1, A_2, respectively, A_1^{+}, A_2^{+} are the pseudoinverse matrices for the submatrices A_1, A_2.

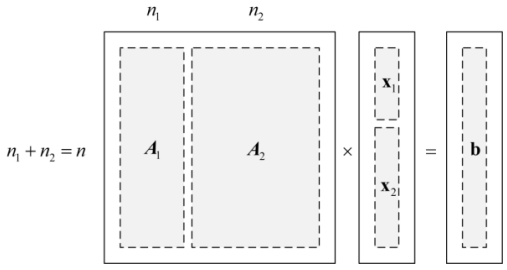

Fig. 1. Block partitioning of matrices and vectors in the matrix equation.

4 Reduction of NEVE based on matrix annihilators

Considering the NEVE (1) in the division into blocks, as it is done in the equation (7), we introduce the designations

$$Y_N = \begin{bmatrix} Y_{NN} \\ \hdashline Y_{MN} \end{bmatrix}, \quad Y_M = \begin{bmatrix} Y_{MN} \\ \hdashline Y_{MM} \end{bmatrix} \tag{11}$$

and write (1), taking into account (11). Obtained equation

$$Y_N \vdots Y_M \begin{bmatrix} U_N \\ \hdashline U_M \end{bmatrix} = \begin{bmatrix} I_N \\ \hdashline I_M \end{bmatrix} \tag{12}$$

Annihilator $Y_M^{\perp}$ is introduced and it satisfies the following conditions:

$$Y_M^{\perp} Y_M = 0_{M\times M}, \quad Y_M^{\perp} Y_M^{\perp \mathsf{T}} = E_{M\times M} \tag{13}$$

Then, according to the first equation (10), from the theorem proved earlier we can write

$$Y_M^{\perp} Y_N U_N = Y_M^{\perp} \begin{bmatrix} I_N \\ \hdashline I_M \end{bmatrix} \tag{14}$$

Introducing new designations

$$\hat{Y}_{NN}^{э} = Y_M^{\perp} Y_N, \; I_N^{э} = Y_M^{\perp} \begin{bmatrix} I_N \\ \hdashline I_M \end{bmatrix} \tag{15}$$

The equation (14) can be rewritten in a generalized form

$$\hat{Y}_{NN}^{э} U_N = \hat{I}_N^{э} \tag{16}$$

The equations (6) and (16) are different, but they have the same solution U_N. The principal difference of the equation (16) is that it allows simultaneously with the equivalence to solve the problem of preconditioning in order to minimize computational errors.

It is well known [6, 11, 12] that in order to reduce the influence of errors in the initial data, to increase the accuracy of the solution, and to accelerate the convergence of the iterative methods, various algorithms are used that usually consist of elementary transformations of rows (columns) of matrices in equation (7): scaling, regularization, balancing, preconditioning (preconditioning, use of spectrally equivalent operators), etc.

With respect to the matrix equation (16), the problem of reducing errors will consist in minimizing the ratio [11]

$$\tau = \frac{\|\Delta U_N\| \cdot \|\hat{I}_N^{э}\|}{\|U_N\| \cdot \|\Delta \hat{I}_N^{э}\|} \tag{17}$$

However, the direct determination of the value t in terms of the coefficients of the matrices of the original equation is difficult due to the nonlinearity of the valuation operation. Therefore, it is preferable to use a qualitative characteristic called the matrix condition number [6, 10, 11]. In the case considered, this number is

$$\text{cond } \hat{Y}_{NN}^{э} = \|\hat{Y}_{NN}^{э}\| \cdot \| \hat{Y}_{NN}^{э} {}^{-1} \| \tag{18}$$

and satisfies the inequality

$$\frac{\|\Delta U_N\|}{\|U_N\|} \le \text{cond } \hat{Y}_{NN}^{э} \le \|\Delta \hat{I}_N^{э}\| / \|\hat{I}_N^{э}\| \tag{19}$$

Taking into account (15), the ratio (19) is transformed to the form

$$\frac{\|\Delta U_N\|}{\|U_N\|} \le \text{cond } Y_M^{\perp} Y_N \le \frac{\left\| \Delta \left(Y_M^{\perp} \begin{pmatrix} I_N \\ I_M \end{pmatrix} \right) \right\|}{\left\| Y_M^{\perp} \begin{pmatrix} I_N \\ I_M \end{pmatrix} \right\|}. \tag{20}$$

The larger the condition number (18), the greater the impact on the decision NEVE errors in the original data.

Reduction of the conditionality number (18) can be achieved by further transformation of the NEVE equivalent system (16) by introducing a new matrix D, which should be [12] as close as possible to $Y_M^{\perp} Y_N {}^{-1}$ easily computable and easily invertible. In this case, the NEVE will be replaced by the equation

$$D \hat{Y}_{NN}^{э} U_N = D \hat{I}_N^{э},$$

where

$$\text{cond } D\hat{Y}_{NN}^{э} < \text{cond } \hat{Y}_{NN}^{э}.$$

5 Reduction of a small NEVE system

Let us consider the computational example [9].

Let the electric network shown in fig. 2 [2]. For convenience of counting, we take all the bond resistances to be the same r_{ij} = 10 Ohm (Y_{ij} = 0,1 S), except for the two bonds r_{13} = 20 Ohm (Y_{13} = 0,05 S), r_{24} = 5 Ohm (Y_{24} = 0,2 S).

The basic mode corresponds to the NEVE system

$$\begin{bmatrix} -0,25 & 0,1 & 0,05 & 0 \\ 0,1 & -0,4 & 0,1 & 0,2 \\ 0,05 & 0,1 & -0,25 & 0 \\ 0 & 0,2 & 0 & -0,2 \end{bmatrix} \begin{bmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \\ U_4^0 \end{bmatrix} = \begin{bmatrix} -8,5 \\ 1 \\ -7 \\ 2 \end{bmatrix} \quad (21)$$

the results of the calculation are shown in Fig. 2.

Let us suppose that the set M of excluded nodes is 3 and 4. Then the matrix (3) has the following block decompositions:

$$\boldsymbol{Y}_N \,\vdots\, \boldsymbol{Y}_M = \left[\begin{array}{cc:cc} -0,25 & 0,1 & 0,05 & 0 \\ 0,1 & -0,4 & 0,1 & 0,2 \\ 0,05 & 0,1 & -0,25 & 0 \\ 0 & 0,2 & 0 & -0,2 \end{array}\right] \quad (22)$$

considering that

$$\boldsymbol{Y}_M^{\perp} = \left(\text{null} \begin{bmatrix} 0,05 & 0 \\ 0,1 & 0,2 \\ -0,25 & 0 \\ 0 & -0,2 \end{bmatrix}^{\mathrm{T}} \right)^{\mathrm{T}} = \begin{bmatrix} 0,9646 & 0,0915 & 0,2295 & 0,0915 \\ -0,1837 & 0,6752 & 0,2333 & 0,6752 \end{bmatrix}$$

Indeed

$$\boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_M = \begin{bmatrix} 0,9646 & 0,0915 & 0,2295 & 0,0915 \\ -0,1837 & 0,6752 & 0,2333 & 0,6752 \end{bmatrix} \begin{bmatrix} 0,05 & 0 \\ 0,1 & 0,2 \\ -0,25 & 0 \\ 0 & -0,2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Performing further calculations, we obtain

$$\hat{\boldsymbol{Y}}_{NN}^{\text{э}} = \boldsymbol{Y}_M^{\perp}\boldsymbol{Y}_N = \begin{bmatrix} -0,2205 & 0,1011 \\ 0,1251 & -0,1301 \end{bmatrix},$$

$$\hat{\boldsymbol{I}}_N^{\text{э}} = \boldsymbol{Y}_M^{\perp} \begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{I}_M \end{bmatrix} = \begin{bmatrix} -9,8979 \\ -0,7473 \end{bmatrix},$$

$$\boldsymbol{U}_N = \hat{\boldsymbol{Y}}_{NN}^{\text{э}\,-1} \hat{\boldsymbol{I}}_N^{\text{э}} = \begin{bmatrix} 85,0 \\ 87,5 \end{bmatrix},$$

which exactly corresponds to the values indicated in fig. 2.

It is important to note that the number of conditionality for the matrix (3) and the number of conductivity for original matrix (22) for the original matrix is $\text{cond}\ \boldsymbol{Y}_N \,\vdots\, \boldsymbol{Y}_M = 15,9373$, and for the equivalent

$$\text{cond}\ \hat{\boldsymbol{Y}}_{NN}^{\text{э}} = \text{cond} \begin{bmatrix} -0,2205 & 0,1011 \\ 0,1251 & -0,1301 \end{bmatrix} = 5,5204,$$

almost in 3 times less. This number can be further reduced if the $\boldsymbol{Y}_M^{\perp}$ will be the matrix

$$\boldsymbol{Y}_M^{\perp} = \begin{bmatrix} 6,6150 & 4,7836 & 3,2365 & 4,7836 \\ 3,5836 & 8,6518 & 4,1774 & 8,6518 \end{bmatrix} \quad (23)$$

The use of the annulant (23) in the calculations provides

$$\hat{\boldsymbol{Y}}_{NN}^{\text{э}} = \begin{bmatrix} -0,2205 & 0,1011 \\ 0,1251 & -0,1301 \end{bmatrix} \quad (24)$$

$\text{cond}\ \hat{\boldsymbol{Y}}_{NN}^{\text{э}} = 1,2444$, which is more than an order of magnitude less than $\text{cond}\ \boldsymbol{Y}_N \,\vdots\, \boldsymbol{Y}_M$. In this case, the matrix

$$\boldsymbol{Y}_{NN}^{\text{э}} = \begin{bmatrix} -0,24 & 0,12 \\ 0,12 & -0,56 \end{bmatrix}$$

calculated by the traditional method, has a 3,5 times greater number of conditionality than the matrix (24).

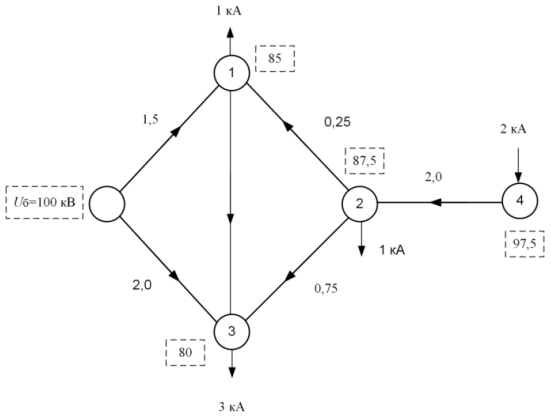

Fig. 2. The scheme of the electrical network in the basic mode.

6 Reduction of a large NEVE system

Let us consider the reduction of a large NEVE system, the basic mode of which corresponds to a size matrix

$$\text{size} \left[\begin{array}{c:c} \boldsymbol{Y}_{NN} & \boldsymbol{Y}_{MN} \\ \hdashline \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM} \end{array}\right] = 1000 \times 1000$$

and vector of dimension

$$\text{size} \left[\begin{array}{c} \boldsymbol{I}_N \\ \hdashline \boldsymbol{I}_M \end{array}\right] = 1000\,.$$

The matrix (3) and vector (2) for the basic mode are densely populated arrays whose elements vary within the following limits: $-4,2; \ldots; 5,7$. The total number of non-zero elements of the matrix from (3) is $\approx 9,91 \cdot 10^5$.

Let the set M of excluded nodes be equal 995, and, accordingly, the number of nodes left $N = 1000 - 995 = 5$.

Calculation by formulas (11) – (16) in the *Matlab* using orthogonal annihilators leads to the following results:

$$\hat{Y}^{\ni}_{NN} = \begin{bmatrix} 1,8571 & -0,7800 & 1,4434 & 0,4049 & -0,4329 \\ -0,7242 & 0,6592 & 1,5200 & 0,5827 & 1,6713 \\ 0,2350 & -0,3492 & -0,6982 & 0,9702 & -1,3692 \\ -0,6484 & -0,6965 & 0,0061 & -1,1241 & 0,5262 \\ -0,8512 & 0,2244 & 0,1312 & 1,1808 & 1,2175 \end{bmatrix}, \quad (25)$$

$$\hat{I}^{\ni}_{N} = \begin{bmatrix} 0,7807 \\ -0,1045 \\ -1,4519 \\ -0,2478 \\ -0,1782 \end{bmatrix}, \quad U_N = \hat{Y}^{\ni}_{NN}{}^{-1} \hat{I}^{\ni}_{N} = \begin{bmatrix} 1,6907 \\ 0,4469 \\ -0,8643 \\ -0,3758 \\ 1,4110 \end{bmatrix} \quad (26)$$

In this case, the number of conditionality of the matrix (25) is 7,8019, and the Euclidean error rate of the solution (26) with respect to the exact value of the solution vector is $3{,}825310^{-13}$.

Calculations using conventional methods allow to obtain following matrix and vector

$$Y^{\ni}_{NN} = \begin{bmatrix} 41,7122 & 19,8072 & -2,6691 & 84,1475 & -29,5413 \\ -55,6173 & -48,4439 & 9,7584 & -51,1095 & 43,1407 \\ -84,5763 & -19,1018 & 14,5854 & -42,1711 & 75,9152 \\ 87,8123 & 63,1374 & -21,0482 & 19,9958 & -59,6218 \\ 49,3494 & -8,4865 & 36,5263 & 64,3591 & -27,9839 \end{bmatrix} \quad (27)$$

$$I^{\ni}_{N} = \begin{bmatrix} 8,3722 \\ -44,0346 \\ -41,1712 \\ 103,2302 \\ -15,6018 \end{bmatrix},$$

giving increased more than 3,5 times the Euclidean norm of the error with respect to the exact value of vector $1{,}465110^{-12}$. The number of the matrix conditionality (27) is 39,4146 and almost 5 times higher than the number of the matrix conditionality (25).

7 Reduction of a very large NEVE system

Let us suppose that a very large NEVE system is given, whose dimension is 10^4. In the matrix (3), there are no zero elements, thus the number of non-zero elements is 10^8, that is one hundred million ($100 \cdot 10^6$). In this case, the elements in the matrix and the vector (2) vary in the range from $-10,7$ to $11,5$. Supposing that the set of excluded nodes M is equal $9 \cdot 10^3$, that means that the number of nodes left is $N = 10^4 - 9 \cdot 10^3 = 100$.

As a result of the calculations, matrix with size 100×100 was obtained and it had a condition number $\text{cond}\ \hat{Y}^{\ni}_{NN} = 47,3464$, while the number of a matrix conditionality $Y^{\ni}_{NN}$ with the same size, was $\text{cond}\left(Y^{\ni}_{NN}\right) = 10390,4112$, that is more than 200 times higher.

We note that it is not possible to directly use the Schur complement algorithm for such a large matrix. In this case, the authors used the parallelization of the computation process.

8 Conclusion

The original algebraic method of the equations reduction of an electric network steady-state regimes on the basis of matrices annihilators with the purpose of obtaining equivalents is considered, allowing to carry out transformations of the substitution scheme and its parameters to a form having a significantly smaller number of nodes and branches. Numerical procedures for computing matrix annihilators are well developed for large ($100 \le n \le 1000$) and very large ($10^3 \le n \le 10^5$) matrices, while the variation of the annihilators properties makes it possible to significantly (by an order or more) improve the conditionality of the resulting equivalents of the nodal stress equations and thereby reduce computational errors and improve the correctness of the solution obtained.

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