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Noise variance estimation for Kalman filter

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Abstract. In this paper, we propose an algorithm that evaluates noise variance with a numerical integration method. For noise variance estimation, we use Krogh method with a variable integration step. In line with common practice, we limit our study to fourth-order method. First, we perform simulation tests for randomly generated signals, related to the transition state and steady state. Next, we formulate three methodologies (research hypotheses) of noise variance estimation, and then compare their efficiency.

1 Introduction

Noise variance estimation is one of key unresolved issues associated with signal filtering [1-4]. If the noise variance is estimated incorrectly, and the estimation output is then used for signal filtering, real time measurement results might be distorted. To estimate the measurement noise one may calculate, for instance, measurement variance, as in [1]. Estimation of the process noise, however, is a way more complicated. The latter might be carried out by means of Bayesian, maximum likelihood, correlation and covariance matching [1, 4, 5]. Another acceptable approach is to assume that the process noise is constant, while taking into account the varying measurement noise.

Kalman filter is one of the most frequently used filters [6-8]. It is an extended version of recursive filtering [1, 6]. The latter is applied only for the measurement noise, while the former can be used also for the process noise. However, to use Kalman filter we need to have precise information about the object (in other words, we might encounter problems resulting from the observer effect). However, due to varying degree of the object's complexity, this is not always possible. Having complete information about the object, we can select the elements of the positive definite covariance matrix after they have been modified. If we lack more specific information about the object, we might define the elements of the covariance matrix incorrectly, and, in extreme cases, the matrix may turn out not positive definite. In consequence, the measurement results might prove imprecise.

This paper presents a method for noise variance estimation based on verification of three hypotheses, with Krogh's [9] method for numerical integration applied in each case. Small degree of complexity allows to apply these methods in real time measurement. We use values corresponding to individual hypotheses to explain the measurement noise **R**. The process noise is assumed to be constant and equal to **Q**.

2 Krogh algorithm

The Krogh algorithm [9] was first described in 1960s and is usually applied for numerical integration. One of its unique features is that it allows for changes in values of determinants in each timestep. At the same time, in spite of the change in determinants, their convergence of the individual processes of differential equation solutions is preserved. The Krogh algorithm is defined as follows [9].

May w determine a polynomial given for time points t_n, $t_{n-1}, \ldots, t_{n-q+1}$. The polynomial is defined by Newton forward differences formulae:

$$\begin{aligned} M_{q-1,n}(t) &= w[t_n] - (t-t_n)w[t_n, t_{n-1}] + \ldots \\ & (t-t_n)(t-t_{n-1})\ldots(t-t_{n-q+2})w[t_n, t_{n-1}, \ldots, t_{n-q+1}], \end{aligned} \tag{1}$$

where

$$w[t_n, t_{n-1}, \ldots, t_{n-i}] = \begin{cases} w(t_n) & i = 0 \\ \dfrac{w[t_n, \ldots, t_{n-i+1}] - w[t_{n-1}, \ldots, t_{n-i}]}{t_n - t_{n-i}} & i = 1,2\ldots \end{cases} \tag{2}$$

Consider a following polynomial $M^*_{q,n}(t)$

$$\begin{aligned} M^*_{q,n}(t) &= M_{q-1,n}(t) + \ldots \\ & (t-t_n)(t-t_{n-1})\ldots(t-t_{n-q+1})w[t_{n+1}, t_n, \ldots, t_{n-q+1}], \end{aligned} \tag{3}$$

where M defines the predictor, while M^* determines the corrector. By inserting additional symbols

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$$\begin{aligned}
&\Delta t_i = t_i - t_{i-1}\\
&\tau = (t - t_n)/\Delta t_{n+1}\\
&\xi_i(n+1) = \Delta t_{n+1} + \Delta t_n + \ldots \Delta t_{n+1-i}\\
&\beta_0(n+1) = 1\\
&\beta_i(n+1) = \left[\xi_0(n+1)\xi_1(n+1)\ldots\xi_{i-1}(n+1)\right]/\left[\xi_0(n)\ldots\xi_{i-1}(n)\right]\\
&\varphi_0(n) = w(t_n)\\
&\varphi_i(n) = \xi_0(n)\xi_1(n)\ldots\xi_{i-1}(n)w[t_n, t_{n-1}, \ldots t_{n-i}]
\end{aligned} \tag{4}$$

where i denotes the concurrent sample numbers from equations (1) to (4), we can yield the dependence which defines the range of the polynomial coefficients

$$\varphi_{i+1}(n+1) = \varphi_i(n+1) - \beta_i(n+1)\varphi_i(n). \tag{5}$$

3 Algorithm

Below we present the algorithm for the measurement variance and the hypotheses we aim to test.

The algorithms consist of five main steps, which three hypotheses are based on:

1. Calculating coefficients ϕ according to (5).
2. Calculating the variance for each of the coefficients ϕ.
3. Estimating the mutual dependencies between the calculated variances and ordering them from the largest (r_{max}) to the smallest one (r_{min}); the number of variances are denoted with l.
4. Calculating the arithmetic mean $\bar{c}$ from the mutual dependencies from the third step.
5. Calculating the variance of the signal measured with noise as σ_{sn}.

Hypothesis 1:

$$h_1 = \frac{1}{l}\bar{c}\sigma_{sn}, \tag{6}$$

Hypothesis 2:

$$h_2 = \begin{cases} (r_{\max} - r_{\min})/r_{\max} & r_{\max} \geq 1 \\ (r_{\max} - r_{\min})\sigma_{sn} & r_{\max} < 1 \end{cases} \tag{7}$$

Hypothesis 3:

$$h_3 = \begin{cases} \sqrt{\left(\frac{1}{l}\sigma_{sn} - \sigma_{10}\right)^2} & n \geq 10 \\ \frac{1}{l}\sigma_{sn} & n < 10 \end{cases} \tag{8}$$

where σ_{10} denotes the variance of the first 10 samples.

In this paper, we use a modified form of Kalman filter, which can be written in short as follows [6, 8]

$$\hat{\mathbf{x}}_{k/k-1} = \mathbf{A}\hat{\mathbf{x}}_{k-1/k-1} + \mathbf{B}\mathbf{u}_k, \tag{9}$$

$$\mathbf{P}_{k/k-1} = \mathbf{A}\mathbf{P}_{k-1/k}\mathbf{A}^T + \mathbf{Q}, \tag{10}$$

$$\hat{\mathbf{x}}_{k/k} = \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k/k-1}\right), \tag{11}$$

$$\mathbf{K}_k = \mathbf{P}_{k/k-1}\mathbf{H}^T\left(\mathbf{H}\mathbf{P}_{k/k-1}\mathbf{H}^T + \mathbf{R}\right)^{-1}, \tag{12}$$

$$\mathbf{P}_{k/k} = \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}\right)\mathbf{P}_{k/k-1}, \tag{13}$$

where **Q** matrix of the process noise, **R** matrix of the measurement noise depending on the hypothesis, **K** denotes the Kalman gain, **A** – state transition matrix, **B** – control matrix, **P** – state variance matrix, **H** – measurement matrix, **z** – measurement variables, **I** – identity matrix, $\hat{\mathbf{x}}$ – estimated state, index k denotes a sample in k-th time point.

Next, in order to verify the proposed hypotheses, we have carried out tests for randomly generated signals, both for steady state and transition states.

4 Simulation results

In this Chapter, we present simulation results for a signal generated randomly for the transition state. The signal was generated by means of the Taylor series

$$x(k) = \sum_{n=0}^{m} \frac{f^{(n)}(a)}{n!}(k-a)^n. \tag{14}$$

We restricted our calculations to the fourth order derivative (m=4). The value of the parameter a was randomly drawn from the interval <-100, 100>. Following that, a white noise is superimposed on the randomly generated signal.

Next, we estimated the noise using the algorithm from Chapter 3 and the proposed hypotheses. The results are presented in Fig. 1. In addition, in this Figure we show the variance of the noise with and without the source signal – σ_{sn}, σ_n, respectively.

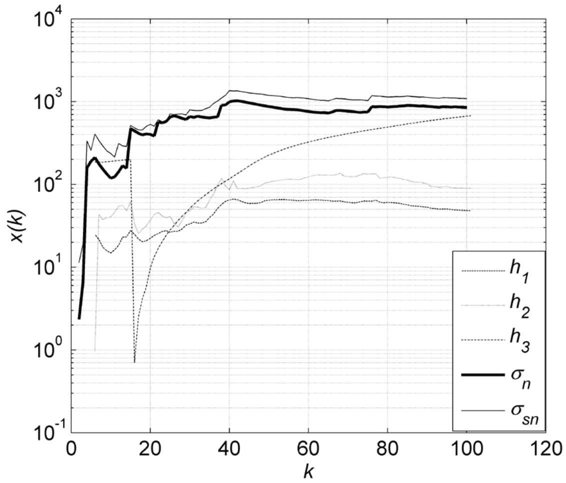

Fig. 1. Verification of the proposed hypotheses for the transition state.

From Fig. 1 we can conclude that, based on Hypothesis 3, we can obtain the most accurate estimation of the noise variance.

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Next, based on Fourier series, we generate the signal for the steady state

$$x(k) = a_0 + \sum_{n=1}^{m} \left(a_n \cos nk + b_n \sin nk \right), \tag{15}$$

where coefficients a_0, a_n and b_n are drawn randomly from the interval <-100, 100> and then, consistently, we restricted the series to a fourth order (m=4).

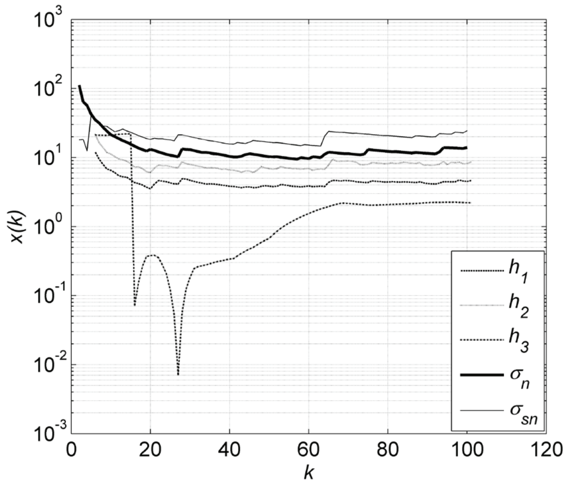

Fig. 2. Verification of the proposed hypotheses for the transition state.

As the former case, we imposed a white noise on the randomly generated signal. The results, along with the variance of the distorted source signal and the variance of the noise itself, are depicted in Fig. 2. The best estimation of the noise variance in the steady state was obtained when Hypothesis 2 was applied.

To confirm that the above conclusions are correct, we carried out additional verification, for distorted signal of sinusoidal, sawtooth, rectangular and constant waveforms. A white noise was superimposed on the sinusoidal signal. Next, we checked the proposed hypotheses. The result is shown in Fig. 3.

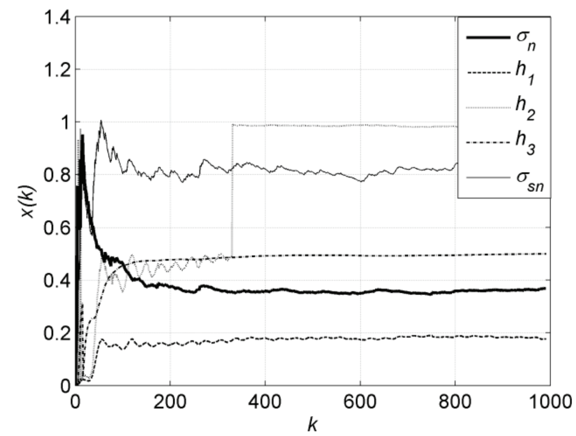

Fig. 3. Verification of the proposed hypotheses for the sinusoidal signal with a superimposed white noise.

We repeated the above procedure for the sawtooth signal with a superimposed white noise. The result is presented in Fig. 4. We conclude that by applying either Hypothesis 2 or 3, we yielded the most accurate estimation for sinusoidal and sawtooth shape. We also verified the rectangular and constant signal. We imposed a white noise on both and checked if the algorithm estimates the noise variance correctly.

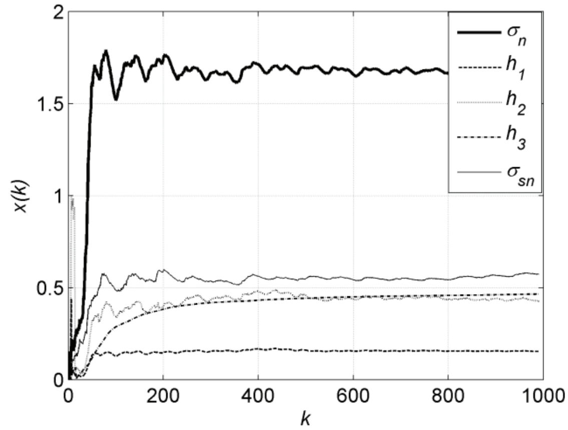

Fig. 4. Verification of the proposed hypotheses for the saw signal with a superimposed white noise.

The results are presented in Fig. 5 (for the rectangular signal) and Fig. 6 (constant signal). Both Hypotheses, 2 and 3, estimated the noise variance best.

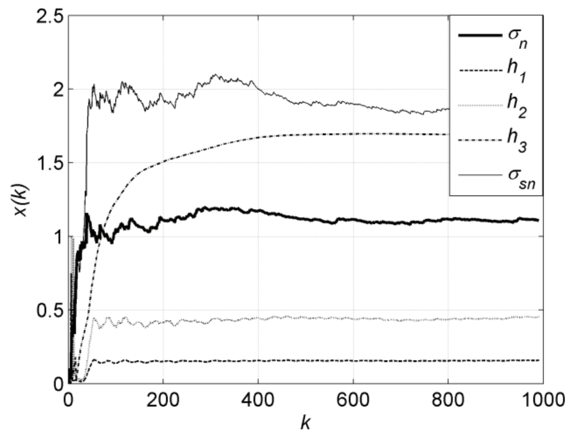

Fig. 5. Verification of the proposed hypotheses for the rectangular signal with a superimposed white noise.

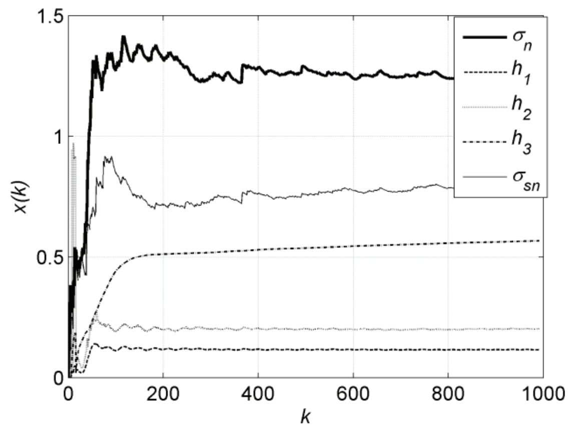

Fig. 6. Verification of the proposed hypotheses for the constant signal with a superimposed white noise.

We can observe that in most cases the proposed hypotheses allow for an accurate estimation of the noise variance, with the best results obtained for Hypotheses 2 and 3. The noise was estimated particularly well in case of randomly generated signals, both for steady state (Hypotheses 2 and 3) and transient state.

Next, we confronted the above results with the results obtained from measurements performed for a real medium-power industrial drive with an SEE355ML4BS frequency inverter. The characteristics of the drive are as follows. U_Δ=400 V, I_Δ=523 A, P_n=315 kW, T_n=2020 Nm, n_n=1489 rpm, η=96.6 %, I_d=6.2 kgm^2, where U_Δ denotes line-to-line voltage, I_Δ line current, P_n nominal power, T_n nominal torque, n_n nominal rotary velocity, I_d moment of inertia on the motor shaft, η efficiency. The considered object also comprised a 20W39MX3GV pomp of capacity equal to 550m^3/h, liquid column of 130 m and rotary velocity equal to 1490 rpm.

Next, we performed measurement of the converter drive described above. We used a measurement device consisting of two NI 6133 measurement cards, which enable 16-channel measuring with a maximum sampling rate of 1 MS/s. We applied CWT30LF Rogowski coils from PEM with measurement error of 0.2%. The sampling rate was lowered to 400 kS/s. We yielded a very distorted result. Hence, we applied linear Kalman filter, assuming that the object's model is completely unknown. Therefore, the filtering depending exclusively on the assumed noise variance and measurement results.

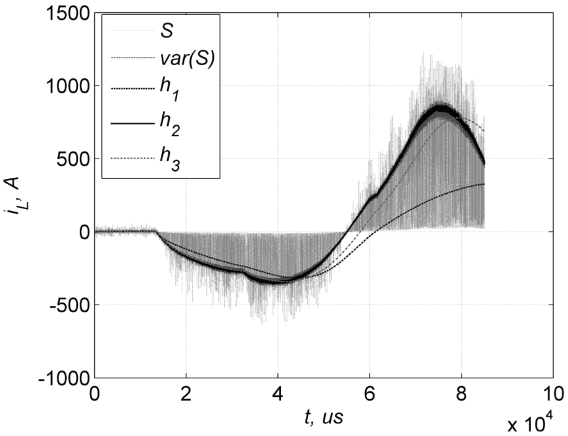

Fig. 7. Measurement results of the line current i_{L1}.

Figs. 7 and 8 show the measurement results *S* of the line current i_{L1} and i_{L2}, both raw and after Kalman filter has been applied. We assumed that the process noise **Q** is constant. The measurement noise **R** is estimated based on the three hypotheses (h_1, h_2 and h_3, respectively) and the variance of the measurement results - *var(S)*. Such an assumption is frequently applied by engineers; yet, as Figs. 7 and 8 show, it proved to be an oversimplification, and to yield inaccurate results. In extreme cases, such an assumption might distort measurement results. However, from Figs. 7 and 8 we may also conclude that the hypotheses, particularly Hypothesis 2, proposed in this paper estimate the measurement noise precisely. For Hypotheses 1 and 3, the results are quite smooth. However, in case of these hypotheses, we can observe a phase shift, which is unwelcome.

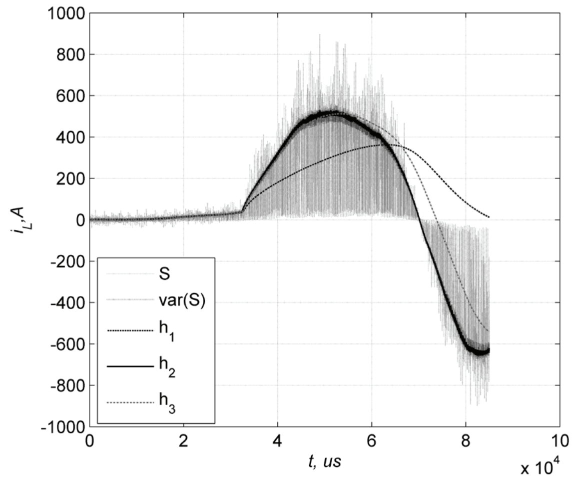

Fig. 8. Measurement results of the line current i_{L2}.

Thus, both hypotheses should not be applied in practice. Therefore, our results indicate that Hypothesis 2 may prove a handy tool when estimating the measurement noise by means of Kalman filter.

5 Conclusions

In this paper, we compared three hypotheses, which facilitate estimating the measurement noise. We applied the estimation results in Kalman filter. Next, confronted the results from theoretical considerations with the results from measurements, carried out in working conditions, and obtain a satisfactory outcome, particularly for one of the hypotheses. Taking into account the above as well as simplicity of our approach, it may be recommend for real-time measurements.

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